9th Class Math Exercise 9.4

Q. 1 (i) What is the sum of the interior angles of a decagon (10 - sided polygon)?

Solution:

Numbers of sides $= \mathrm{n} = 10$

Sum of interior angles $= (n-2) \times 180^\circ$

$= (10-2) \times 180^\circ$

$= 8 \times 180^\circ$

$= 1440^\circ$

(ii) Calculate the measure of each interior angle of a regular hexagon.

Solution:

In regular hexagon

Numbers of sides $= n = 6$

Each interior angel $= \dfrac{(n-2) \times 180^\circ}{n}$

$= \dfrac{(6-2) \times 180^\circ}{6}$

$= \dfrac{4 \times 180^\circ}{6}$

$= 4 \times 30^\circ$

$= 120^\circ$

(iii) What is each exterior angle of a regular pentagon?

Solution

In Regular pentagon: $\mathrm{n} = 5$

Number of sides $= n = 5$

Each exterior angle

$= \dfrac{360^\circ}{\mathrm{n}} = \dfrac{360^\circ}{5}$

$= 72^\circ$

(iv) If the sum of the interior angles of a polygon is 1260°, how many sides does the polygon have?

Solution:

Sum $= 1260^\circ$

Sum of interior angles $= (n-2) \times 180^\circ$

$$1260^\circ = (n-2) \times 180^\circ$$
$$\frac{1260^\circ}{180^\circ} = n - 2^\circ$$
$$7 = n - 2$$
$$7 + 2 = n$$
$$9 = n$$
$$n = 9$$

Thus number of sides of regular polygon is 9

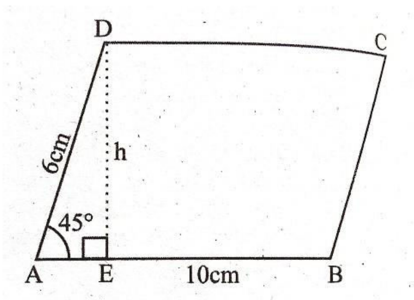
Q. 2 In a parallelogram ABCD , m $\overline{\mathrm{AB}} = 10$ cm, $\mathrm{m}\overline{\mathrm{AD}} = 6$ cm and $\mathrm{m}\angle\mathrm{BAD} = 45^\circ$. Calculate area of $ABCD$

Solution:

<u>Given :</u>

<u>$\mathrm{m}\widehat{\mathrm{AB}} = 10\ \mathrm{cm}$</u>

<u>$\mathrm{mAD} = 6\mathrm{cm}$</u>



We know that

Area of $\|\mathrm{gm}$ = base × height

Area of $\|\mathrm{gmABCD} = \mathrm{m}\overline{\mathrm{AB}} \times \mathrm{mDE}$ In right angled $\triangle\ \mathrm{AED}$,

$$\mathrm{Sinm}\angle\ \mathrm{A} = \frac{\mathrm{m}\overline{\mathrm{DE}}}{\mathrm{mAD}}$$
$$\mathrm{Sin}45^\circ = \frac{\mathrm{h}}{6}$$
$$\frac{1}{\sqrt{2}} = \frac{\mathrm{h}}{6} \qquad \left(\because \sin\ 45^\circ = \frac{1}{\sqrt{2}}\right)$$

$$6 \times \frac{1}{\sqrt{2}} = \mathrm{h}$$

$$3 \times 2 \times \frac{1}{\sqrt{2}} = \mathrm{h}$$

$$3\sqrt{2} = \mathrm{h} \ \left(\because \frac{a}{\sqrt{a}} = \sqrt{a}\right)$$

$$\Rightarrow \mathrm{h} = 3\sqrt{2} \Rightarrow \mathrm{mDE} = 3\sqrt{2}\ \mathrm{cm}$$

Now, using formula (i)

Area of parallelogram ABCD

$$= 10\ \mathrm{cm} \times 3\sqrt{2}\ \mathrm{cm}$$

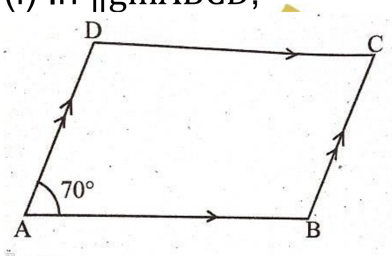
$$= 30\sqrt{2}\ \mathrm{cm}^2 = 42.43\ \mathrm{cm}^2$$

Q. 3 In a parallelogram ABCD if $\mathrm{m}\angle\mathrm{DAB} = 70^\circ$, find the measures of all other angles in the parallelogram.

Solution:

m $\angle\mathrm{BAD} = 70^\circ$ (i) (given)

(i) In $\|\mathrm{gm}\mathrm{ABCD}$,



$\mathrm{m}\angle\mathrm{A} = \mathrm{m}\angle\mathrm{C}$ [opposite angles of $\|\mathrm{gm}$] $\mathrm{m}\angle\mathrm{C} = 70^\circ$

(ii) In parallelogram ABCD ,

$\mathrm{m}\angle\mathrm{A} + \mathrm{m}\angle\mathrm{B} = 180^\circ$ [anlges on end points of a side

$$70^\circ + \mathrm{m}\angle\ \mathrm{B} = 180^\circ$$

$$\mathrm{m}\angle\ \mathrm{B} = 180^\circ - 70^\circ$$

$$\mathrm{m}\angle\ \mathrm{B} = 110^\circ$$

(iii) In parallelogram ABCD ,

$\mathrm{m}\angle\mathrm{D} = \mathrm{m}\angle\mathrm{B}$ (opposite angles)

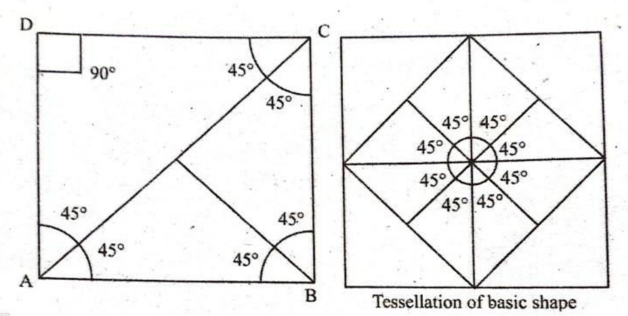
$$\mathrm{m}\angle\mathrm{D} = 110^\circ$$

Q. 4 A shape is created by cutting a square in half diagonally and then attaching a right-angled triangle to the hypotenuse of each half. Explain why this shape can tessellate and calculate the interior angles of the new shape.

Solution:

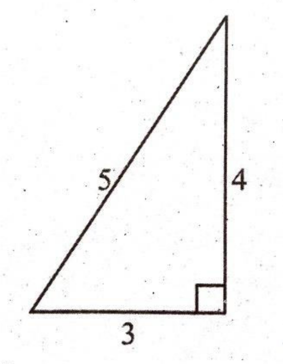
Tessellation of the New Shape:

By cutting a square in half diagonally we get two right-angled isosceles triangles, each with angles of $90^\circ, 45^\circ$, and 45 . When two right-angled triangles are attached to the bypotenuses they form a new shape that is a square. Squares can tessellate perfectly because each square has an internal angle of $90°$ and four squares meet at a point to form a $360°$ angle.



Tessellation of basic shape

Angle at A:
The $45°$ angle of the original triangle and the $45°$ angle of the attached triangle at the point of attachment sum to $90°$.
Angle at B:
The sum of the two $45°$ angles at the attachment point from two attached triangles is $90°$.
Angle at C:
The $45°$ angle of the original triangle and the $45°$ angle of the attached triangle at the point of attachment sum to $90°$.
Angle at D:
One $90°$ angle from original $\triangle ADC$
Since all interior angles are $90°$ and their sum is $360°$. The new shape is a square which naturally tessellates the plane and it can cover the plane without gaps or overlaps.
Q. 5 A tessellation is created by repeatedly reflecting a basic shape. The basic shape is a right angled triangle with sides of length 3,4 , and 5 units. Find: The minimum number of reflections needed to create a tessellation that covers a square with an area of 3600 square units.
Solution.



Basic shape is right angled triangle with sides 3,4 and 5 units.

Area of basic shape $= \frac{1}{2}(\ b \times h)$

Area of $A = a = \frac{1}{2}(3 \times 4) = \frac{1}{2}(12)$

$= 6$ square units

Area to be covered $= A = 3600$ square units

$$\text{Number of reflections } = \frac{\text{Total area}}{\text{Area of } \Delta}$$

$$= \frac{3600}{6} = 600$$

Thus minimum 600 reflections are needed to cover the area of 3600 square units.

Q. 6 A tessellation is created using regular hexagons. Each hexagon has a side length of 5 cm. Find the total area of the tessellation if it consists of 25 hexagons and total perimeter of the outer edge of the tessellation, assuming it's a perfect hexagon.

Solution:

Each side of hexagon $= s = 5$ cm

Area of an equilateral $\Delta = \frac{\sqrt{3}}{4} \cdot\ s^2$

Area of 1 regular hexagon $= 6 \times \frac{\sqrt{3}}{4} \cdot\ s^2$

$$= \frac{3\sqrt{3}}{2} \cdot s^2$$

Area of 25 regular hexagon

$$= 25 \times \frac{3\sqrt{3}}{2} \times s^2$$

$$= \frac{75\sqrt{3}}{2} \times (5)^2$$

$$= \frac{75\sqrt{3} \times 25}{2}$$

$= 1623.8$ sq.unit.

Thus total area of tessellation is 1623.8 square unit.
Given that tessellation is in the form of regular hexagon.
Let length of its each side be x units.

Area of tessellation (hexagon) $= \frac{3\sqrt{3}}{2} \times x^2$

$$1623.8 = \frac{3\sqrt{3}}{2} x^2$$

$$\frac{2 \times 1623.8}{3\sqrt{3}} = x^2$$

$$625 = x^2$$

$$\Rightarrow x^2 = 625$$

$$\sqrt{x^2} = \sqrt{625}$$

$$x = 25 \text{ cm}$$

The length of each side of tessellation

$$= x = 25 \text{ cm}$$

Perimeter of tessellation $= 6 \times x$

$$= 6 \times 25 = 150 \text{ cm}$$

Q. 7 A rectangular floor is 12 m by 15 m . How many square tiles, each 1 m by 1 m , are needed to cover the floor?

Solution:

Dimension of floor $= 12$ m by 15 m
Area of floor $= 12 \times 15 = 180 \text{ m}^2$
Dimension of tile $= 1$ m by 1 m
Area of 1 tile $= 1 \text{ m} \times 1 \text{ m} = 1 \text{ m}^2$
Number of tiles to cover floor =

$$= \frac{\text{total area of floor}}{\text{Area of } 1 \text{ tile}}$$

$$= \frac{180 \text{ m}^2}{1 \text{ m}^2} = 180$$

Thus 180 tiles are needed to cover the floor.

Q. 8 A rectangular wall is 10 m tall and 120 m wide. How many

gallons of paint are needed to cover the wall, if one gallon covers $35\ \mathrm{m}^2$?
Solution:
Length of wall $= \mathrm{L} = 10\ \mathrm{m}$
Width of wall $= \mathrm{W} = 120\ \mathrm{m}$
Area of wall $= \mathrm{L} \times \mathrm{W}$

$$= 10\ \mathrm{m} \times 120\ \mathrm{m} = 1200\ \mathrm{m}^2$$

Given that 1 gallon covers $= 35\ \mathrm{m}^2$
No, of gallons to cover $35\ \mathrm{m}^2 = 1$
No. of gallons to cover $1\ \mathrm{m}^2 = \frac{1}{35}$
No. of gallons cover $1200\ \mathrm{m}^2 = \frac{1}{35} \times 1200 = 34.29$

$$\approx 35$$

Thus 35 gallons of paint are required to paint the wall.
Q. 9 A rectangular wall has a length of 10 m and a width of 4 meters. If 1 litre of paint covers $7\ \mathrm{m}^2$, how many liters of paint are needed to cover the wall? 09309072
Solution:
Length of wall $= \mathrm{L} = 10\ \mathrm{m}$
Width of wall $= \mathrm{W} = 4\ \mathrm{m}$
Area of wall $= \mathrm{L} \times \mathrm{W}$

$$= 10\ \mathrm{m} \times 4\ \mathrm{m} = 40\ \mathrm{m}^2$$

Given that 1 liter of paint covers $= 7\ \mathrm{m}^2$
No. of liters to cover $7\ \mathrm{m}^2 = 1$ litre
No. of liters of paint to cover $1\ \mathrm{m}^2$

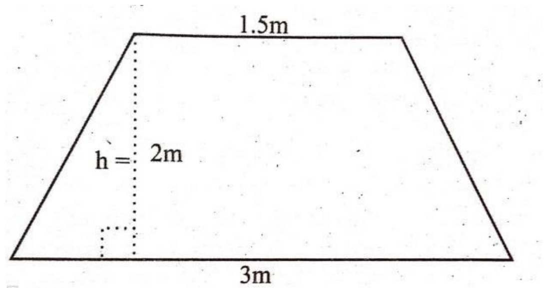
$$= \frac{1}{7} \text{ litres}$$

No. of liters of paint to cover $40\ \mathrm{m}^2$

$$= \frac{1}{7} \times 40 \text{ litres}$$

$$= 5.71 \approx 6 \text{ litres}$$

Thus 6 liters of paint are required to cover area of $40\ \mathrm{m}^2$.
Q. 10 A window has a trapezoidal shape with parallel sides of 3 m and 1.5 m and a height of 2 m . Find the area of the window.
Solution:



Length of 1^{st} side $= \mathrm{s}_1 = 3\ \mathrm{m}$
Length of 2^{nd} side $= \mathrm{s}_2 = 1.5\ \mathrm{m}$
Height of trapezium $\mathrm{h} = 2\ \mathrm{m}$
Area of trapezoidal window $= \left(\frac{s_1+s_2}{2}\right) \times h$

$$= \left(\frac{3+1.5}{2}\right) \times 2 = \frac{(4.5)}{2} \times 2 = 4.5\ \mathrm{m}^2$$

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