9th Math Exercise No 3.3

Q. 1 For $\mathrm{A} = \{1,2,3,4\}$, find the following relations in A. State the domain and range of each relation.

Solution:

$$A = \{1,2,3,4\}$$

$$A \times A = \{1,2,3,4\} \times \{1,2,3,4\}$$
$$A \times A = \{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),$$
$$(2,3),(2,4),(3,1),(3,2),(3,3),(3,4),(4,1),$$
$$(4,2),(4,3),(4,4)\}$$

(i) $\{(x,y) \mid y = x\}$

Solution:

$$R = \{x,y) \mid y = x\}$$
$$R = \{(1,1),(2,2),(3,3),(4,4)\}$$
$$\mathrm{Dom}(R) = \{1,2,3,4\}$$
range $(R) = \{1,2,3,4\}$

(ii) $\{(x,y) \mid y + x = 5\}$

Solution:

$$\{(x,y) \mid y + x = 5\}$$
$$R = \{(1,4),(2,3),(3,2),(4,1)\}$$
Range $(R) = \{4,3,2,1\}$

(iii) $\{(x,y) \mid x + y < 5\}$

Solution:

$$\{(x,y) \mid (x + y < 5\}$$
$$\mathrm{R} = \{x,\mathrm{y}), \mid x + \mathrm{y} < 5\}$$

$R = \{1,1),(1,2),(1,3),(2,1),(2,2),(3,1)\}$

$\mathrm{Dom}(R) = \{1,2,3\}$

Range $(R) = \{1,2,3\}$

(iv) $\mathbb{R} = \{(x,y) \mid x + y > 5\}$

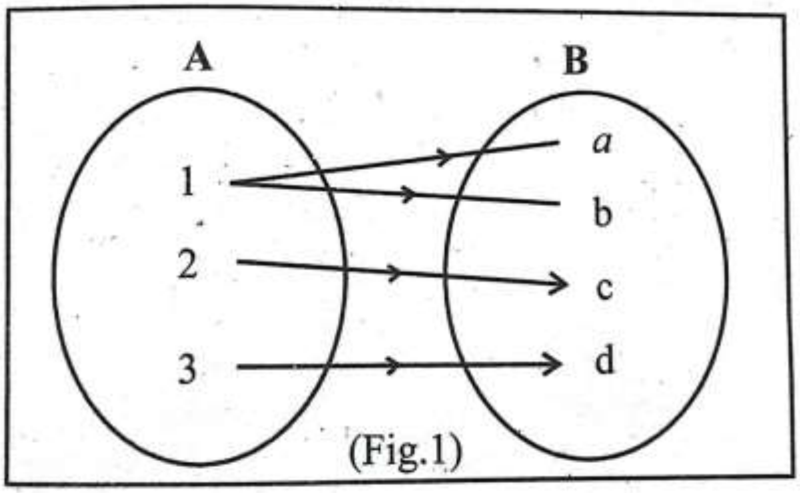
Solution:

$$\mathrm{R} = \{(x,y) \mid x + y > 5\}$$
$$\mathrm{R} = \{(2,4),(3,3),(3,4),(4,2),(4,3),(4,4)\}$$

$\mathrm{Dom}(R) = \{2,3,4\}$

Range $(R) = \{4,3,2\}$

Q. 2 Which of the following diagrams represent functions and of which type?



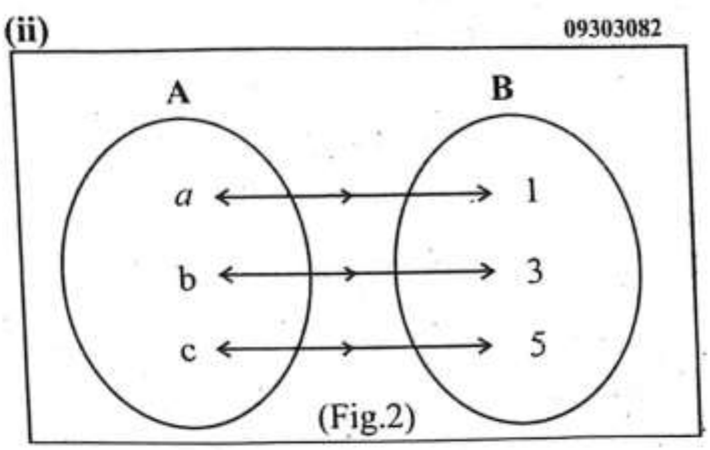
(Fig.1)

(i)

Solution:

$$A = \{1,2,3\}, B = \{a, b, c, d\},$$
$$R = \{(1, a), (1, b), (2, c), (3, d)\}$$

Since, first elements in first two ordered pairs are same i.e. 1, 1, so the relation is Not a function.



(Fig.2)

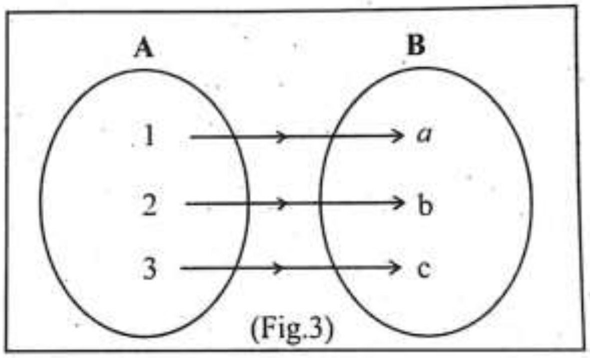
(ii)

Solution:

$$A = \{a, b, c\}, B = \{1,3,5\}$$
$$R = \{(a, 1), (b, 3), (c, 5)\}$$

Range $(R) = \{1,3,5\} = B$

Since relation is an onto and one-one function, so relation is a bijective function.



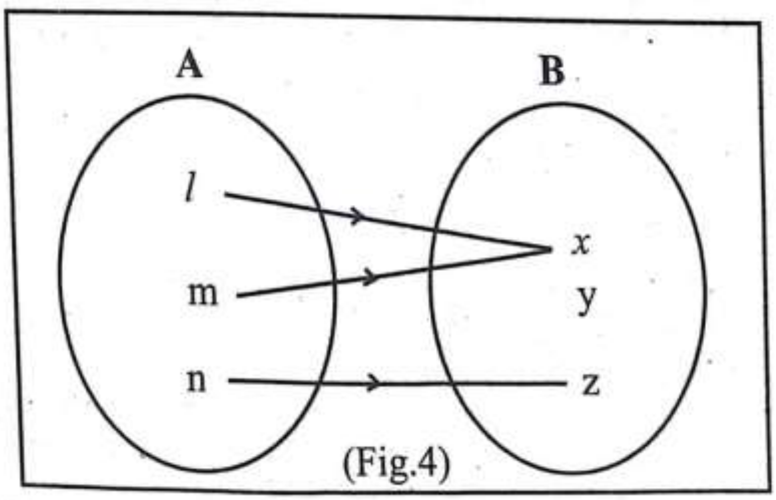
(Fig.3)

(iii)

$$R = \{(1, a), (2, b), (3, c)\}$$
$$A = \{1,2,3\}, B = \{a, b, c\}$$
$$\text{Range } = \{a, b, c\} = B$$

Since, relation is onto function and one one function.So relation is a bijective function.

iv)



(Fig.4)

Solution:

$$A = \{l, m, n\}, B = \{x, y, z\}$$
$$R = \{(l, x), (m, x), (n, z)\}$$

Range $(f) = \{x, z\} \neq B$

Since, range $(f) \subseteq B$, so function into function.

Q. 3 If $g(x) = 3x + 2$ and $h(x) = x^2 + 1$, then find:

(i) $g(0)$

Solution:

Put $x = 0$ in $g(x)$

$$g(0) = 3(0) + 2 = 0 + 2 = 2$$

(ii) $g(-3)$

Solution

Put $x = -3$ in $g(x)$

$$g(-3) = 3(-3) + 2 = -9 + 2 = -7$$

(iii) $g\left(\frac{2}{3}\right)$

Solution:

Put $x = \frac{2}{3}$ in $g(x)$

$$g\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right) + 2 = 2 + 2 = 4$$

(iv)h(1)

Solution:

Put $x = 1$ in $h(x)$

$$h(1) = (1)^2 + 1 = 1 + 1 = 2$$

(v) $h(-4)$

Solution:

Put $x = -4$ in $h(x)$

$$h(-4) = (-4)^2 + 1 = 16 + 1 = 17$$

(vi) $h\left(-\frac{1}{2}\right)$

Solution:

Put $x = -\frac{1}{2}$ in $h(x)$

$$h\left(-\frac{1}{2}\right) = \left(-\frac{1}{2}\right)^2 + 1$$
$$= \frac{1}{4} + 1$$
$$= \frac{1+4}{4} = \frac{5}{4}$$

Q. 4 Given that $f(x) = ax + b + 1$, where a and b are constant numbers. If $f(3)$=8 and $f(6)$=14, then find the values of and b .

Solution:

Given $f(x) = ax + b + 1$

$f(3)$=8 ………..(i)

$f(6)$=14………….(ii)

Put x=3 in f(x)

f(3)=3a+b+1

From (i).
$8 = 3a + b + 1$
$8 - 1 = 3a + b$
$7 = 3a + b$
$\Rightarrow 3a + b = 7$
Now, put $x = 6$ in $\mathrm{f(x)}$,

$$f(\mathrm{x}) = a(6) + b + 1$$
$$f(6) = 6a + b + 1$$

From (ii) put $f(6) = 14$

$$14 = 6a + b + 1$$
$$14 - 1 = 6a + b$$
$$13 = 6a + b$$
$$\Rightarrow 6a + b = 13$$

Subtracting (iii), from (iv)

$$6a + b = 13$$
$$+3a \pm b = \pm 7$$
$$3a = 6$$
$$a = \frac{6}{3}$$
$$a = 2$$

Put $a = 2$ in (iii)

$$3(2) + b = 7$$
$$6 + b = 7$$
$$b = 7 - 6$$
$$b = 1$$

Thus $a = 2, b = 1$
Q. 5 Given that $\mathrm{g}(x) = ax + \mathrm{b} + 5$, where a and b are constant numbers. If $g(-1) = 0$ and $g(2) = 10$, find the values of a and b.
Solution:

$$g(-1) = 0$$
$$g(2) = 10$$
$$g(x) = ax + b + 5$$

Put $x = -1$ in $g(x)$

$$g(-1) = a(-1) + b + 5$$
$$g(-1) = -a + b + 5$$

From put (i) put $g(-1) = 0$

$$\Rightarrow 0 = -a + b + 5$$
$$\Rightarrow a - b = 5$$

Now, put $x = 2$ in $\mathrm{g}(x)$

$$g(2) = a(2) + b + 5$$
$$g(2) = 2a + b + 5$$

From (ii) put $g(2) = 10$

$$\Rightarrow 10 = 2a + b + 5$$
$$10 - 5 = 2a + b$$
$$5 = 2a + b$$
$$\Rightarrow 2a + b = 5$$

Adding eq. (iii) and (iv)

$$a - \cancel{b} = 5$$
$$2a + \cancel{b} = 5$$
$$\overline{3a = 10}$$
$$a = \frac{10}{3}$$

Put it in equation (iii)

$$\frac{10}{3} - b = 5$$
$$\frac{10}{3} - b = 5$$
$$\frac{10}{3} - 5 = b$$
$$\frac{10 - 15}{3} = b$$
$$b = \frac{-5}{3}$$

Thus $a = \frac{10}{3}$, b $= \frac{-5}{3}$

Q. 6 Consider the function defined by $f(x) = 5x + 1$. If $f(x) = 32$, find the x value.

Solution:

$$f(x) = 5x + 1$$
$$f(x) = 32$$

By comparing, we get

$$5x + 1 = 32$$
$$5x = 32 - 1$$
$$5x = 31$$
$$x = \frac{31}{5}$$

Q. 7 Consider the function $f(x) = cx^2 + d$, where c and d are constant numbers If $f(1) = 6$ and $f(-2) = 10$, then find the values of c and d.

Solution

$$f(1) = 6$$
$$f(-2) = 10$$
$$f(x) = cx^2 + \mathrm{d}$$

Put $x = 1$ in $f(x)$

$$f(1) = \mathrm{c}(1)^2 + \mathrm{d}$$
$$f(1) = \mathrm{c} + \mathrm{d}$$

From (i) put $f(1) = 6$

$$6 = c + d$$
$$\Rightarrow c + d = 6$$

Now, put $x = -2$ in $f(x)$

$$f(-2) = c(-2)^2 + d$$
$$f(-2) = c(4) + d$$
$$f(-2) = 4c + d$$

From (ii) put $f(-2) = 10$

$$\Rightarrow 10 = 4c + d$$
$$\Rightarrow 4c + d = 10$$

$$f(-2) = c(4) + d$$
$$f(-2) = 4c + d$$

From (ii) put $f(-2) = 10$

$$\Rightarrow 10 = 4c + d$$
$$\Rightarrow 4c + d = 10$$

Subtracting (iii) from (iv)

$$4c + d = 10$$
$$\pm c \pm d = \pm 6$$
$$3c = 4$$
$$c = \frac{4}{3}$$

Put it in equation (iii)

$$c + d = 6$$
$$\frac{4}{3} + d = 6$$
$$d = 6 - \frac{4}{3}$$
$$d = \frac{18 - 4}{3}$$
$$d = \frac{14}{3}$$

Thus $c = \frac{4}{3}$, $d = \frac{14}{3}$