9TH Class Math Exercise 9.3

Q. 1 The radii of two spheres are in the ratio $3:4$. What is the ratio of their volumes?

Solution:

Ratio of radii of spheres $= r_1 : r_2 = 3:4$ let V_1 and V_2 be the volumes of these spheres.

Since spheres are similar,

So,

$$\frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3$$

$$\frac{V_1}{V_2} = \left(\frac{3}{4}\right)^3$$

$$\frac{V_1}{V_2} = \frac{27}{64}$$

$$\Rightarrow V_1 : V_2 = 27:64$$

Q. 2 Two regular tetrahedrons have volumes in the ratio $8:27$. What is the ratio of their sides?

Solution:

Ratio of volumes $= V_1 : V_2 = 8:27$

Let ℓ_1 and ℓ_2 be the sides of these tetrahedrons respectively.

Since regular tetrahedrons are similar

So,

$$\because \frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$$

$$\Rightarrow \left(\frac{l_1}{l_2}\right)^3 = \frac{V_1}{V_2}$$

$$\left(\frac{l_1}{l_2}\right)^3 = \frac{8}{27}$$

Taking cube root on B.S

$$\sqrt[3]{\left(\frac{\ell_1}{\ell_2}\right)^3} = \sqrt[3]{\frac{8}{27}}$$

$$\frac{\ell_1}{\ell_2} = \frac{2}{3}$$

$$\Rightarrow \ell_1 : \ell_2 = 2:3$$

Q. 3 Two right cones have volumes in the ratio $64:125$. What is the ratio of:

(a) Their heights

Solution:

64:125

let h_1 and h_2 be the heights of these cones, since right cones are similar.

So,

$$\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)'$$

$$\Rightarrow \left(\frac{h_1}{h_2}\right)' = \frac{V_1}{V_2}$$

$$\left(\frac{h_1}{h_2}\right)^3 = \frac{64}{125}$$

$$\sqrt[3]{\left(\frac{h_1}{h_2}\right)'} = \sqrt[3]{\frac{64}{125}}$$

$$\frac{h_1}{h_2} = \frac{\sqrt[3]{64}}{\sqrt[3]{125}}$$

$$\frac{h_1}{h_2} = \frac{\sqrt{4^3}}{\sqrt{5^3}} \Rightarrow \frac{h_1}{h_2} = \frac{4}{5} \Rightarrow h_1 : h_2 = 4:5$$

(b) Their base radii?

Solution:

Let r_1 and r_2 be the radius of bases of these cones.

$$\because \frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3 \Rightarrow \left(\frac{r_1}{r_2}\right)^3 = \frac{V_1}{V_2}$$

$$\left(\frac{r_1}{r_2}\right)^3 = \frac{64}{125}$$

$$\sqrt[3]{\left(\frac{r_1}{r_2}\right)^3} = \sqrt{\frac{64}{125}}$$

$$\frac{r_1}{r_2} = \frac{\sqrt[3]{64}}{\sqrt[3]{125}}$$

$$\frac{r_1}{r_2} = \frac{4}{5} \Rightarrow r_1 : r_2 = 4:5$$

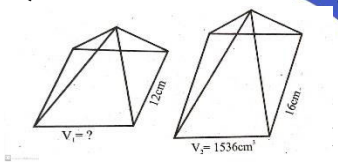
$$\frac{A_1}{A_2} = \left(\frac{r_1}{r_2}\right)^2$$

$$\frac{A_1}{A_2} = \left(\frac{4}{5}\right)^2$$

$$\frac{A_1}{A_2} = \frac{16}{25}$$

$$\Rightarrow A_1 : A_2 = 16:25$$

Q. 4 Find the missing volume in the following similar figures.



$$V_1 = ?$$

Solution:

Let V_1 and V_2 be the volume of smaller and larger pyramids and ℓ_1 and ℓ_2 are their corresponding lengths.

Volume of smaller pyramid $= V_1 = ?$

Volume of larger pyramid $= V_2 = 1536 \text{ cm}^3$

Length of smaller pyramid $= \ell_1 = 12 \text{ cm}$

Length of larger pyramid $= \ell_2 = 16 \text{ cm}$

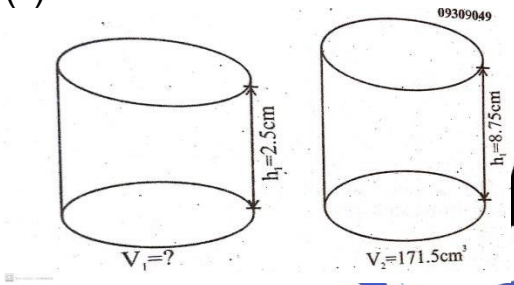
Since pyramids are similar,

so,

$$\frac{V_1}{V_2} = \left(\frac{\ell_1}{\ell_2}\right)^3$$
$$\frac{V_1}{V_2} = \left(\frac{12}{16}\right)^3$$
$$\frac{V_1}{V_2} = \left(\frac{312}{16}\right)^3$$
$$\frac{V_1}{1536} = \left(\frac{3}{4}\right)^3$$
$$V_1 = \frac{27}{64} \times 1536$$
$$V_1 = 27 \times 24$$
$$V_1 = 648 \text{ cm}^3$$

Thus volume of smaller pyramid is 648 cm^3.

(ii)



Solution:

volume of smaller cylinder $= V_1 = ?$

volume of larger cylinder $= V_1 = 171.5 \text{ m}$

Height of smaller cylinder $= h_1 = 2.5 \text{ m}$

Height of larger cylinder $= \text{h}_2 = 8.75 \text{ m}$

Since, cylinders are similar,

So,

$$\frac{V_1}{V_2} = \left(\frac{h_1}{h_2}\right)'$$
$$\frac{V_1}{171.5} = \left(\frac{2.5}{8.75}\right)^3$$
$$\frac{V_1}{171.5} = \left(\frac{10}{35}\right)^3$$

$$\frac{V_1}{171.5} = \left(\frac{2}{7}\right)^3$$

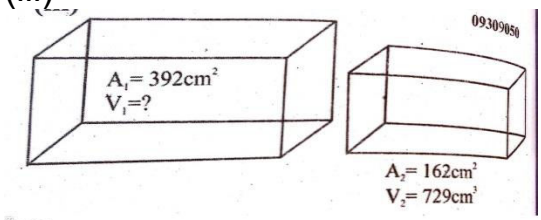
$$V_1 = \frac{8}{343} \times 171.5$$

$$V_1 = \frac{1372}{343}$$

$$V_1 = 4 \text{ m}^3$$

Thus volume of smaller cylinder is 4 m^3.

(iii)



$$A_2 = 162 \text{ cm}^3$$

$$V_3 = 729 \text{ cm}^3$$

Solution:

Surface area of larger cuboid $= A_1 = 392 \text{ cm}^2$

Surface area of smaller cuboid $= A_2 = 162 \text{ cm}^2$

Volume of larger cuboid $= V_1 = ?$

Volume of smaller cuboid $= V_2 = 729 \text{ cm}^3$

Let ℓ_1 and ℓ_2 be the lengths larger and smaller cuboid respectively.

(iii) Using formula of areas of similar figures.

figures.

$$\frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$$

$$\Rightarrow \sqrt{\left(\frac{\ell_1}{\ell_2}\right)^2} = \sqrt{\frac{A_1}{A_2}}$$

$$\frac{\ell_1}{\ell_2} = \sqrt{\frac{392}{162}}$$

$$\frac{\ell_1}{\ell_2} = \sqrt{\frac{196}{81}}$$

$$\frac{\ell_1}{\ell_2} = \sqrt{\frac{(14)^2}{(9)^2}}$$

$$\left(\frac{\ell_1}{\ell_2}\right) = \sqrt{\left(\frac{14}{9}\right)^2}$$

$$\frac{l_1}{l_2} = \frac{14}{9}$$

Now, using formula of volumes for similar figures.

$$\frac{V_1}{V_2} = \left(\frac{\ell_1}{\ell_2}\right)^3$$

$$\frac{V_1}{V_2} = \left(\frac{14}{9}\right)^3$$

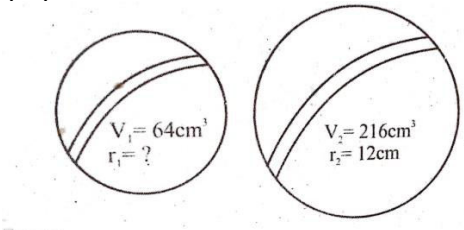
$$\frac{V_1}{729} = \frac{2744}{729}$$

$$V_1 = \frac{2744}{729} \times 729$$

$$V_1 = 2744 \text{ cm}^3$$

Thus volume of larger cuboid is 2744 cm^3.

(iv)



Solution:
We know that

$$\left(\frac{V_1}{V_2}\right) = \left(\frac{r_1}{r_2}\right)^3$$

$$\frac{64}{216} = \left(\frac{r_1}{12}\right)^3$$

$$\frac{64}{216} = \frac{r_1^3}{(12)^3}$$

$$(12)^3 \times \frac{8}{27} = r_1^3$$

$$\Rightarrow r_1^3 = \frac{(12)^3 \times 2^3}{3^3}$$

$$r_1^3 = \left(\frac{12^4 \times 2}{\not{3}_1}\right)^3$$

$$r_1^3 = (8)^3$$

Taking cube root B.S

$$r_1 = 8$$

Q.5 The ratio of corresponding lengths of two similar canonical cans is 3:2.
(i)The larger canonical can have surface area of the smaller canonical can.
(ii)The smaller canonical can have volume of 240m^3.Find the volume of large canonical can.
Solution:
We know that

$$\frac{A_1}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$$
$$\frac{96}{A_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$$
$$\frac{96}{A_2} = \left(\frac{3}{2}\right)^2$$
$$\frac{A_2}{96} = \frac{9}{4}$$
$$\Rightarrow \frac{A_2}{96} = \frac{4}{9}$$
$$A_2 = \frac{4}{9} \times 9.6$$
$$A_2 = \frac{384}{9}$$
$$A_2 = 42.67 \text{ m}^2$$

Solution (ii):

(ii) let V_1 and V_2 be the volume of larger and smaller can respectively. Volume of smaller canonical can $= V_2 = 240 \text{ m}^2$ Volume of larger can $V_1 = ?$

We know that

$$\left(\frac{V_1}{V_2}\right) = \left(\frac{\ell_1}{\ell_2}\right)^3$$
$$\frac{V_1}{240} = \left(\frac{3}{2}\right)^3$$
$$V_1 = \frac{27}{8} \times 240$$
$$V_1 = 27 \times 30$$
$$V_1 = 810 \text{ m}^3$$

Thus volume of larger canonical can is 810 m^3.

Q. 6 The ratio of the heights of two similar cylindrical water tanks is $5:3$.

(i) If the surface area of the larger tank is 250 square metres, find the surface area of the smaller tank.

Solution:

Ratio of height of water tanks $= \ell_1 = \ell_2 = 5:3$

let A_1 and A_2 be the surface areas of larger and smaller cylindrical water tanks respectively.

Surface area of larger tank $= \mathrm{A}_1 = 250\ \mathrm{m}^2$
Surface area of smaller tank $= A_2 = ?$
We know that

$$\frac{\mathrm{A}_1}{\mathrm{A}_2} = \left(\frac{\ell_1}{\ell_2}\right)^2$$
$$\frac{250}{\mathrm{A}_2} = \left(\frac{5}{3}\right)^2$$
$$\frac{250}{\mathrm{A}_2} = \frac{25}{9}$$
$$\Rightarrow \frac{\mathrm{A}_2}{250} = \frac{9}{25} \quad \text{(reciprocal)}$$
$$\mathrm{A}_2 = \frac{9}{25} \times 250$$
$$\mathrm{A}_2 = 9 \times 10$$
$$\mathrm{A}_2 = 90\ \mathrm{m}^2$$

Thus surface area of smaller water tank is $90\ \mathrm{m}^2$.
(ii)If the volume of the smaller tank is 270 cubic metres, find the volume of the larger tank.
Solution
Let V_1 and V_2 be the volumes of larger and smaller tanks respectively.
Let volume of smaller tank $V_2 = 270\ \mathrm{m}^3$ Volume of larger tank $= \mathrm{V}_1 =$?
We know that

$$\left(\frac{\mathrm{V}_1}{\mathrm{V}_2}\right) = \left(\frac{\ell_1}{\ell_2}\right)^3$$
$$\frac{\mathrm{V}_1}{270} = \left(\frac{5}{3}\right)^3$$
$$\frac{\mathrm{V}_1}{270} = \frac{125}{27}$$
$$\mathrm{V}_1 = \frac{125}{27} \times 270$$
$$\mathrm{V}_1 = 125 \times 10$$
$$\mathrm{V}_1 = 1250\ \mathrm{m}^3$$

$$\mathrm{V}_1 = 125 \times 10$$
$$\mathrm{V}_1 = 1250\ \mathrm{m}^3$$

Thus volume of larger water tank is $1250\ \mathrm{m}^3$.