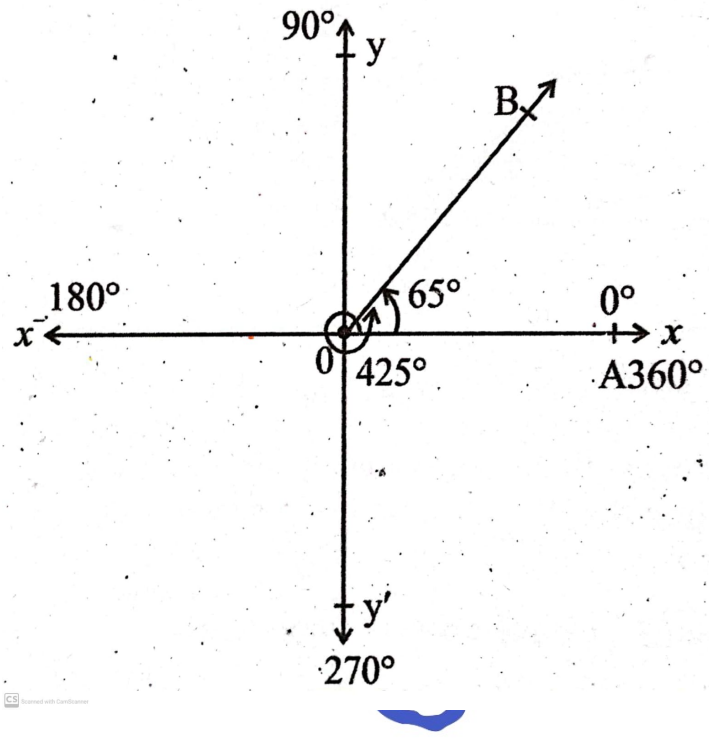
9th Math Exercise 6.1

Q. 1 Find in which quadrant the following angel lies. Write a co-terminal angle for each.

(i) 65°

Solution

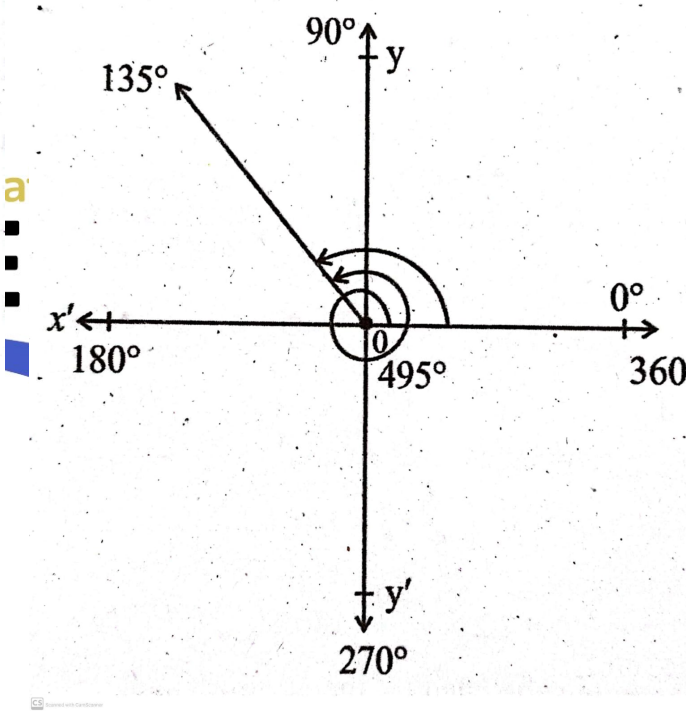


The angle 65° lies in quadrant I.

The angle 65° lies in I quadrant.

The coterminal angle of $65^\circ = 65^\circ + 360^\circ = 425^\circ$

(ii) 135°



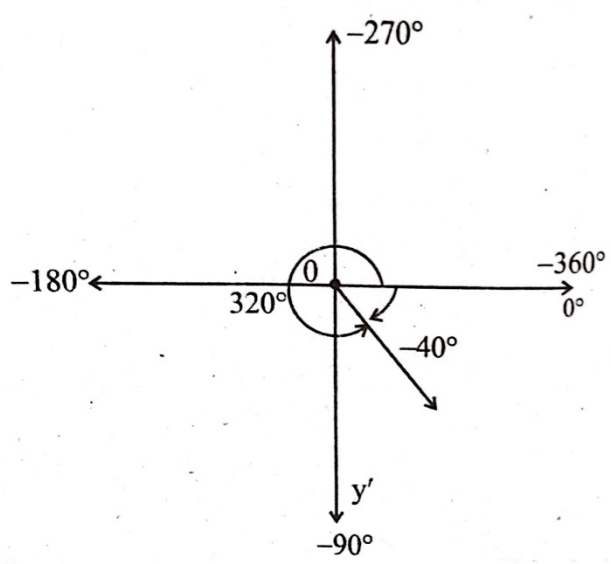
Solution:

The angle of 135° lies in Q.II.

The coterminal angle of $135^\circ = 135^\circ + 360^\circ = 495^\circ$

(iii) -40°

Solution:

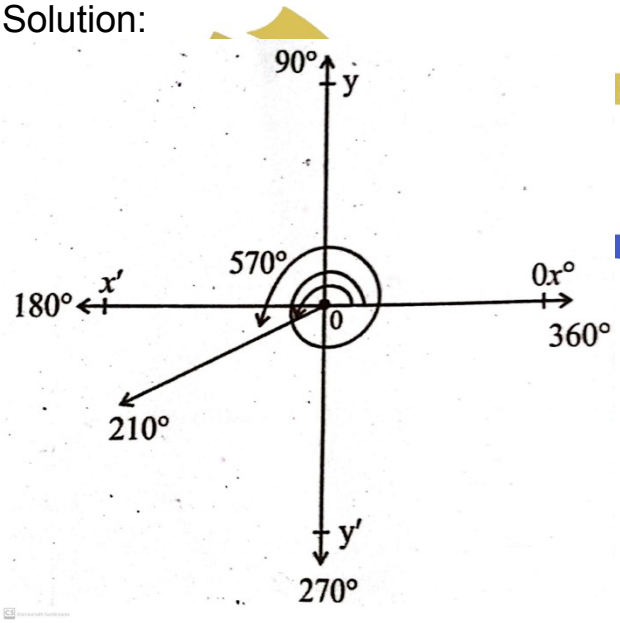


The angle of -40° lies in Q.IV.

The coterminal angle of $-40^\circ = -40^\circ + 360^\circ = 320^\circ$

(iv) 210°

Solution:



The angle of -40° lies in Q.IV.

The coterminal angle of $-40^\circ = -40^\circ + 360$

(iv) 210°

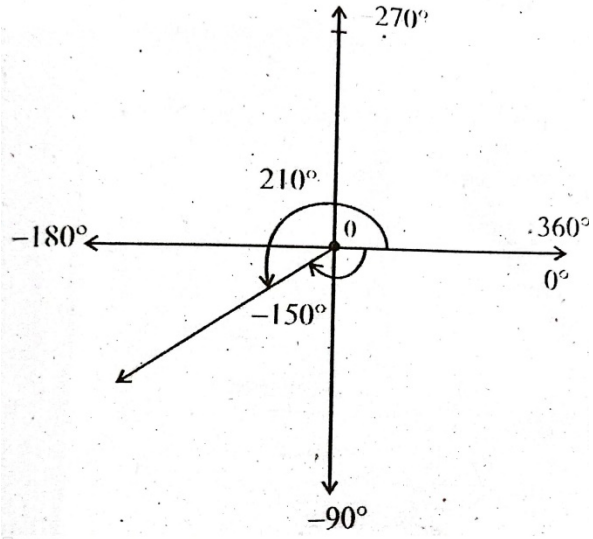
Solution:

The angle of 210° lies in Q.III.

The coterminal angle of $210^\circ = 210^\circ + 360^\circ = 570^\circ$

(v) -150°

Solution:



The angle of -150° lies in Q.III.

The coterminal angle of -150°

$$= -150^\circ + 360^\circ = 210^\circ$$

Q. 2 Convert the following into to degrees, minutes, and seconds.

(i) 123.456°

Solution:

$$\begin{aligned}
123.456^\circ &= 123^\circ + 0.456^\circ \\
&= 123^\circ + (0.456 \times 60)' \\
&= 123^\circ + 27.36' \\
&= 123^\circ + 27'' + 0.36' \\
&= 123^\circ + 27' + (0.36 \times 60)'' \\
&= 123^\circ + 27' + 21.6'' \\
&= 123^\circ 27' 21.6''
\end{aligned}$$

(ii) 58.7891°

Solution:

$$\begin{aligned}
58.7891^\circ &= 58^\circ + 0.7891^\circ \\
&= 58^\circ + (0.7891 \times 60)' \\
&= 58^\circ + 47.346' \\
&= 58^\circ + 47' + 0.346' \\
&= 58^\circ + 47' + (0.346 \times 60)'' \\
&= 58^\circ + 47' + 21'' \\
&= 58^\circ 47' 21''
\end{aligned}$$

(iii) 90.5678°

Solution:

$$= 90.5678^\circ = 90^\circ + 0.5678^\circ$$
$$= 90^\circ + (0.5678 \times 60)'$$
$$= 90^\circ + 34.068'$$
$$= 90^\circ + 34' + 0.068'$$
$$= 90^\circ + 34' + (0.068 \times 60)''$$
$$= 90^\circ + 34' + 4.08''$$
$$= 90^\circ 34' 4.08''$$

Q. 3 Convert the following into decimal degrees.

(i) $65^\circ 32' 15''$

Solution

$$65^\circ 32' 15''$$
$$= 65^\circ + \left(\frac{32}{60}\right)^\circ + \left(\frac{15}{3600}\right)^\circ$$
$$= 65^\circ + 0.5333^\circ + 0.0042^\circ$$
$$= 65.5375^\circ$$

(ii) $42^\circ 18' 45''$

Solution

$$42^\circ 18' 45''$$
$$= 42^\circ + \left(\frac{18}{60}\right)^\circ + \left(\frac{45}{3600}\right)^\circ$$
$$= 42^\circ + 0.3^\circ + 0.0125^\circ$$
$$= 42.3125^\circ$$

(iii) $78^\circ 45' 36''$

Solution:

$$78^\circ 45' 36''$$
$$= 78^\circ + \left(\frac{45}{60}\right)^\circ + \left(\frac{36}{3600}\right)^\circ$$
$$= 78^\circ + 0.75^\circ + 0.01^\circ = 78.76^\circ$$

Q. 4 Convert the following into radian.

(i) 36°

Solution:

$$36^\circ = 36 \times 1^\circ \quad \left(\because 1^\circ = \frac{\pi}{180}\text{rad}\right)$$
$$= 36 \times \frac{\pi}{180}\text{rad}$$
$$= \frac{\pi}{5}\text{rad}$$

(ii) 22.5°

Solution:

$$22.5^\circ = 22.5 \times 1^\circ$$

$$= 22.5 \times \frac{\pi}{180}\text{rad}$$

$$= \frac{\pi}{8}\text{rad}$$

(iii) 67.5°

Solution

$$67.5^\circ = 67.5 \times 1^\circ$$

$$= 67.5 \times \frac{\pi}{180}\text{rad} = \frac{3\pi}{8}\text{rad}$$

Q. 5 Convert the following into degrees.

(i) $\frac{\pi}{16}\text{rad}$

Solution:

$$\frac{\pi}{16}\text{rad}$$

$$= \left(\frac{\pi}{16} \times \frac{180}{\pi}\right)^\circ$$

$$= 11.25^\circ$$

(ii) $\frac{11\pi}{5}\text{rad}$

Solution:

$$\frac{11\pi}{5}\text{rad}$$

$$= \frac{11\pi}{5}\text{rad}$$

$$= \frac{11\pi}{5}\text{rad}$$

$$= \left(\frac{11\pi}{5} \times \frac{180}{\pi}\right)^\circ$$

$$= 396^\circ$$

(iii) $\frac{7\pi}{6}\text{rad}$

Solution:

$$\frac{7\pi}{6}\text{rad}$$

$$= \left(\frac{7\pi}{6} \times \frac{180}{\pi}\right)^\circ$$

$$= 210^\circ$$

Q. 6 Find the arc length and area of a sector with:

(i) $r = 6$ cm and $\theta = \frac{\pi}{3}\text{rad}$

Solution:

Area of sector $= A = ?$

Arc length $= \ell = ?$

$r = 6$ cm and $\theta = \frac{\pi}{3}$ rad

Arc length $= \rho = r\theta$

$$\ell = 6 \times \frac{\pi}{3} \text{ cm}$$

$$\ell = 6.28 \text{ cm}$$

Area of sector $= A = \frac{1}{2} r^2 \theta$

$$= \frac{1}{2} \times (6)^2 \times \frac{\pi}{3}$$

$$= \frac{1}{6} \times 36 \times \pi$$

$$= 6\pi \text{ cm}^2$$

$$= 18.85 \text{ cm}^2$$

(iii) $r = \frac{4.8}{\pi}$ cm and central angle $\theta = \frac{5\pi}{6}$ radians.

Solution:

$r = \frac{4.8}{\pi}$ cm and $\theta = \frac{5\pi}{6}$ rad.

Arc length $= \ell = r\theta$

$$r = \frac{4.0}{\pi} \text{ cm}$$

Arc length $= \ell = r\theta$

$$\ell = \frac{4.8}{\pi} \times \frac{5\pi}{6}$$

$$= 4 \text{ cm}$$

Area of sector $= A = \frac{1}{2} r^2 \theta$

$$= \frac{1}{2} \left(\frac{4.8}{\pi} \right)^2 \frac{5\pi}{6}$$

$$= \frac{1}{12} \times \frac{23.04}{\pi^2} \times 5\pi$$

$$= \frac{1}{12} \times \frac{115.2}{\pi}$$

$$= 3.056 \text{ cm}^2$$

Q. 7 If the central angle of a sector is 60° and the radius of the circle is 12 cm , find the area of the sector and the percentage of the total area of the circle it represents.

Solution

Radius $= r = 12$ cm

Central angle $= \theta = 60^\circ$

$\theta = 60 \times \frac{\pi}{180} \text{rad} = \frac{\pi}{3} \text{rad}$

Area of sector $= A_1 = \frac{1}{2} r^2 0$

$= \frac{1}{2} \times (12 \text{ cm})^2 \times \frac{\pi}{3}$

$= \frac{1}{6} \times 144\pi$

$= 24\pi \ (\because \pi \approx 3.1416)$

$(\because \pi \approx 3.1416)$

$= 24 \times 3.1416$

$= 75.39 \approx 75.4 \text{ cm}^2$

Area of circle $= A = \pi r^2$

$= 3.1416 \times (12 \text{ cm})^2$

$= 3.1416 \times 144 \text{ cm}^2$

$= 452.39 \approx 452.4 \text{ cm}^2$

Percentage of sector area $= \frac{A_1}{A} \times 100\%$

$= \frac{75.4}{452.4} \times 100\%$

$= 16.67\%$

Q. 8 Find the percentage of the area of sector subtending an angle $\frac{\pi}{8}$ radians.

Solution:

Angle of sector $= \theta = \frac{\pi}{8} \text{rad}$

$= \frac{\pi}{8} \text{rad}.$

Let radius of circle be r units

$= \pi r^2$ square unit

Now area of sector $= \frac{1}{2} r^2 0$

$= \frac{1}{2} r^2 \times \frac{\pi}{8}$ square unit

$= \frac{\pi}{16} r^2$ square unit

Area of circle

Percentage $= \frac{\text{Area of sector}}{\text{Area of circle}} \times 100\%$

$= \frac{\pi}{16} r^2 \div \pi r^2 \times 100\%$

$= \frac{\pi}{16} r^2 \times \frac{1}{\pi r^2} \times 100\%$

$= \frac{1}{16} \times 100\% = 6.25\%$

Q. 9 A circular sector of radius $r = 12$ cm has an angle of 150°. This sector is cut out and then bent to form a cone. What is the slant height and radius of the base of cone.
cone.

Solution:

Radius of sector $= \mathrm{r} = 12$ cm

Angle of sector $= 150?$

$= 150 \times \frac{\pi}{180} \mathrm{rad}$

$= \frac{5\pi}{6} \mathrm{rad}$

(i) The radius of sector is slant height of cone $= \ell$

Slant height of cone $= \ell = 12$ cm

Also

(ii) We know that

The area of sector = curved surface area or cone

Now area of sector $= \frac{1}{2} \mathrm{r}^2 \theta$

$= \frac{1}{2} (12)^2 \frac{5\pi}{6}$

$= \frac{1}{2} \times 144 \times \frac{5\pi}{6}$

$= 60\pi \text{ cm}^2$

Let radius of cone $= \mathrm{R} = ?$

Curved surface area of cone $= \pi \mathrm{R} \ell$

$\because$ using eq. (i)

$\Rightarrow \pi \mathrm{R} \ell = 60\pi$

$$R = \frac{60\pi}{\hbar \ell}$$

$$R = \frac{60}{12} \quad (\because \ell = 12 \text{ cm})$$

$$R = 5 \text{ cm}$$