9^{th} Math Exercise 6.3

Q. 1 If θ lies in first quadrant, find the remaining trigonometric ratios of θ.

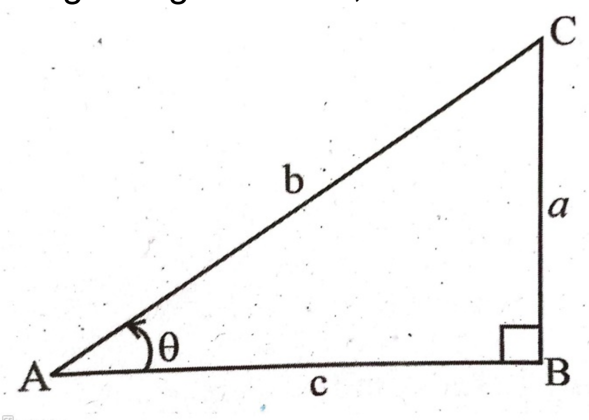
(i) $\sin\,\theta = \frac{2}{3}$

Solution:

$$\sin\,\theta = \frac{2}{3}$$

Let $\triangle ABC$ be a right angled triangle in which $\mathrm{m}\angle\mathrm{B} = 90^\circ$, and $\mathrm{m}\angle\mathrm{A} = \theta$

In right-angled $\triangle\mathrm{ABC}$,



$\sin\,\theta = \frac{a}{b}$

From (i) and (ii) $\frac{a}{b} = \frac{2}{3} \Rightarrow a = 2, b = 3$

By Pythagoras, theorem

$b^2 = a^2 + c^2$

$(3)^2 = (2)^2 + c^2$

$9 = 4 + c^2$

$9 - 4 = c^2$

$5 = c^2$

$\sqrt{c^2} = \sqrt{5}$

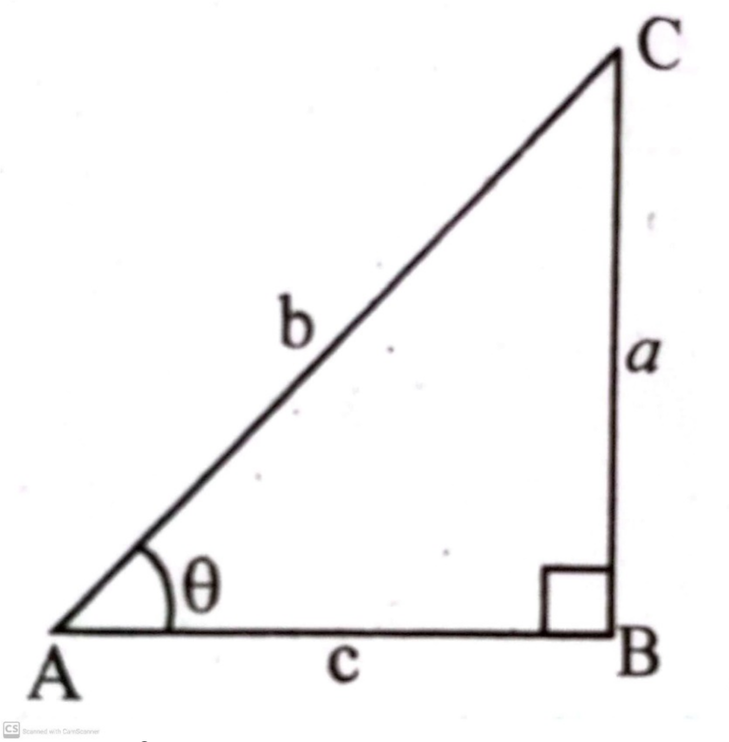
$c = \sqrt{5}$

Remaining ratios:

$$\cos\ \theta = \frac{c}{b} = \frac{\sqrt{5}}{3}$$
$$\tan\ \theta = \frac{a}{c} = \frac{2}{\sqrt{5}}$$
$$\operatorname{cosec}\theta = \frac{b}{a} = \frac{3}{2}$$
$$\sec\ \theta = \frac{b}{c} = \frac{3}{\sqrt{5}}$$
$$\cot = \frac{c}{a} = \frac{\sqrt{5}}{2}$$

(ii) $\cos\ \theta = \frac{3}{4}$

Let $\triangle ABC$ be a right angled triangle is which $\mathrm{m}\angle\mathrm{B} = 90^\circ, \mathrm{m}\angle\mathrm{A} = \theta$, In $\triangle$ ABC



$$\cos\ \theta = \frac{c}{b}$$

from (i) and (ii)

$$\frac{c}{b} = \frac{3}{4}$$
$$c = 3, b = 4$$

By pythagoras theorem

$$b^2 = c^2 + a^2$$
$$4^2 = 3^2 + a^2$$
$$16 = 9 + a^2$$
$$16 = 9 = a^2$$
$$a^2 = 7$$
$$\sqrt{a^2} = \sqrt{7}$$
$$a = \sqrt{7}$$

Remaining ratios

$$\sin\ \theta = \frac{a}{b} = \frac{\sqrt{7}}{4}$$

$$\tan\ \theta = \frac{a}{c} = \frac{\sqrt{7}}{3}$$

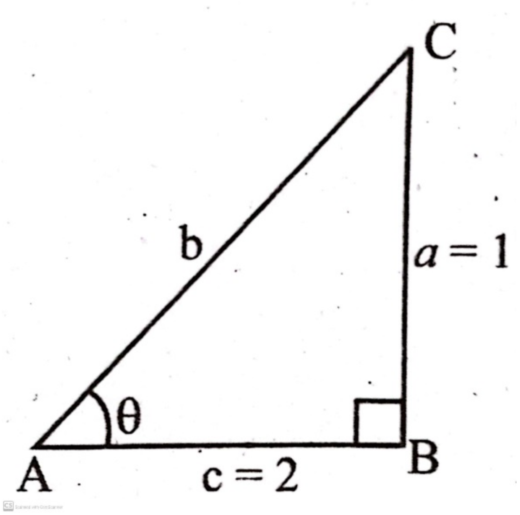
$$\text{cosec}\theta = \frac{b}{a} = \frac{4}{\sqrt{7}}$$

$$\sec\ \theta = \frac{b}{c} = \frac{4}{3}$$

$$\cot\ \theta = \frac{c}{a} = \frac{3}{\sqrt{7}}$$

(iii) $\tan\ \theta = \frac{1}{2}$

Solution:



$$\tan\ \theta = \frac{1}{2}$$

Let $\triangle \mathrm{ABC}$ is a right - angled triangle in which $\mathrm{m}\angle \mathrm{B} = 90^{\circ}, \mathrm{m}\angle \mathrm{A} = \theta$

In $\triangle \mathrm{ABC}, \tan\,\theta = \frac{a}{\mathrm{c}} =$

By comparing (i) & (ii)

$$\frac{a}{c} = \frac{1}{2}$$

$$\Rightarrow a = 1, c = 2$$

By Pythagoras theorem

$$b^2 = c^2 + a^2$$

$$b^2 = 2^2 + 1^2$$

$$b^2 = 4 + 1$$

$$b^2 = 5$$

$$\sqrt{b^2} = \sqrt{5}$$

$$\Rightarrow b = \sqrt{5}$$

Remaining Ratios

$$\sin\,\theta = \frac{a}{b} = \frac{1}{\sqrt{5}}$$

$$\cos\,\theta = \frac{c}{b} = \frac{2}{\sqrt{5}}$$

$$\mathrm{cosec}\theta = \frac{b}{a} = \frac{\sqrt{5}}{1} = \sqrt{5}$$

$$\sec\,\theta = \frac{b}{c} = \frac{\sqrt{5}}{2}$$

$$\cot\,\theta = \frac{c}{a} = \frac{2}{1} = 2$$

sind $= \frac{a}{b} = \frac{1}{\sqrt{5}}$

cosect $\theta = \frac{b}{b} = \frac{\sqrt{5}}{1} = \sqrt{5}$

sect $= \frac{b}{c} = \frac{\sqrt{5}}{2}$

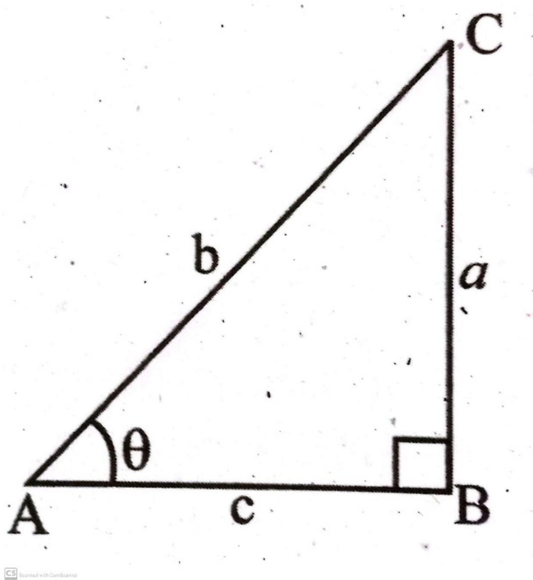
(iv) $\sec\ \theta = 3$

Solution:

$\sec\ \theta = 3$

$\sec\ \theta = \frac{3}{1}$

Let $\triangle ABC$ be a right-angled triangle in which $\mathrm{m}\angle\mathrm{B} = 90^{\circ}, \mathrm{m}\angle\mathrm{A}\theta$ In $\triangle$ ABC,



$$\sec\ \theta = \frac{b}{c}$$

From (i) and (ii)

$$\frac{b}{c} = \frac{3}{1}$$

$$\Rightarrow b = 3, c = 1$$

By Pythagoras theorem

$$b^2 = a^2 + c^2$$
$$3^2 = a^2 + (i)^2$$
$$9 = a^2 + 1$$
$$9 - 1 = a^2$$
$$8 = a^2$$
$$\sqrt{a^2} = \sqrt{8}$$
$$a = \sqrt{4 \times 2}$$
$$a = 2\sqrt{2}$$

Remaining Ratios

$$\sin\ \theta = \frac{a}{b} = \frac{2\sqrt{2}}{3}$$

$$\cos\ \theta = \frac{c}{b} = \frac{1}{3}$$

$$\tan\ \theta = \frac{a}{c} = \frac{2\sqrt{2}}{1} = 2\sqrt{2}$$

$$\text{cosec}\theta = \frac{b}{a} = \frac{3}{2\sqrt{2}} = \frac{3}{2\sqrt{2}}$$

$$\cot\ \theta = \frac{c}{a} = \frac{1}{2\sqrt{2}}$$

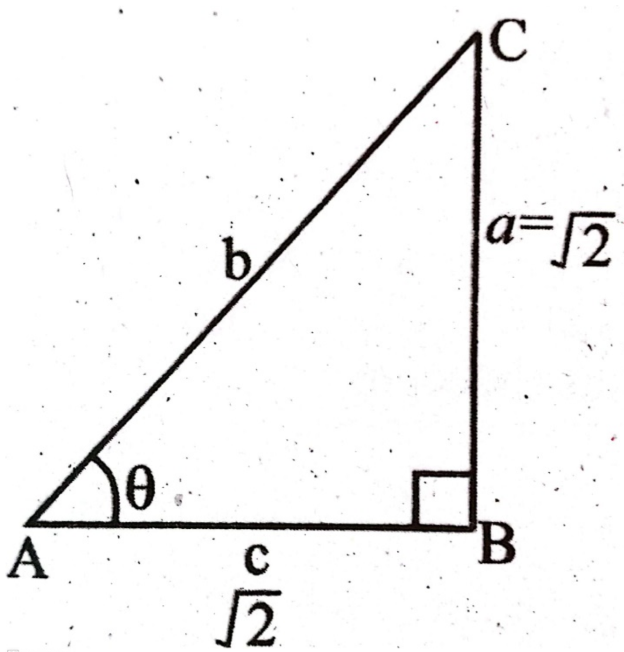
(v) $\cot\ \theta = \sqrt{\frac{3}{2}}$

Solution:

$$\cot\ \theta = \sqrt{\frac{3}{2}}$$

$$\cot\ \theta = \frac{\sqrt{3}}{\sqrt{2}}$$

Let $\triangle \mathrm{ABC}$ is a right angled in which $= 90^\circ \mathrm{m}\angle \mathrm{B} = \theta$



In $\triangle ABC$

$\cot\,\theta = \frac{c}{a}$

By comparing (i) and (iii)

$$\frac{c}{a} = \frac{\sqrt{3}}{\sqrt{2}}$$

$$\Rightarrow \begin{array}{l} c = \sqrt{3} \\ a = \sqrt{2} \end{array}$$

By Pythagoras theorem

$$b^2 = c^2 + a^2$$

$$b^2 = (\sqrt{3})^2 + (\sqrt{2})^2$$

$$b^2 = 3 + 2 = 5$$

$$\sqrt{b^2} = \sqrt{5}$$

$$h = \sqrt{5}$$

$a = \sqrt{2}$

By Pythagoras theorem

$b^2 = c^2 + a^2$

$b^2 = (\sqrt{3})^2 + (\sqrt{2})^2$

$b^2 = 3 + 2 = 5$

$\sqrt{b^2} = \sqrt{5}$

$b = \sqrt{5}$

Remaining Ratios

$\sin\ \theta = \frac{a}{b} = \frac{\sqrt{2}}{\sqrt{5}} = \sqrt{\frac{2}{5}}$

$\cos\ \theta = \frac{c}{b} = \frac{\sqrt{3}}{\sqrt{5}} = \sqrt{\frac{3}{5}}$

$\tan\ \theta = \frac{a}{c} = \frac{\sqrt{2}}{\sqrt{b}} = \sqrt{\frac{2}{3}}$

$\text{cosec}\theta = \frac{b}{a} = \frac{\sqrt{5}}{\sqrt{2}} = \sqrt{\frac{5}{2}}$

sec $\theta = \frac{b}{c} = \frac{\sqrt{5}}{\sqrt{3}} = \sqrt{\frac{5}{3}}$

prove the following trigonometric identities:

Q. 2 $(\sin\ \theta + \cos\ \theta)^2 = 1 + 2\sin\ \theta\cos\ \theta$

$(\sin\ \theta + \cos\ \theta)^2 = 1 + 2\sin\ \theta\cos\ \theta$

L.H.S $= (\sin\ \theta + \cos\ \theta)^2\ \left[\because (a+b)^2 = a^2 + b^2 + 2ab\right]$

$= (\sin\ \theta)^2 + (\cos\ \theta)^2 + 2\sin\ \theta\cos\ \theta$

$= \sin^2\ \theta + \cos^2\ \theta + 2\sin\ \theta\cos\ \theta$

$= 1 + 2\sin\ \theta\cos\ \theta$

$=$ R.H.S $\left[\because \sin^2\ \theta + \cos^2\ \theta = 1\right]$

Hence proved

Q. 3 $\frac{\cos\ \theta}{\sin\ \theta} = \frac{1}{\tan\ \theta}$

Solution:

$$\frac{\cos\ \theta}{\sin\ \theta} = \frac{1}{\tan\ \theta}$$

$$\text{L.H.S} = \frac{\cos\ \theta}{\sin\ \theta}$$

$$= \cot\ \theta$$

$$= \frac{1}{\tan\ \theta}$$

I.H.S = R.H.S

$$= \frac{\cot\ \theta}{\frac{1}{\tan\ \theta}}$$

LHS=RH.S

Q. $4\frac{\sin\ \theta}{\text{cosec}\theta} + \cos\ \theta$

$$\frac{\sin\ \theta}{\text{cosec}\theta} + \frac{\cos\ \theta}{\sec\ \theta} - 1$$

$$\text{L.H.S} = \frac{\sin\ \theta}{\text{cosec}\theta} + \frac{\cos\ \theta}{\sec\ \theta}$$

$$= \sin\ \theta \cdot \frac{1}{\text{cose}\theta} + \cos\ \theta \cdot \frac{1}{\text{Sec}\theta}$$

$$= \sin\ \theta \cdot \sin\ \theta \cdot + \cos\ \theta \cdot \cos\ \theta$$

$$= \sin^2\ \theta + \cos^2\ \theta$$

$$= 1$$

$= \text{ R.H.S } \left[\because \sin^2\ \theta + \cos^2\ \theta = 1\right]$

Hence proved

Q. $5\cos^2\ \theta - \sin^2\ \theta = 2\cos^2\ \theta - 1$ оэцасаво

Solution:

$\cos^2\theta - \sin^2\theta = 2\cos^2\theta$

L.H.S $= \cos^2\theta - \sin^2\theta$

$= \cos^2\theta - \left(1 - \cos^2\theta\right)$

$= \cos^2\theta - 1 + \cos^2\theta$

$= 2\cos^2\theta - 1$

L.H.S = R.H.S

Q. $\sigma \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta$

$\cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta$

L.H.S $= \cos^2\theta - \sin^2\theta$

$= 1 - \sin^2\theta - \sin^2\theta \left[\because \sin^2\theta + \cos^2\theta = 1\right]$

$= 1 - 2\sin\theta$

L.H.S

Q. 7 $\frac{1-\sin\theta}{\cos\theta} = \frac{\cos\theta}{1+\sin\theta}$

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Solution:

$$\frac{1-\sin\theta}{\cos\theta} = \frac{\cos\theta}{1+\sin\theta}$$

L.H.S $= \dfrac{1-\sin\theta}{\cos\theta}$

$$= \frac{1-\sin\theta}{\cos\theta} \times \frac{1+\sin\theta}{1+\sin\theta}$$

$$= \frac{(1)^2 - (\sin\theta)^2}{\cos\theta(1+\sin\theta)}$$

$$= \frac{1-\sin^2\theta}{\cos\theta(1+\sin\theta)}$$

$$= \frac{\cos^2\theta}{\cos\theta(1+\sin\theta)} \left[\because \sin^2\theta + \cos^2\theta = 1\right]$$

$$= \frac{\cos\theta}{1+\sin\theta}$$

L.H.S = R.H.S

Hence proved

Q. $8(\sec\theta - \tan\theta)^2 = \dfrac{1-\sin\theta}{1+\sin\theta}$

Solution：

$$(\sec\theta-\tan\theta)^2=\frac{1-\sin\theta}{1+\sin\theta}$$

L.H.S $=(\sec\theta-\tan\theta)^2$

$$=\left[\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}\right]^2 \quad \because \sec\theta=\frac{1}{\cos\theta}$$

$$=\left[\frac{1-\sin\theta}{\cos\theta}\right]^2 \quad \left(\because \frac{\sin\theta}{\cos\theta}=\tan\theta\right)$$

$$=\frac{(1-\sin\theta)^2}{\cos^2\theta}$$

$$=\frac{(1-\sin\theta)^2}{1-\sin^2\theta} \quad \left(\because \cos^2\theta=1-\sin^2\theta\right)$$

$$=\frac{(1-\sin\theta)^2}{(1)^2-(\sin\theta)^2} \quad \left[\because a^2-b^2=(a+b)(a-b)\right]$$

$$=\frac{(1-\sin\theta)(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$$

$=\frac{1-\sin\theta}{1+\sin\theta}=$ R.H.S

Hence proved.

Q. 9 $(\tan\theta+\cot\theta)^2=\sec^2\theta\operatorname{cosec}^2\theta$

Solution:

$$(\tan\theta+\cot\theta)^2=\sec^2\theta\operatorname{cosec}^2\theta$$

L.H.S $=(\tan\theta+\cot\theta)^2$

$$=\left[\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}\right]^2$$

$$=\left[\frac{\sin^2\theta+\cos^2\theta}{\cos\theta\sin\theta}\right]^2$$

$$=\left(\frac{1}{\cos\theta\sin\theta}\right)^2$$

$$=(\sec\theta\cdot\operatorname{cosec}\theta)^2$$

$$=\sec^2\theta\cdot\operatorname{coseccos}^2\theta$$

$=$ R.H.S

Hence proved

Q. 10 $\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}=\tan\theta+\sec\theta$ 09306085

Solution:

L.H.S $= \frac{\tan\ \theta + \sec\ \theta - 1}{\tan\ \theta - \sec\ \theta + 1}$

$$= \frac{(\tan\ \theta + \sec\ \theta) - (1)}{\tan\ \theta - \sec\ \theta + 1}$$

$$\left| \begin{array}{l} \because 1 + \tan^2\ \theta - \sec^2\ \theta \\ 1 = \sec^2\ \theta - \tan^2\ \theta \end{array} \right|$$

$$= \frac{}{(\tan\ \theta + \sec\ \theta) - \left(\sec^2\ \theta - \tan^2\ \theta\right)}$$

$$\therefore \text{ L.H.S } = \frac{(\tan\ \theta - \sec\ \theta + 1}{(\tan\ \theta + \sec\ \theta) - (\sec\ \theta + \tan\ \theta)(\sec\ \theta - \tan\ \theta)}$$

$$= \frac{(\tan\ \theta + \sec\ \theta)[1 - (\sec\ \theta - \tan\ \theta)]}{(\tan\ \theta - \sec\ \theta + 1)}$$

$$= \frac{(\tan\ \theta + \sec\ \theta)[1 - \sec\ \theta + \tan\ \theta]}{(\tan\ \theta - \sec\ \theta + 1)}$$

$$= \frac{(\tan\ \theta + \sec\ \theta)[\tan\ \theta - \sec\ \theta + 1]}{(\tan\ \theta - \sec\ \theta + 1)}$$

L.H.S $= \tan\ \theta + \sec\ \theta$

L.H.S = R.H.S

Hence proved.

Q. 11 $\sin^3\ \theta - \cos^3\ \theta = (\sin\ \theta - \cos\ \theta)(1 + \sin\ \theta\cos\ \theta)$

Solution:

$\sin^3\ \theta - \cos^3\ \theta = (\sin\ \theta - \cos\ \theta)(1 + \sin\ \theta\cos\ \theta)$

L.H.S $= \sin^3\ \theta - \cos^3\ \theta$

$\because a^3 - b^3 = (a - b)\left(a^2 + ab + b^2\right)$

$= (\sin\ \theta - \cos\ \theta)\left[\sin^2\ \theta + \cos^2\ \theta + \sin\ \theta\right.$

$= (\sin\ \theta - \cos\ \theta)(1 + \sin\ \theta \cdot \cos\ \theta)$

L.H.S = R.H.S

Q. 12 $\sin^6\ \theta - \cos^6\ \theta$

$= \left(\sin^2\ \theta - \cos^2\ \theta\right)\left(1 - \sin^2\ \theta\cos^2\ \theta\right)$

$\sin^6\theta - \cos^6\theta = \left(\sin^2\theta - \cos^2\theta\right)\left(1 - \sin^2\theta\cos^2\theta\right)$

L.H.S $= \sin^6\theta - \cos^6\theta$

$= \left(\sin^2\theta\right)^3 - \left(\cos^2\theta\right)^3$

$\because a^3 - b^3 = (a-b)\left(a^2 + ab + b^2\right)$

$= \left(\sin^2\theta - \cos^2\theta\right)$

$\left[\left(\sin^2\theta\right)^2 + \left(\cos^2\theta\right)^2 + \sin^2\theta \cdot \cos^2\theta\right]$

$= \left(\sin^2\theta - \cos^2\theta\right)$

$\left[\left(\sin^2\theta\right)^2 + \left(\cos^2\theta\right)^2 + 2\sin^2\theta \cdot \cos^2\theta - \sin^2\theta\right.$

$\left.\cos^2\theta\right]$

$= \left(\sin^2\theta - \cos^2\theta\right)\left[\left(\sin^2\theta + \cos^2\theta\right]^2 - \sin^2\theta\cos^2\theta\right]$

$= \left(\sin^2\theta - \cos^2\theta\right)\left[(1)^2 - \sin^2\theta \cdot \cos^2\theta\right]$

$= \left(\sin^2\theta - \cos^2\theta\right)\left(1 - \sin^2\theta \cdot \cos^2\theta\right)$

$=$ R.H.S

Hence proved.