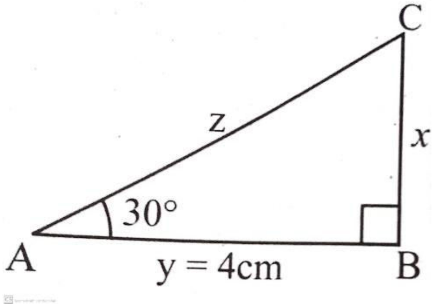
# $9^{th}$ Math Exercise 6.5

Q. 1 Find the values of $x, y$ and $z$ from the following right angled triangles.

Solution

From the figure in right angle $\triangle ABC$




$$m\angle B = 90^\circ$$
$$m\angle A = \theta = 30^\circ$$
$$y = 4 \text{ cm}$$
$$x = ? \text{ and } z = ?$$

Using $\tan\theta = \dfrac{\text{Perp}}{\text{Base}}$

$$\tan 30^\circ = \frac{x}{y}$$
$$\Rightarrow x = y \tan 30^\circ$$
$$x = 4 \times \frac{1}{\sqrt{3}}$$
$$x = \frac{4}{\sqrt{3}} \text{ cm}$$

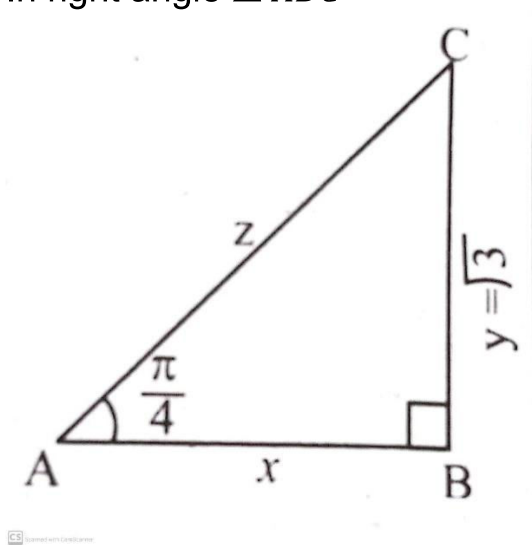
Now,

$$\cos\theta = \frac{\text{Base}}{\text{Hyp.}}$$
$$\cos 30^\circ = \frac{y}{z}$$
$$\frac{\sqrt{3}}{2} = \frac{4}{z} \left(\cos 30^\circ = \frac{\sqrt{3}}{2}\right)$$
$$\Rightarrow z\sqrt{3} = 2 \times 4$$
$$\Rightarrow z\sqrt{3} = 8$$
$$\Rightarrow z = \frac{8}{\sqrt{3}} \text{ cm}$$

(ii)

Solution:

In right angle $\triangle ABC$




$m\angle B = 90^\circ$

$m\angle A = \frac{\pi}{4}$

or

By using

$$\tan\ \theta = \frac{\text{Perp}}{\text{Base}}$$

$$\tan \frac{\pi}{4} = \frac{y}{x}$$

$$\tan\ 45^\circ = \frac{y}{x}$$

$$\Rightarrow 1 = \frac{\sqrt{3}\ \text{cm}}{x} \left(\because \frac{\pi}{4} = 45^\circ\right)$$

$$x = \sqrt{3}\ \text{cm}$$

now, $\sin\ \theta = \frac{\text{Per.}}{\text{Hyp.}}$

$$\sin \frac{\pi}{4} = \frac{y}{z}$$

$$\sin\ 45^\circ = \frac{y}{z}$$

$$\frac{1}{\sqrt{2}} = \frac{\sqrt{3}}{z}$$

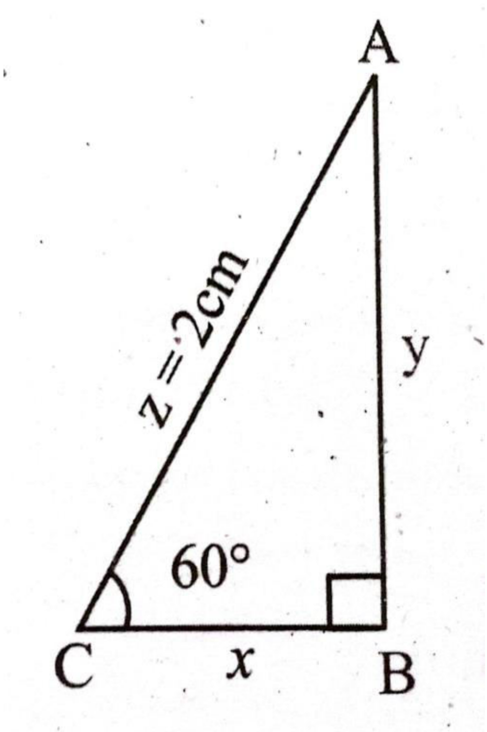
$$\Rightarrow z = \sqrt{2} \times \sqrt{3}$$

$$\Rightarrow z = \sqrt{2 \times 3}$$

$$\Rightarrow z = \sqrt{6}\ \text{cm}$$

(iii)

Solution:
In right angle $\triangle ABC$






$m\angle B = 90^\circ$
$m\angle C = 60^\circ$
$z = 2$ cm
$x = ?$ and $y = ?$

Using $\cos\,\theta = \frac{\text{Base}}{\text{Hyp.}}$

$$\cos\,60^\circ = \frac{x}{2}$$
$$\Rightarrow\; x = z\cos\,60^\circ$$
$$\Rightarrow\; x = 2 \times \frac{1}{2} \qquad \cos\,60^\circ = \frac{1}{2}$$
$$x = 1\text{ cm}$$

now, $\sin\,\theta = \frac{\text{Perp}}{\text{Hyp}}$

$$\sin\,60^\circ = \frac{y}{z}$$

$$\Rightarrow \quad y = z\sin\ 60^\circ$$

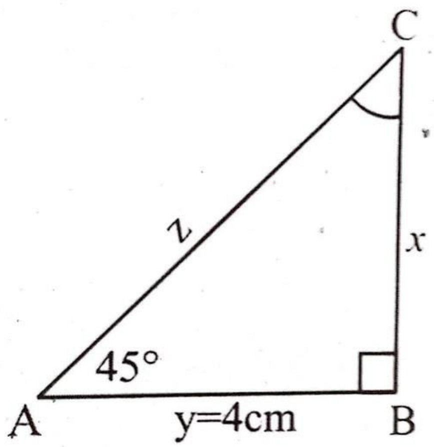
$$\Rightarrow \quad y = 2 \times \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad y = \sqrt{3}\text{ cm}$$

(iv)

Solution:

In right angled $\triangle ABC$.




$$\cos\ 45^\circ = \frac{y}{z}$$

$$\frac{1}{\sqrt{2}} = \frac{4}{z}$$

$$\Rightarrow z = 4\sqrt{2}\text{ cm}$$



In right-angled $\triangle ABC$

$$\tan\ 45^\circ = \frac{x}{y}$$

$$1 = \frac{x}{4}$$

$$\Rightarrow 4 = x$$

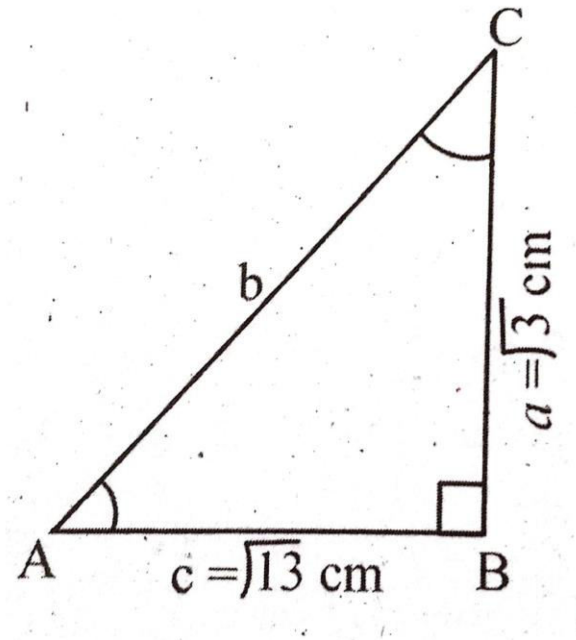
$$\Rightarrow x = 4\text{ cm}$$

Q. 2 Find the unknown side and angles of the following triangles.

(i)

Solution:

In right angled $\triangle ABC$




In right angled $\triangle \mathrm{ABC}$,
by Pythagoras theorem

$$b^2 = c^2 + a^2$$
$$b^2 = \sqrt{(13)^2} + \sqrt{(3)^2}$$
$$b^2 = 13 + 3$$
$$b^2 = 16$$
$$\sqrt{b^2} = \sqrt{16}$$
$$\Rightarrow 6 = 4 \text{ cm}$$



In right $\triangle ABC$,

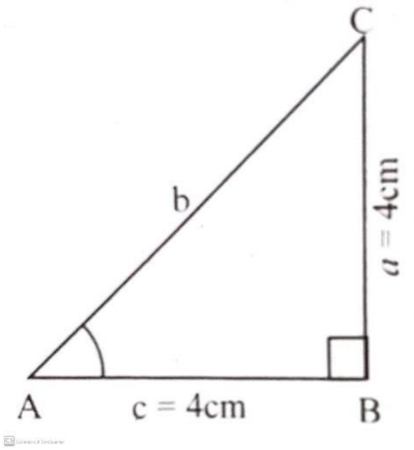
$$\tan\ m\angle\ \mathrm{A} = \frac{a}{\mathrm{c}}$$
$$\tan\ m\angle\ \mathrm{A} = \frac{\sqrt{3}}{\sqrt{13}}$$
$$m\angle\ \mathrm{A} = \tan^{-1}\left(\frac{\sqrt{3}}{\sqrt{13}}\right) = 25.658^\circ$$
$$m\angle\ \mathrm{A} = 25.7^\circ$$

In right-angled $\triangle ABC$,

$$m\angle\ \mathrm{A} + m\angle \mathrm{C} = 90^\circ$$
$$m\angle \mathrm{C} = 90^\circ - m\angle\ \mathrm{A}$$
$$m\angle \mathrm{C} = 90^\circ - 25.7^\circ$$
$$m\angle \mathrm{C} = 64.3^\circ$$

(ii)

Solution




In right angle $\triangle ABC$ by Pythgoras theorem

$$(\text{ Hyp. })^2 = (\text{ Base })^2 + (\text{ Per. })^2$$
$$b^2 = c^2 + a^2$$
$$b^2 = (4)^2 + (4)^2$$
$$b^2 = 16 + 16$$
$$b^2 = 32$$

Taking square root

$$b = \sqrt{32}$$
$$b = \sqrt{16 \times 2}$$
$$b = 4\sqrt{2} \text{ cm}$$



In right $\triangle$ ABC.

$$\tan \ m\angle \ A = \frac{a}{c} = \frac{4}{4} = 1$$
$$\tan \ \ m\angle \ A = \tan^{-1}(1)$$
$$\angle A = 45°$$

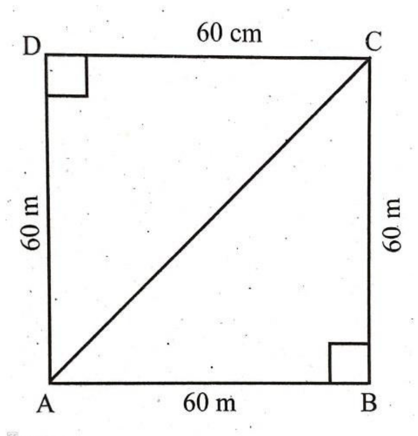
We know that in right $\triangle$ ABC

$$m\angle \ A + m\angle C = 90 = 90°$$
$$45° + m\angle C = 90°$$
$$m\angle \ A = 90° - 45° = 45$$

Q. 3 Each side of a square field is 60 m long. Find the lengths of the diagonals of the field.

Solution:

Let ABCD be the square filed with each side 60 m long. Let AC is the one of diagonals.




Applying Pythagora's theorem in right angle $\triangle$ ABC.

$$(\text{ Hyp. })^2 = (\text{ Base })^2 + (\text{ Per. })^2$$
$$(\overline{AC})^2 = (60)^2 + (60)^2$$
$$(\overline{AC})^2 = 3600 + 3600$$
$$= 7200$$

Taking square root

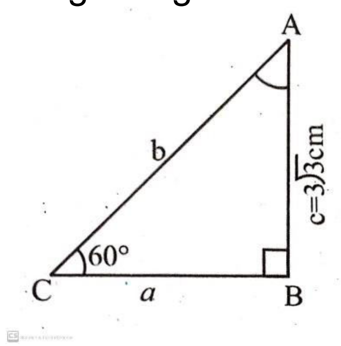
$$(\overline{AC})^2 = \sqrt{7200}$$
$$\overline{AC} = \sqrt{3600 \times 2}$$
$$\overline{AC} = 60\sqrt{2} \text{ m}$$



as length each diagonal of square is same.
So length of each diagonal is $60\sqrt{2}$ m
Q. 4 Solve the following triangles when $m\angle B = 90^\circ$ :
(i) $m\angle C = 60^\circ, c = 3\sqrt{3}$ cm
Solution

$$m\angle C = 60^\circ . c = 3\sqrt{3} \text{ cm}$$

In right angle $\triangle$ ABC




$m\angle$ B $= 90^\circ$ and
$m\angle C = 60^\circ$
$c = 3\sqrt{3}$ cm
$a = ?$ b $= ?$

and $m\angle A = ?$

As

$$m\angle A + m\angle B + m\angle C = 180$$
$$m\angle A + 90^\circ + 60^\circ = 180^\circ$$
$$m\angle A + 150^\circ = 180^\circ$$
$$m\angle A = 180^\circ - 150^\circ = 30^\circ$$
$$m\angle A = 30^\circ$$

Now, using

$$\tan\ \theta = \frac{\text{Perp.}}{\text{Base}}$$
$$\tan\ 60^\circ = \frac{c}{a}$$
$$\sqrt{3} = \frac{3\sqrt{3}}{a}\ (\because \tan\ 60^\circ = \sqrt{3})$$
$$\Rightarrow \sqrt{3}a = 3\sqrt{3}$$
$$a = \frac{3\sqrt{3}}{\sqrt{3}}$$
$$a = 3 \text{ cm}$$



Now, $\sin\theta = \frac{\text{Perp.}}{\text{Hyp.}}$

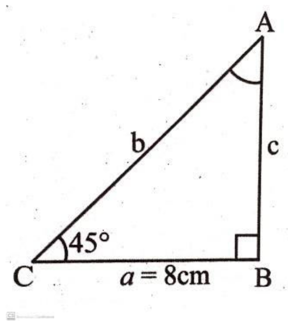
$$\sin\ 60^\circ = \frac{c}{b}$$
$$\frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{b}\left(\because \sin\ 60^\circ = \frac{\sqrt{3}}{2}\right)$$
$$\Rightarrow \sqrt{3}\ b = 2 \times 3\sqrt{3}$$
$$\Rightarrow \ b = \frac{6\sqrt{3}}{\sqrt{3}}$$
$$b = 6 \text{ cm}$$

(ii) $m\angle C = 45^\circ, a = 8 \text{ cm}$

Solution

$$m\angle C = 45^\circ, a = 8 \text{ cm}$$

In right angle $\triangle ABC$




$$\text{m}\angle\ \text{B} = 90^\circ \text{ and } \text{m}\angle\text{C} = 45^\circ$$
$$a = 8\ \text{cm}$$
$$\text{m}\angle\ \text{A} = ?$$
$$\text{b} = ? \text{ and } \text{c} = ?$$

We know that in a $\triangle\ \text{ABC}$

$$\text{m}\angle\ \text{A} + \text{m}\angle\ \text{B} + \text{m}\angle\text{C} = 180^\circ$$
$$\text{m}\angle\ \text{A} + 90^\circ + 45^\circ = 180^\circ$$
$$\text{m}\angle\ \text{A} + 135^\circ = 180$$
$$\text{m}\angle\ \text{B} = 180^\circ - 135^\circ$$
$$\text{m}\angle\ \text{A} = 45^\circ$$

Now, using

$$\tan\ \theta = \frac{\text{Perp.}}{\text{Base}}$$
$$\tan\ 45^\circ = \frac{c}{a} \qquad (\because \tan\ 45^\circ = 1)$$
$$\Rightarrow 1 = \frac{c}{8\ \text{cm}}$$
$$\Rightarrow c = 8\ \text{cm}$$

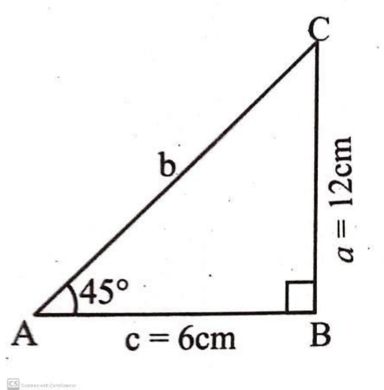
Now, $\cos\ \theta = \dfrac{\text{Base}}{\text{Hyp.}}$

$$\cos\ 45^\circ = \frac{a}{b}$$
$$\frac{1}{\sqrt{2}} = \frac{8\ \text{cm}}{b} \qquad \left(\because \cos\ 45^\circ = \frac{1}{\sqrt{2}}\right)$$
$$\Rightarrow b = 8\sqrt{2}\ \text{cm}$$

(iii) $a = 12\ \text{cm}, \text{c} = 6\ \text{cm}$

Solution:

$$m\angle\ \text{B} = 90^\circ, a = 12\ \text{cm}, \text{c} = 6\ \text{cm}$$




In right angled $\triangle \mathrm{ABC}$ by Pythagoras theorem

$$b^2 = c^2 + a^2$$
$$b^2 = (6)^2 + (2)^2$$
$$b^2 = 36 + 144$$
$$b^2 = 180$$
$$\sqrt{b} = \sqrt{180} = \sqrt{36 \times 5}$$
$$b = 6\sqrt{5}\ \mathrm{cm}$$

In right angled $\triangle \mathrm{ABC}$,

$$\tan\ \mathrm{m}\angle\ \mathrm{A} = \frac{a}{c}$$
$$\tan\ \mathrm{m}\angle\ \mathrm{A} = \frac{12}{6} = 2$$
$$\Rightarrow \mathrm{m}\angle\ \mathrm{A} = \tan^{-1}(2)$$
$$\mathrm{m}\angle\ \mathrm{A} = 63.4^\circ$$

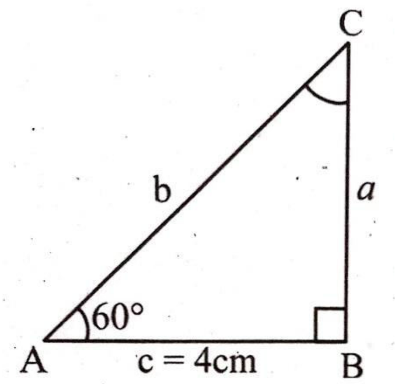
In right-angled $\triangle ABC$,

$$\mathrm{m}\angle\ \mathrm{A} + \mathrm{m}\angle\mathrm{C} = 90^\circ$$
$$\mathrm{m}\angle\mathrm{C} = 90^\circ - \mathrm{m}\angle\ \mathrm{A}$$
$$\mathrm{m}\angle\mathrm{C} = 90^\circ - 63.43^\circ$$
$$\mathrm{m}\angle\mathrm{C} = 26.6^\circ$$

(iv) $m\angle A = 60^\circ, c = 4\ \mathrm{cm}$

Solution

In right angle $\triangle \mathrm{ABC}$




$m\angle B = 90°$ and $m\angle A = 60°$
c $= 4$ cm
$\mathrm{m}\angle\mathrm{C} = ?$
$a = ?$ and $\mathrm{b} = ?$
We know that

As $\mathrm{m}\angle\ \mathrm{A} + \mathrm{m}\angle\ \mathrm{B} + \mathrm{m}\angle\mathrm{C} = 180°$

$$60° + 90° + \mathrm{m}\angle\mathrm{C} = 180°$$

$$150° + \mathrm{m}\angle\mathrm{C} = 180°$$

$$\mathrm{m}\angle\mathrm{C} = 180° - 150°$$

$$\mathrm{m}\angle\mathrm{C} = 30°$$

Now using

$$\tan\ \theta = \frac{\text{Perp.}}{\text{Base}}$$

$$\Rightarrow \tan\ 60° = \frac{a}{\mathrm{c}} \left(\because \tan\ 60° = \sqrt{3}\right)$$

$$\Rightarrow \sqrt{3} = \frac{a}{4}$$

$$a = 4\sqrt{3}\ \mathrm{cm}$$

Now $\cos\ \theta = \frac{\text{Base}}{\text{Hyp.}}$

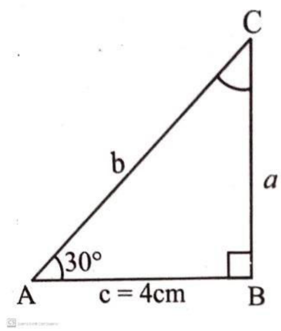
$$\cos\ 60° = \frac{c}{b}$$

$$\frac{1}{2} = \frac{4}{b}$$

$$\Rightarrow b \times 1 = 4 \times 2\ \left(\because \cos\ 60° = \frac{1}{2}\right)$$

$$b = 8\ \mathrm{cm}$$

(v) $m\angle A = 30°, c = 4$ cm
Solution:
In right angle $\triangle\ \mathrm{ABC}$




$m\angle B = 90^\circ$ and $m\angle A = 30^\circ$
c $= 4$ cm
$\mathrm{m}\angle \mathrm{C} = ?$
$a = ?$ and $\mathrm{b} = ?$
We know that

$$\mathrm{m}\angle\ \mathrm{A} + \mathrm{m}\angle\ \mathrm{B} + \mathrm{m}\angle \mathrm{C} = 180^\circ$$
$$30^\circ + 90^\circ + \mathrm{m}\angle \mathrm{C} = 180^\circ$$
$$120^\circ + \mathrm{m}\angle \mathrm{C} = 180^\circ$$
$$\mathrm{m}\angle \mathrm{C} = 180^\circ - 120^\circ$$
$$\mathrm{m}\angle \mathrm{C} = 60^\circ$$

Now using

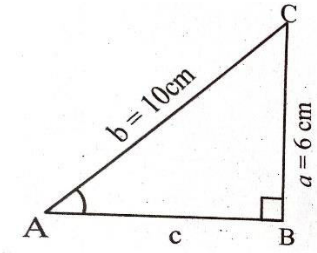


$$\tan\theta = \frac{\text{Per.}}{\text{Base}}$$
$$\tan 30^\circ = \frac{a}{\mathrm{c}}$$
$$\frac{1}{\sqrt{3}} = \frac{a}{4}$$
$$\frac{1}{\sqrt{3}} \times 4 = a$$
$$\Rightarrow a = \frac{4}{\sqrt{3}}\ \mathrm{cm}$$

Now $\cos\theta = \frac{\mathrm{B}}{\mathrm{I}}$

$$\cos 30^\circ = \frac{\mathrm{c}}{\mathrm{b}}$$
$$\frac{\sqrt{3}}{2} = \frac{4}{\mathrm{b}}$$
$$\Rightarrow \sqrt{3}\ \mathrm{b} = 2 \times 4$$
$$\sqrt{3}\ \mathrm{b} = 8$$
$$\mathrm{b} = \frac{8}{\sqrt{3}}\ \mathrm{cm}$$

(vi) $\mathrm{b}=10 \mathrm{~cm}, a=6 \mathrm{~cm}$
Solution:




In right - angled $\triangle \mathrm{ABC}$, By Pythagoras theorem

$$b^2=a^2+c^2$$
$$(10)^2=(6)^2+c^2$$
$$100=36+c^2$$
$$100-36=c^2$$
$$64=c^2$$
$$\Rightarrow c^2=64$$
$$\sqrt{c^2}=64$$
$$c=8 \mathrm{~cm}$$

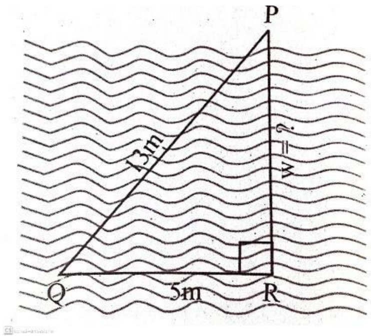
In right angled $\triangle \mathrm{ABC}$,

$$\sin \mathrm{m}\angle \mathrm{~A}=\frac{a}{\mathrm{b}}$$
$$\sin \mathrm{m}\angle \mathrm{~A}=\frac{6}{10}=\frac{3}{5}$$
$$\mathrm{m}\angle \mathrm{~A}=\sin^{-1}\left(\frac{3}{5}\right)$$
$$\mathrm{m}\angle \mathrm{~A}=36.869$$
$$\mathrm{m}\angle \mathrm{~A}=36.9^{\circ}$$

In right-angled $\triangle \mathrm{ABC}$,

$$\mathrm{m}\angle \mathrm{~A}+\mathrm{m}\angle \mathrm{C}=90^{\circ}$$
$$\mathrm{m}\angle \mathrm{C}+=90^{\circ}-\mathrm{m}\angle \mathrm{~A}$$
$$\mathrm{m}\angle \mathrm{C}+=90^{\circ}-36.9^{\circ}$$
$$\mathrm{m}\angle \mathrm{C}=53.1^{\circ}$$

Q. 5 Let $Q$ and $R$ be the two points on the same bank of a canal. The point $P$ is placed on the other bank straight to point $R$. Find the width of the canal and angle PQR in radians.
Solution




From the figure width of canal $= \overline{\mathrm{PR}} = \mathrm{w} = ?$ $\mathrm{m}\overline{\mathrm{PQ}} = 13\ \mathrm{m}$ and $\mathrm{m}\overline{\mathrm{QR}} = 5\ \mathrm{m}$

In right angle $\triangle\ \mathrm{PQR}$ By Pythagora's theorem $(m\overline{PR})^2 + (m\overline{QR})^2 = (m\overline{PQ})^2$

$$(\mathrm{m}\overline{\mathrm{PR}})^2 + (5)^2 = (13)^2$$
$$(\ \mathrm{m}\overline{\mathrm{PR}})^2 = 169 - 25$$
$$(\ \mathrm{m}\overline{\mathrm{PR}})^2 = 144$$

Taking square root of both sides

$$\mathrm{m}\overline{\mathrm{PR}} = 12\ \mathrm{cm}$$

In $\triangle\ \mathrm{PQR}$

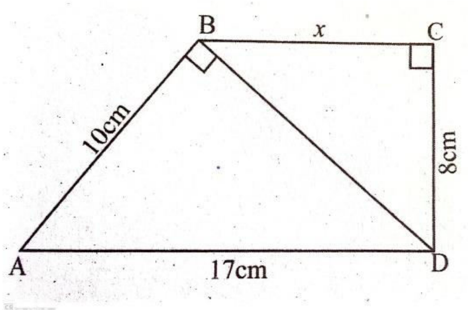


$$\cos\theta = \frac{\overline{\mathrm{QR}}}{\mathrm{QP}}$$
$$\cos\theta = \frac{5}{13}$$
$$\theta = \cos^{-1}\left(\frac{5}{13}\right)$$
$$\theta = 67.38^\circ$$
$$\theta = 67.38 \times \frac{\pi}{180}\,\mathrm{rad}$$
$$\theta = 1.8\mathrm{rad}$$

So, width of canal is $w = 12\ \mathrm{m}$

Q. 6 Calculate the length $x$ in the adjoining figure.

Solution:




In right angle $\triangle$ ABD By Pythagora's theorem.

$$(m\overline{BD})^2 + (m\overline{AB})^2 = (m\overline{AD})^2$$
$$(m\overline{BD})^2 + (10)^2 = (17)^2$$
$$(m\overline{BD})^2 + 100 = 289$$
$$(m\overline{BD})^2 = 289 - 100 = 189$$
$$\Rightarrow (m\overline{BD})^2 = 189$$

Now in $\triangle BCD$, by Pythagoras theorem

$$(\mathrm{m}\overline{\mathrm{BC}})^2 + (\mathrm{m}\overline{\mathrm{CD}})^2 = (\mathrm{m}\overline{\mathrm{BD}})^2$$
$$x^2 + (8)^2 = 189$$
$$x^2 = 189 - 64$$
$$x^2 = 125$$
$$\sqrt{x^2} = \sqrt{125} = \sqrt{25 \times 5} = 5\sqrt{5}\ \text{cm}$$

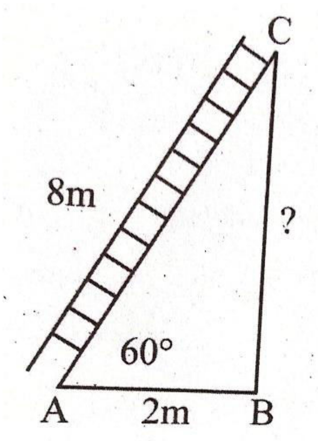
Taking square root

$$x = \sqrt{125}$$
$$= 11.18\ \text{cm}$$



Q. 7 If the ladder is placed along the wall such that the foot of the ladder is 2 m away from the wall. If the length of the ladder is 8 m , find the height of the wall?

Solution



Let length of ladder $= m\overline{AC} = 8m$

Distance of ladder from wall $= \overline{\mathrm{AB}} = 2\ \mathrm{m}$

Height of wall $= \mathrm{mBC} =$ ?

In right angle $\triangle\ \mathrm{ABC}$

By Pythagora's theorem

$$(m\overline{BC})^2 + (m\overline{AB})^2 = (m\overline{AC})^2$$

$$(m\overline{BC})^2 + (2)^2 = (8)^2$$

$$(m\overline{BC})^2 + 4 = 64$$

$$(\overline{BC})^2 = 64 - 4$$

$$(m\overline{BC})^2 = 60$$

Taking square root

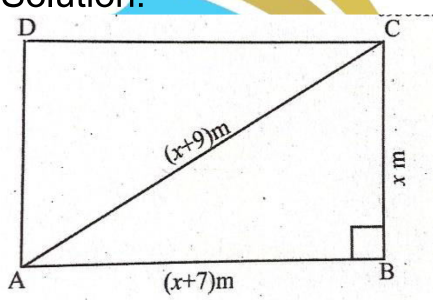
$$\sqrt{(\mathrm{mBC})^2} = \sqrt{60}$$

$$\mathrm{mBC} = 7.75\ \mathrm{m}$$

So, height of wall is 7.75 m

Q. 8 The diagomal of a rectangular field $ABCD$ is $(x+9)m$ and the sides are $(x+7)\mathrm{m}$ and $x\ \mathrm{m}$. Find the value of $x$.

Solution:






ABCD is a rectangular filed.

Diagonal $= m\overline{AC} = (x+9)\mathrm{cm}$

Sides are

$\mathrm{m}\overline{\mathrm{AB}} = (x+7)\mathrm{cm}$ and $\mathrm{m}\overline{\mathrm{BC}} = x\ \mathrm{cm}$

Applying Pythagora's theorem in right angled $\triangle\ \mathrm{ABC}$.

$$(\mathrm{m}\overline{\mathrm{AB}})^2 + (\mathrm{m}\overline{\mathrm{BC}})^2 = (\mathrm{m}\overline{\mathrm{AC}})^2$$
$$(x+7)^2 + (x)^2 = (x+9)^2$$
$$(x)^2 + 2(x)(7) + (7)^2 + x^2 = (x^2) + 2(x)(9) + (9)^2$$
$$x^2 + 14x + 49 + x^2 = x^2 + 18x + 81$$
$$2x^2 + 14x + 49 - x^2 - 18x - 81 = 0$$
$$x^2 - 4x - 32 = 0$$
$$x^2 - 8x + 4x - 32 = 0$$
$$x(x-8) + 4(x-8) = 0$$
$$(x-8)(x+4) = 0$$
$\Rightarrow x - 8 = 0$ or $x + 4 = 0$

$x = 8$ or $x = -4$

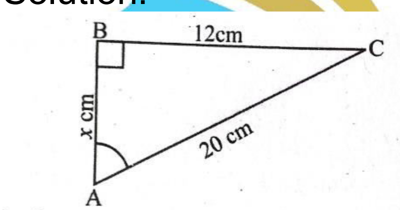
Since $x$ is a length of a side of rectangle which cannot be negative, so we ignore $x = -4$

So required values is $x = 8$ cm.

Q. 9 Calculate the value of ' $x$ '
case.

(i)

Solution:





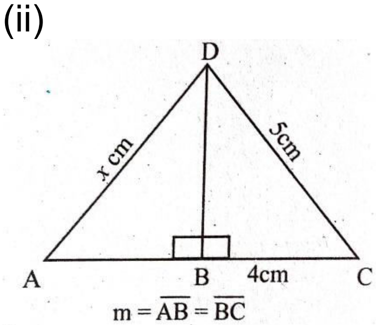

In right angle $\triangle \mathrm{ABC}$ By Pythagora's theorem

$$(\mathrm{m}\overline{\mathrm{AB}})^2 + (\mathrm{m}\overline{\mathrm{BC}})^2 = (\mathrm{m}\overline{\mathrm{AC}})^2$$
$$x^2 + (12)^2 = (20)^2$$
$$x^2 = 400 - 144$$
$$\sqrt{x^2} = \sqrt{256}$$

Taking square root

$$x = 16 \text{ cm}$$

(ii)



$$m = \overline{AB} = \overline{BC}$$

Solution

In $\triangle DBC$

$$(m\overline{BD})^2 + (m\overline{BC})^2 = (m\overline{CD})^2$$
$$(m\overline{BD})^2 + (4)^2 = (5)^2$$
$$(m\overline{BD})^2 + 16 = 25$$
$$(m\overline{BD})^2 = 25 - 16$$
$$(m\overline{BD})^2 = 9$$

Given that

$$m\overline{AB} = m\overline{BC}$$

So, $m\overline{AB} = 4\ \text{cm}$

In right angle $\triangle ABD$ By Pythagora's theorem

$$(m\overline{AD})^2 = (m\overline{AB})^2 + (m\overline{BD})^2$$
$$x^2 = (4)^2 + 9$$
$$= 16 + 9$$
$$x^2 = 25$$

Taking square root

$$\sqrt{x^2} = \sqrt{25}$$
$$x = 5\ \text{cm}$$