Exercise 1.2

Q.1 Rationalize the denominator of following：

(i) $\frac{13}{4+\sqrt{3}}$

Solution：

$$\frac{13}{4+\sqrt{3}}$$

Multiplying and dividing by $4-\sqrt{3}$

$$=\frac{13}{(4+\sqrt{3})}\times\frac{(4-\sqrt{3})}{(4-\sqrt{3})}$$

$$=\frac{13(4-\sqrt{3})}{(4)^2-(\sqrt{3})^2}$$

$$=\frac{13(4-\sqrt{3})}{16-3}$$

$$=\frac{13(4-\sqrt{3})}{13}$$

$$=4-\sqrt{3}$$

(ii) $\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}$

$$\frac{\sqrt{2}+\sqrt{5}}{\sqrt{3}}$$

Multiplying and dividing by $\sqrt{3}$

$$=\frac{(\sqrt{2}+\sqrt{5})}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$$

$$=\frac{\sqrt{2}\times\sqrt{3}+\sqrt{5}\times\sqrt{3}}{(\sqrt{3})^2}$$

$$=\frac{\sqrt{6}+\sqrt{15}}{3}$$

(iii) $\frac{\sqrt{2}-1}{\sqrt{5}}$

Solution：

$\frac{\sqrt{2}-1}{\sqrt{5}}$

Multiplying and dividing by $\sqrt{5}$ "

$$=\frac{(\sqrt{2}-1)}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}$$

$$=\frac{(\sqrt{2}-1)\sqrt{5}}{(\sqrt{5})^2}$$

$$=\frac{\sqrt{5}(\sqrt{2}-1)}{5}$$

$$=\frac{\sqrt{10}-\sqrt{5}}{5}$$

(iv) $\frac{6-4\sqrt{2}}{6+4\sqrt{2}}$

Solution:

$=\frac{6-4\sqrt{2}}{6+4\sqrt{2}}$

Multiplying and divided by $(6-4\sqrt{2})$

$$=\frac{(6-4\sqrt{2})}{(6+4\sqrt{2})}X\frac{(6-4\sqrt{2})}{6-4\sqrt{2}}$$

$$=\frac{(6-4\sqrt{2})^2}{(6)^2-(4\sqrt{2})^2}$$

$[(a-b)^2=a^2+b^2-2ab]$

$$=\frac{(6)^2+(4\sqrt{2})^2-2(6)(4\sqrt{2})}{36-16(2)}$$

$$=\frac{36+16(2)-48\sqrt{2}}{36-32}$$

$$=\frac{36+32-48\sqrt{2}}{4}$$

$$=\frac{68-48\sqrt{2}}{4}$$

$$=\frac{4(17-12\sqrt{2})}{4}$$

$$=17-12\sqrt{2}$$

(v) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

Selverion:

$$\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

Multrplying and dividing by $(\sqrt{3}-\sqrt{2})$

$$= \frac{(\sqrt{3}-\sqrt{2})}{\sqrt{3}+\sqrt{2}} \times \frac{(\sqrt{3}-\sqrt{2})}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{(\sqrt{3}-\sqrt{2})^2}{(\sqrt{3})^2 - (\sqrt{2})^2}$$

$(a-b)^2 = a^2 + b^2 - 2ab)$

$$= \frac{(\sqrt{3})^2+(\sqrt{2})^2-2(\sqrt{3})(\sqrt{2})}{3-2}$$

$$= \frac{3+2-2\sqrt{6}}{1}$$

$$= \frac{3+2-2\sqrt{6}}{1}$$

$$= \frac{5-2\sqrt{6}}{1}$$

$$= 5-2\sqrt{6}$$

vi) $\frac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}}$

Solution:

$$\frac{4\sqrt{3}}{\sqrt{7}+\sqrt{5}} \times \frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}-\sqrt{5}}$$

$$= \frac{4\sqrt{3}(\sqrt{7}-\sqrt{3})}{(\sqrt{7})^2-(\sqrt{5})^2}$$

$$= \frac{4\sqrt{3}(\sqrt{7}-\sqrt{5})}{7-5}$$

$$= \frac{4\sqrt{3}(\sqrt{7}-\sqrt{5})}{2}$$

$$= 2\sqrt{3}(\sqrt{7}-\sqrt{5})$$

Q. 2 Simplify the following

(i) $\left(\frac{81}{16}\right)^{-\frac{3}{4}}$

Solution:

$$=\left(\frac{81}{16}\right)^{-\frac{3}{4}}$$

$$\because \left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^{m}$$

$$= \left(\frac{16}{81}\right)^{\frac{3}{4}}$$

$$= \left(\frac{2^4}{3^4}\right)^{\frac{3}{4}}$$

$$= \frac{2^{4\frac{3}{4}}}{3^{4\times\frac{3}{4}}}$$

$$= \frac{2^3}{3^3} = \frac{2\times 2\times 2}{3\times 3\times 3}$$

$$= \frac{8}{27}$$

(ii) $\left(\frac{3}{4}\right)^{-2} \div \left(\frac{4}{9}\right)^{3} \times \frac{16}{27}$

Solution:

$$=\left(\frac{3}{4}\right)^{-2} \div \left(\frac{4}{9}\right)^{3} \times \frac{16}{27}$$

$$=\left(\frac{4}{3}\right)^{2} \times \left(\frac{9}{4}\right)^{3} \times \frac{2^4}{3^3}$$

$$=\left(\frac{2^2}{3}\right)^{2} \times \left(\frac{3^2}{2^2}\right)^{3} \times \frac{2^4}{3^3}$$

$$=\frac{2^4}{3^2} \times \frac{3^6}{2^6} \times \frac{2^4}{3^3}$$

$$=\frac{2^{4+4} \times 3^6}{2^6 \times 3^{2+3}}$$

$$=\frac{2^8 \times 3^6}{2^6 \times 3^5}$$

$$=2^8 \times 3^6 \times 2^{-6} \times 3^{-5}$$

$$=2^{8-6} \times 3^{6-5}$$

$$=2^2 \times 3^1$$

$$=4 \times 3$$

$$=12$$

2	16
2	8
2	4
2	2

	1

3	27
3	9
3	3
	1

(iii) $(0.027)^{-\frac{1}{3}}$

Solution:

$=(0.027)^{-\frac{1}{3}}$

$=\left(\frac{27}{1000}\right)^{\frac{1}{3}}$

$=\left(\frac{1000}{27}\right)^{\frac{1}{3}} \because \left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^{m}$

$=\left(\frac{10^3}{3^3}\right)^{\frac{1}{3}}$

$=\frac{10^{3\times\frac{1}{3}}}{3^{3\times\frac{1}{3}}}$

$=\frac{10}{3} = 3\frac{1}{3}$

(iv) $\sqrt[7]{\frac{x^{14}\times y^{21}\times z^{35}}{y^{14}z^{7}}}$

$=\sqrt[7]{\frac{x^{14}\times y^{21}\times z^{35}}{y^{14}z^{7}}}$

$=\sqrt[7]{x^{14}\times y^{21}\times z^{35}\times y^{-14}\times z^{-7}}$

$=(x^{14}\times y^{14}\times y^{21-14}\times z^{35-7}$

$=x^{14\times\frac{1}{7}}\times y^{7\times\frac{1}{7}}\times z^{28^{\times\frac{1}{7}}}$

$=x^2yz^4$

(v) $\frac{5\times(25)^{n+1}-25\times(5)^{2n}}{5\times(5)^{2n+3}-(25)^{n+1}}$ 09301056

Solution:

$$=\frac{5\times(25)^{n+1}-25\times(5)^{2n}}{5\times(5)^{2n+3}-(25)^{n+1}}$$

$$=\frac{5\cdot(5^2)^{n+1}-(5^2)(5^{2n})}{5\cdot(5)^{2n+3}-(5^2)^{n+1}}$$

$$=\frac{5\times(5^{2n+2})-5^{2n+2}}{5\times(5^{2n+2}\times5^1)-5^{2n+2}}$$

$$=\frac{5\times5^{2n+2}-5^{2n+2}}{5^{2n+2}\times25-5^{2n+2}}$$

$$=\frac{5^{2n+2}(5-1)}{5^{2n+2}\times25-5^{2n+2}}$$

$$=\frac{5^{2n+2}(4)}{5^{2n+2}(25-1)}$$

$$=\frac{4}{24}$$

$$=\frac{1}{6}$$

(vi) $\frac{(16)^{x+1}+20(4^{2x})}{2^{x-3}\times8^{x+2}}$

Solution:

$$=\frac{(2^4)^{x+1}+(5\times2^2)(2^2)^{2x}}{2^{x-3}\times(2^3)^{x+2}}$$

$$=\frac{2^{4x+4}+(5\times2^2)(2^{4x})}{2^{x-3}\times2^{3x+6}}$$

$$=\frac{2^{4x+4}+5\times2^{4x+2}}{2^{3x+6+x-3}}$$

$$=\frac{2^{4x+4}+5\times2^{4x+2}}{2^{4x+3}}$$

$$=\frac{2^{4x+2}\times2^2+5\times2^{4x+2}}{2^{4x+2}\times2^1}$$

$$=\frac{2^{4x+2}\times(2^2+5)}{2^{4x+2}\times(2)}$$

$$=\frac{4+5}{2}.$$

$$=\frac{9}{2}$$

(vii) $\left(64^{-\frac{2}{3}}\div(9)^{\frac{-3}{2}}\right)$

Solution:

$=(64)^{-\frac{2}{3}} \div (9)^{-\frac{3}{2}}$

$$=\frac{(64)^{-\frac{2}{3}}}{(9)^{-\frac{3}{2}}}$$

$$=\frac{9^{\frac{3}{2}}}{64^{\frac{2}{3}}}\left(\because \frac{a^{-m}}{b^{-n}}=\frac{b^{n}}{a^{m}}\right)$$

$$=\frac{(3^2)^{\frac{3}{2}}}{(2^6)^{\frac{2}{3}}}$$

$$=\frac{3^{2\times\frac{3}{2}}}{2^{6\times\frac{2}{3}}}$$

$$=\frac{3^3}{2^{2\times 2}}$$

$$=\frac{27}{2^4}$$

$$=\frac{27}{16}$$

(viii) $\frac{3^n\times 9^{n+1}}{3^{n-1}\times 9^{n-1}}$

Solution:

$$=\frac{3^n\times 9^{n+1}}{3^{n-1}\times 9^{n-1}}$$

$$=\frac{3^n\times (3^2)^{n+1}}{3^{n-1}\times (3^2)^{n-1}}$$

$$=\frac{3^n\times 3^{2n+2}}{3^{n-1}\times 3^{2n-2}}$$

$$=\frac{3^{n+2n+2}}{3^{n-1+2n-2}}$$

$$=\frac{3^{3n+2}}{3^{3n-3}}$$

$=3^{3n+2}\times 3^{-3n+3}$

$$= 3^{3n+2-3n+3} \left(\because \frac{a^m}{a^n} = a^{m-n}\right)$$
$$= 3^5$$
$$= 3 \times 3 \times 3 \times 3 \times 3$$
$$= 243$$
(ix) $\frac{5^{n+3}-6.5^{n+1}}{9\times5^n-4\times5^n}$

Solution:

$$=\frac{5^{n+3}-6.5^{n+1}}{9\times5^n-4\times5^n}$$
$$=\frac{5^n\times5^3-6.5^n.5^1}{9\times5^n-4\times5^n}$$
$$=\frac{5^n(5^3-6\times5)}{5^n(9-4)}$$
$$=\frac{5^3-6\times5}{9-4}$$
$$=\frac{125-30}{9-4}$$
$$=\frac{95}{5}$$
$$=19$$
Q. 3 If $x = 3+\sqrt{8}$ then find the value of

(i) $x+\frac{1}{x}$ (ii) $x-\frac{1}{x}$ (iii) $x^2+\frac{1}{x^2}$ (iv) $x^2-\frac{1}{x^2}$ (v) $x^4+\frac{1}{x^4}$ (vi) $\left(x-\frac{1}{x}\right)^2$

Solution:

$$x = 3+\sqrt{8} \quad ----------- (i)$$
$$\frac{1}{x} = \frac{1}{3+\sqrt{8}}$$
$$\frac{1}{x} = \frac{1}{(3+\sqrt{8})}\times\frac{(3-\sqrt{8})}{(3-\sqrt{8})}$$
$$\frac{1}{x} = \frac{3-\sqrt{8}}{(3)^2-(\sqrt{8})^2}$$

$$\frac{1}{x}=\frac{3-\sqrt{8}}{9-8}$$
$$\frac{1}{x}=\frac{3-\sqrt{8}}{1}$$
$$\frac{1}{x}=3-\sqrt{8}\ -----------(ii)$$
(i) Finding $x+\frac{1}{x}$

Adding eq. (i) and (ii)
$$x+\frac{1}{x}=3+\sqrt{8}+3-\sqrt{8}$$
$$x+\frac{1}{x}=6$$
(ii) Finding $x-\frac{1}{x}$

Subtracting eq.(i) from (ii)
$$x-\frac{1}{x}=(3+\sqrt{8})-(3-\sqrt{8})$$
$$x-\frac{1}{x}=3+\sqrt{8}-3+\sqrt{8}$$
$$x-\frac{1}{x}=2\sqrt{8}$$
(iii) Finding $x^2+\frac{1}{x^2}$

Taking square of eq. (iii)
$$\left(x+\frac{1}{x}\right)^2=(6)^2$$
$$(x)^2+\left(\frac{1}{x}\right)^2+2(x)\left(\frac{1}{x}\right)=36$$
$$x^2+\frac{1}{x^2}=36-2$$
$$x^2+\frac{1}{x^2}=34\ldots(v)$$
(iv) Finding $x^2-\frac{1}{x^2}$

We know that
$$x^2-\frac{1}{x^2}=(x)^2-\left(\frac{1}{x}\right)^2$$
$$x^2-\frac{1}{x^2}=\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)$$

Putting the values from (iii) and (iv)

$$x^2 - \frac{1}{x^2} = (6)(2\sqrt{8}) = 12\sqrt{8}$$

(v) Finding $x^4 + \frac{1}{x^4}$

Taking square of equation (v)

$$\left(x^2 + \frac{1}{x^2}\right)^2 = (34)^2$$

$$(x^2)^2 + \left(\frac{1}{x^2}\right)^2 + 2(x^2)\left(\frac{1}{x^2}\right) = 1156$$

$$x^4 + \frac{1}{x^4} + 2 = 1156$$

$$x^4 + \frac{1}{x^4} = 1156 - 2$$

$$x^4 + \frac{1}{x^4} = 1154$$

(vi) Finding $\left(x - \frac{1}{x}\right)^2$

Taking square of equation (iv)

$$\left(x - \frac{1}{x}\right)^2 = (2\sqrt{8})^2$$

$$\left(x - \frac{1}{x}\right)^2 = (2)^2(\sqrt{8})^2$$

$$\left(x - \frac{1}{x}\right)^2 = 4(8)$$

$$\left(x - \frac{1}{x}\right)^2 = 32$$

Q. 4 Find the rational numbers p and q such that $\frac{8-3\sqrt{2}}{4+3\sqrt{2}} = p + q\sqrt{2}$

Solution:

$$p + q\sqrt{2} = \frac{8 - 3\sqrt{2}}{4 + 3\sqrt{2}}$$

$$P + q\sqrt{2} = \frac{(8 - 3\sqrt{2})}{(4 + 3\sqrt{2})} \times \frac{(4 - 3\sqrt{2})}{(4 - 3\sqrt{2})}$$

$$= \frac{32 - 24\sqrt{2} - 12\sqrt{2} + (3\sqrt{2})^2}{(4)^2 - (3\sqrt{2})^2}$$

$$= \frac{32 - 36\sqrt{2} + 9(2)}{16 - 9(2)}$$

$$= \frac{32 - 36\sqrt{2} + 18}{16 - 18}$$

$$= \frac{50 - 36\sqrt{2}}{-2}$$

$$= \frac{50}{-2} - \frac{36\sqrt{2}}{-2}$$

$\mathrm{p} + \mathrm{q}\sqrt{2} = -25 + 18\sqrt{2}$

By comparing both sides.

$\Rightarrow p = -25, q = 18$

Q. 5 Simplify the following:

(i) $\frac{(25)^{\frac{3}{2}} \times (243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}} \times (8)^{\frac{4}{3}}}$

Solution

$$=\frac{(25)^{\frac{3}{2}}\times(243)^{\frac{3}{5}}}{(16)^{\frac{5}{4}}\times(8)^{\frac{4}{3}}}$$

$$=\frac{(5^2)^{\frac{3}{2}}\times(3^5)^{\frac{3}{5}}}{(2^4)^{\frac{5}{4}}\times(2^3)^{\frac{4}{3}}}$$

$$=\frac{5^{2\times\frac{3}{2}}\times 3^{5\times\frac{3}{5}}}{2^{4\times\frac{5}{4}}\times 2^{3\times\frac{4}{3}}}$$

$$=\frac{5^3\times 3^3}{2^5\times 2^4}$$

$$=\frac{(5\times 3)^3}{2^{5+4}}$$

$$=\frac{(15)^3}{2^9}$$

$$=\frac{15\times 15\times 15}{2\times 2\times 2\times 2\times 2\times 2\times 2\times 2\times 2}$$

$$=\frac{3375}{512}$$

3	243
3	81
3	27
3	9
3	3
	1

(ii) $\frac{54\times\sqrt[3]{(27)^{2x}}}{9^{x+1}+216(3^{2x-1})}$

Solution:

$$\frac{54 \times \sqrt[3]{(27)^{2x}}}{9^{x+1} + 216(3^{2x-1})}$$

$$= \frac{(2 \times 3^3) \times \sqrt[3]{(3^3)^{2x}}}{(3^2)^{x+1} + 2^3 \times 3^3(3^{2x-1})}$$

$$= \frac{(2 \times 3^3) \times \sqrt[3]{(3^{2x})^3}}{3^{2x+2} + 2^3 \times 3^{2x-1+3}}$$

$$= \frac{2 \times 3^3 \times 3^{2x}}{3^{2x+2} + 2^3 \times 3^{2x+2}}$$

$$= \frac{2 \times 3^{2x+3}}{3^{2x+2}(1 + 2^3)}$$

$$= \frac{2 \times 3^{2x+3-2x-2}}{(1+8)} \left(\because \frac{a^m}{a^n} = a^{m-n}\right)$$

$$= \frac{2 \times 3^1}{9}$$

$$= \frac{6}{9}$$

$$= \frac{2}{3}$$

(iii) $\sqrt{\frac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{\frac{-3}{2}}}}$

Solution:

$$= \sqrt{\frac{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}}}{(0.04)^{\frac{-3}{2}}}}$$

$$= \sqrt{(216)^{\frac{2}{3}} \times (25)^{\frac{1}{2}} \times (0.04)^{\frac{3}{2}}}$$

2	216
2	108
2	54
3	27
3	9
3	3
	1

$$= \sqrt{(2^3 \times 3^3)^{\frac{2}{3}} \times (5^2)^{\frac{1}{2}} \times \left(\frac{4}{100}\right)^{\frac{3}{2}}}$$

$$= \sqrt{2^{3\times\frac{2}{3}} \times 3^{3\times\frac{2}{3}} \times 5^{2\times\frac{1}{2}} \times \left(\frac{1}{25}\right)^{\frac{3}{2}}}$$

$$= \sqrt{2^2 \times 3^2 \times 5^1 \times \frac{1}{(5^2)^{\frac{3}{2}}}}$$

$$= \sqrt{(2 \times 3)^2 \times 5^1 \times \frac{1}{5^3}}$$

$$= \sqrt{(6)^2 \times \frac{1}{5^{3-1}}}$$

$$= \sqrt{\frac{6^2}{5^2}}$$

$$= \sqrt{\left(\frac{6}{5}\right)^2}$$

$$= \frac{6}{5}$$

(iv) $\left(a^{\frac{1}{3}} + b^{\frac{2}{3}}\right) \times \left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{2}{3}} + b^{\frac{4}{3}}\right)$

Solution:

$$\left(a^{\frac{1}{3}} + b^{\frac{2}{3}}\right) \times \left(a^{\frac{2}{3}} - a^{\frac{1}{3}}b^{\frac{2}{3}} + b^{\frac{4}{3}}\right)$$

$$=\left(a^{\frac{1}{3}} + b^{\frac{2}{3}}\right) \times \left[\left(a^{\frac{1}{3}}\right)^2 - \left(a^{\frac{1}{3}}\right)\left(b^{\frac{2}{3}}\right) + \left(b^{\frac{2}{3}}\right)^2\right]$$

$$\because (x + \mathrm{y})(x^2 - x\mathrm{y} + \mathrm{y}^2) = x^3 + \mathrm{y}^3$$

$$\therefore = \left(a^{\frac{1}{3}}\right)^3 + \left(b^{\frac{2}{3}}\right)^3$$

$$=a^{\frac{1}{3}X3} + b^{\frac{2}{3}\times 3}$$

$$=a + b^2$$