Exercise 4.3

Q. 1 Find HCF by factorization method.

(i) $21x^2y, 35xy^2$

Solution

$$21x^2y, 35x^2$$

Factorization

$$21x^2y = 3 \times 7x \cdot x \cdot y$$
$$35xy^2 = 5 \times 7x \cdot y \cdot y$$

Common factors $= 7, x, y$

H.C.F $= 7xy$ (product of common factors)

(ii) $4x^2 - 9y^2, 2x^2 - 3xy$

Solution:

$$4x^2 - 9y^2, 2x^2 - 3xy$$

Factorization

$$4x^2 - 9y^2 = (2x)^2 - (3y)^2$$
$$\because a^2 - b^2 = (a+b)(a-b)$$
$$= (2x + 3y)(2x - 3y)$$

Now,

$$2x^2 - 3xy = x(2x - 3y)$$

Common factor $= (2x - 3y)$

$$\text{H.C.F} = (2x - 3y)$$

(iii) $x^3 - 1, x^2 + x + 1$

Solution:

$$x^3 - 1, x^2 + x + 1$$

Factorization

$$x^3 - 1 = (x)^3 - (1)^3$$
$$\because a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right)$$
$$= (x-1)\left[(x)^2 + (x)(1) + (1)\right]$$
$$= (x-1)\left(x^2 + x + 1\right)$$

Now,

$$x^2 + x + 1 = \left(x^2 + x + 1\right)$$

Common factor $= \left(x^2 + x + 1\right)$

$$HCF = \left(x^2 + x + 1\right)$$

(iv) $a^3 + 2a^2 - 3a, 2a^3 + 5a^2 - 3a$

Solution:

$$a^3 + 2a^2 - 3a, 2a^3 + 5a^2 - 3a$$

Factorization

$$\begin{aligned} a^3 + 2a^2 - 3a &= a\left(a^2 + 2a - 3\right) \\ &= a\left[a^2 + 3a - a - 3\right] \\ &= a[a(a+3) - 1(a+3)] \\ &= a(a+3)(a-1) \end{aligned}$$

Now,

$$\begin{aligned} 2a^3 + 5a^2 - 3a &= a\left(2a^2 + 5a - 3\right) \\ &= a\left[2a^2 + 6a - a - 3\right) \\ &= a[2a(a+3) - 1(a+3) \\ &= a(a+3)(2a-1) \end{aligned}$$

Common factors $= a(a+3)$

H.C.F $= a(a+3)$

(v) $t^2 + 3t - 4, t^2 + 5t + 4, t^2 - 1$ (statement in text)

Solution: (Correction)

$$t^2 + 3t + 4, t^2 + 5t + 4, t^2 - 1$$

Factorization

$$\begin{aligned} t^2 + 3t + 4 &= t^2 + 4t - t + 4 \\ &= t(t+4) - 1(t+4) \\ &= (t+4)(t+1) \\ t + 5t + 4 &= t^2 + 4t + t + 4 \\ &= t(t+4) + 1(t+4) \\ &= (t+4)(t-1) \\ t - 1 &= (t)^2 - (1)^2 \\ & (t+1)(t-1) \end{aligned}$$

Common factor $-(t+1)$

$$\begin{aligned} &= a\left[2a^2 + 6a - a - 3\right) \\ &= a[2a(a+3) - 1(a+3) \\ &= a(a+3)(2a-1) \end{aligned}$$

Common factors $= a(a+3)$

H.C.F $= a(a+3)$

(v) $t^2 + 3t - 4, t^2 + 5t + 4, t^2 - 1$

Solution: (Correction)

$$t^2+3t+4, t^2+5t+4, t^2-1$$

Factorization

$$\begin{aligned} t^2+3t+4 &= t^2+4t-t+4 \\ &= t(t+4)-1(t+4) \\ &= (t+4)(t+1) \end{aligned}$$

$$\begin{aligned} t^2+5t+4 &= t^2+4t+t+4 \\ &= t(t+4)+1(t+4) \\ &= (t+4)(t+1) \end{aligned}$$

$$\begin{aligned} t^2-1 &= (t)^2-(1)^2 \\ &= (t+1)(t-1) \end{aligned}$$

Common factor $= (t+1)$

H.C.F $= (t+1)$

(vi) $x^2+15x+56, x^2+5x-24, x^2+8x$

$$x^2+15x+56, x^2+5x-24, x^2+8x$$

Factorization

$$\begin{aligned} x^2+15x+56 &= x^2+7x+8x+56 \\ &= x(x+7)+8(x+7) \\ &= (x+7)(x+8) \end{aligned}$$

$$\begin{aligned} x^2+5x-24 &= x^2+8x-3x-24 \\ &= x(x+8)-3(x+8) \\ &= (x+8)(x-3) \end{aligned}$$

Now,

$$x^2+8x = x(x+8)$$

Common factor $= (x+8)$

H.C.F $= (x+8)$

Q. 2 Find HCF of the following expressions by using division method:

(i) $27x^3+9x^2-3x-10, 3x-2$ (correction)

Solution:

$$27x^3+9x^2-3x-10, 3x-2$$

H.C.F by Division Method

Solution:

$27x^3 + 9x^2 - 3x - 10,\ \ 3x - 2$

H.C.F by Division Method

$$
\begin{array}{r|l}
 & 9x^2+9x+5 \\
\hline
3x-2 & 27x^3+9x^2-3x-10 \\
 & \oplus 27x^3 \ominus 18x^2 \\
 & \underline{-\quad\quad +\quad\quad} \\
 & \quad 27x^2 - 3x - 10 \\
 & \quad \oplus 27x^2 \ominus 18x \\
 & \quad \underline{-\quad\quad +\quad\quad} \\
 & \quad\quad 15x - 10 \\
 & \quad\quad \ominus 15x \ominus 10 \\
 & \quad\quad \underline{+\quad\quad +} \\
 & \quad\quad\quad 0
\end{array}
$$

Thus, H.C.F = $3x-2$

(ii) $x^3 - 9x^2 + 21x - 9, x^2 - 4x + 3$

Solution:

$x^3 - 9x^2 + 21x - 9, x^2 - 4x + 3$

Solution:

$x^3 - 9x^2 + 21x - 9\ ,\ x^2 - 4x + 3$

H.C.F by Division Method

$$
\begin{array}{r|l}
 & x-5 \\
\hline
x^2-4x+3 & x^3-9x^2+21x-9 \\
 & x^3 - 4x^2 + 3x \\
 & \underline{-\quad\quad +\quad\quad -\quad\quad} \\
 & \quad -5x^2+18x-9 \\
 & \quad -5x^2+20x-15 \\
 & \quad \underline{+\quad\quad -\quad\quad +} \\
 & \quad\quad -2x+6 \\
 & \quad\quad -2(x-3)
\end{array}
$$

$$
\begin{array}{r|l}
 & x-1 \\
\hline
x-3 & x^2-4x+3 \\
 & x^2-3x \\
 & -\quad + \\
\hline
 & \quad -x+3 \\
 & \quad -x+3 \\
 & \quad +\quad - \\
\hline
 & \qquad 0
\end{array}
$$

H.C.F $= x-3$.

H.C.F $= x - 3$

(iii) $2x^3 + 2x^2 + 2x + 2, 6x^3 + 12x^2 + 6x + 12$

Solution:

Solution:

$2x^3 + 2x^2 + 2x + 2,\ 6x^3 + 12x^2 + 6x + 12$

H.C.F by Division Method

$$
\begin{array}{r|l}
 & \quad 3 \\
\hline
2x^3+2x^2+2x+2 & 6x^3+12x^2+6x+12 \\
 & \underline{\underset{-}{\oplus} 6x^3 \underset{-}{\oplus} 6x^2 \underset{-}{\oplus} 6x \underset{-}{\oplus} 6} \\
 & \qquad 6x^2+6 \\
 & \qquad 3(2x^2+2)
\end{array}
$$

$$
\begin{array}{r|l}
 & \quad x+1 \\
\hline
2x^2+2 & 2x^3+2x^2+2x+2 \\
 & \underline{\underset{-}{\oplus} 2x^3 \qquad \underset{-}{\oplus} 2x} \\
 & \qquad 2x^2+2 \\
 & \qquad \underline{\underset{-}{\oplus} 2x^2 \underset{-}{\oplus} 2} \\
 & \qquad\quad 0
\end{array}
$$

Thus H.C.F $= 2x^2+2 = 2(x^2+1)$

(iv) $2x^3 - 4x^2 + 6x, x^3 - 2x, 3x^2 - 6x$

Solution:

$2x^3 - 4x^2 + 6x,\ x^3 - 2x,\ 3x^2 - 6x$

First we find H.C.F of $x^3 - 2x$, $3x^2 - 6x$

$$\begin{array}{r|l} & \quad 3 \\ \hline (x^2 - x) & 3x^2 - 6x \\ & \oplus 3x^2 \ominus 3x \\ & \ \ - \qquad + \\ \hline & \qquad -3x \end{array}$$

(Taking 6 as common) $6x^2 - 6x$

$$\begin{array}{r|l} & \quad 3 \\ \hline (x^2 - x) & 3x^2 - 6x \\ & \oplus 3x^2 \ominus 3x \\ & \ \ - \qquad + \\ \hline & \qquad -3x \end{array}$$

($\because -3$ is not a common factor, we ignore it)

$$\begin{array}{r|l} & \quad x - 1 \\ \hline x & x^2 - x \\ & \underset{-}{\oplus} x^2 \\ \hline & \quad -x \\ & \quad \overset{-}{\underset{\oplus}{}} x \\ \hline & \quad\ 0 \end{array}$$

Thus H.C.F is "x".

Q-3 Find LCM of the following expressions by using prime factorization method.

(i) $2a^2b, 4ab^2, 6ab$

Solution:

$$2a^2b, 4ab^2, 6ab$$
$$2a^2b = (2)\, X a X a X b$$
$$4ab^2 = 2 \times 2 \times \mathrm{a} \times b X b$$
$$6ab = (2) \times 3 \text{ X a Xb}$$

Product of common factors = 2ab

Product of non-common factors

$$= (a)(2b)(3) = 6ab$$

$$\text{L.CM} = (2ab)(6ab) = 12a^2b^2$$
$$\text{LCM} = (2ab)(6ab) = 12a^2b^2$$

(ii) $x^2 + x, x^3 + x^2$

Solution:

$$x^2 + x, x^3 + x^2$$

Factorization

$$x^2 + x = (x + 1)$$
$$x^3 + x^2 = x^2(x + 1)$$
$$= (1).\ x(x + 1)$$

Product of common factors = $x(x + 1)$

Product of non-common factors = x

$$\text{L.C.M} = x(x + 1) \times x$$
$$\text{L.C.M} = x^2(x + 1)$$

(iii) $a^2 - 4a + 4, a^2 - 2a$

Solution:

$$a^2 - 4a + 4, a^2 - 2a$$

Factorization

$$a^2 - 4a + 4 = (a)^2 - 2(a)(2) + (2)^2$$
$$= (a - 2)^2$$
$$= (a - 2)(a - 2)$$
$$a^2 - 2a = a(a - 2)$$

Product of common factors = $(a - 2)$

Product of non-common factors = $a(a - 2)$

$$\text{L.C.M} = (a - 2) \cdot a(a - 2)$$
$$= a(a - 2)^2$$

(iv) $x^4 - 16, x^3 - 4x$

Solution:

$$x^4 - 16, x^3 - 4x$$

$$\begin{aligned} x^4 - 16 &= \left(x^2\right)^2 - (4)^2 \\ &= \left(x^2 + 4\right)\left(x^2 - 4\right) \\ &= \left(x^2 + 4\right)\left[x^2 - 2^2\right] \\ &= \left(x^2 + 4\right)(x + 2)(x - 2) \end{aligned}$$

Now, $x^3 - 4x = x\left(x^2 - 4\right) = x\left(x^2 - 2^2\right)$

$$= x(x + 2)(x - 2)$$

$$\begin{aligned} \text{L.C.M} &= (x + 2)(x - 2)x\left(x^2 + 4\right) \\ &= (x \quad + 2)(x - 2)x\left(x^2 + 4\right) \\ &= x\left(x^2 - 2^2\right)\left(x^2 + 4\right) \\ &= x\left(x^2 - 4\right)\left(x^2 + 4\right) \\ &= x\left(x^2\right)^2 - (4)^2 \\ &= x\left(x^4 - 16\right) \end{aligned}$$

(v) $16 - 4x^2, x^2 + x - 6, 4 - x^2$

Solution:

$$16 - 4x^2, x^2 + x - 6, 4 - x^2$$

Factorization

$$\begin{aligned} 16 - 4x^2 &= 4\left(4 - x^2\right) \\ &= 4\left[2^2 - x^2\right] \\ &= 4(2 + x)(2 - x) \\ &= 4(x + 2)(-1)(-2 + x) \\ &= (-1)(4)(x + 2)(x - 2) \\ &= -4(x + 2)(x - 2) \end{aligned}$$

$$\begin{aligned} x^2 + x - 6 &= x^2 + 3x - 2x - 6 \\ &= x(x + 3) - 2(x + 3) \\ &= (x + 3)(x - 2) \end{aligned}$$

$$\begin{aligned} 4 - x^2 &= (2)^2 - (x)^2 \\ &= (2 + x)(2 - x) \\ &= (x + 2)(-1)(-2 + x) \\ &= -1(x + 2)(x - 2) \end{aligned}$$

Product of common factors

$$(-1)(x-2)$$

Product of non-common factors $= 4(x+3)$(x+2)

$$\text{L.C. } M = 4\left(4-x^2\right)(x+3)$$

Q. 4 The HCF of two polynomials is $y-7$ and their LCM is $y^3-10y^2+11y+70$. If one of the polynomials is $y^2-5y-14$, find the other.

Solution:

H.C.F $= y-7$

$$\text{L.CM } = y^3-10y^2+11y+70$$

$$P(y) = y^2-5y-14$$

We know that

$$p(y)q(y) = L \cdot CM \times H \cdot CF$$

$$q(y) = \frac{L \cdot M \times HCF}{p(y)}$$

$$q(y) = \frac{\left(y^3-10y^2+11y+70\right) \times (y-7)}{y^2-5y-14}$$

$$q(y) = (y-5)(y-7)$$

$$q(y) = y^2-12y+35$$

Q. 5 The LCM and HCF of two polynomials $p(x)$ and $q(x)$ are $36x^3(x+a)\left(x^3-a^3\right)$ and $x^2(x-a)$ respectively. If $p(x) = 4x^2\left(x^2-a^2\right)$, find $q(x)$.

Solution:

$$\text{L.C. } M = 36x^3(x+a)\left(x^3-a^3\right)$$

$$\text{H.C. } F = x^2(x-a)$$

$$P(x) = 4x^2\left(x^2-a^2\right)$$

We know that

$$\mathrm{p}(x).\mathrm{q}(x) = \text{L.C.M} \times \text{H.C.F}$$

$$\mathrm{q}(x) = \frac{\text{L.C.M} \times \text{H.CF}}{\mathrm{p}(x)}$$

$$\mathrm{q}(x) = \frac{36x^3(x+a)\left(x^3-a^3\right) \times x^2(x-a)}{4x\left(x^2-a^2\right)}$$

$$\mathrm{q}(x) = \frac{9x^3\left(x^3-a^3\right)x^2\left(x^2-a^2\right)}{x^2\left(x^2-a^2\right)}$$

$$q(x) = \frac{\text{L.C.M} \times \text{HC.F}}{p(x)}$$

$$q(x) = \frac{36x^3(x+a)\left(x^3-a^3\right) \times x^2(x-a)}{4x^2\left(x^2-a^2\right)}$$

$$q(x) = \frac{9x^3\left(x^3-a^3\right) \cdot x^2\left(x^2-a^2\right)}{x^2\left(x^2-a^2\right)}$$

$$q(x) = 9x^3\left(x^3-a^3\right)$$

Q. 6 The HCF and LCM of two polynomials is $(x+a)$ and $12x^2(x+a)\left(x^2-a^2\right)$ respectively. Find the product of the two polynomials.

Solution:

$$\text{H.C. } F = (x+a)$$

$$\text{L.C. } M = 12x^2(x+a)\left(x^2-a^2\right)$$

Let $p(x)$ and $q(x)$ be required polynomials We know that

$$p(x) \times q(x) = \text{L.C.M} \times \text{H.C.F}$$

$$p(x) \times q(x) = 12x^2(x+a)\left(x^2-a^2\right) \times (x+a)$$

$$= 12x^2(x+a)^2(x+a) \times (x-a)$$

$$= 12x^2(x+a)^3(x-a)$$