9^{th} Math Exercise 7.1

Q. 1 Describe the location in the plane of the point $\mathbb{P}(x, y)$, for which
(i) $x > 0$
Solution:
$x > 0$

The open right half of Cartesian plane.
(ii) $x > 0$ and $y > 0$
Solution
$x > 0$ and $y > 0$
The Set of all the points in 1^{st} quadrant
(iii) $\boldsymbol{x} = \mathbf{0}$
Solution:
$x = 0$, set of all points
on y-axis
(iv) $y = 0$
Solution:
$y = 0$, set of all points on x-axis.
x-axis
(v) $x > 0$ and $y \leq= 0$
Solution:
$x > 0$ and $y \leq 0$
set of all points in 4^{th} quadrant.
The 4^{th} quadrant including negative y -axis.
(vii) $y = 0, x = 0$
Solution:
$y = 0, x = 0$
The origin
viii) $x = y$
Solution:
$\Rightarrow x = \mathrm{y}$
It is a line bisecting the 1^{st} and 3^{rd} quadrant.
(viii) $x \geq 3$
Solution:
$x \geq 3$
The set of points lying on and right side of the line $x = 3$ in Cartesian plane.
(ix) $y > 0$
Solution

$y > 0$

Set of all the points lying above the line of x -axis.
(x) x and y have opposite signs.
Solution
The set of all the points in the 2^{nd} and 4^{th} quadrants.
Q. 2 Find the distance between the points.
(i) $\mathrm{A}(6,7), \mathrm{B}(0,-2)$
Solution
$\mathrm{A}(6,7), \mathrm{B}(0,-2)$
We know that

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
putting values
$|\mathrm{AB}| = \sqrt{(0-6)^2 + (-2-7)^2}$
$= \sqrt{(-6)^2 + (-9)^2}$
$= \sqrt{36 + 81}$
$= \sqrt{117}$
$|\mathrm{AB}| = \sqrt{9 \times 13}$
$= 3\sqrt{13}$ units
(ii) $\mathrm{C}(-5,-2), \mathrm{D}(3,2)$
Solution:
$C(-5,-2), D(3,-2)$
We know that
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
putting values
$|CD| = \sqrt{[3-(-5)]^2 + [2-(-2)]^2}$
$|CD| = \sqrt{(3+5)^2 + (2+2)^2}$
$|CD| = \sqrt{(8)^2 + (4)^2}$
$= \sqrt{64 + 16}$
$= \sqrt{80}$
$= \sqrt{16 \times 5}$
$= 4\sqrt{5}$ units

$=\sqrt{36+81}$

$= \sqrt{117}$

$|\mathrm{AB}| = \sqrt{9 \times 13}$

$= 3\sqrt{13}$ units

(ii) $\mathrm{C}(-5,-2), \mathrm{D}(3,2)$

Solution:

$C(-5,-2), D(3,-2)$

We know that

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

putting values

$|CD| = \sqrt{[3-(-5)]^2 + [2-(-2)]^2}$

$|CD| = \sqrt{(3+5)^2 + (2+2)^2}$

$|CD| = \sqrt{(8)^2 + (4)^2}$

$= \sqrt{64+16}$

$= \sqrt{80}$

$= \sqrt{16 \times 5}$

$= 4\sqrt{5}$ units

(iii) $\mathrm{L}(0,3), \mathrm{M}(-2,-4)$

Solution:

$\mathrm{L}(0,3), \mathrm{M}(-2,-4)$

We know that

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

putting values

$|\mathrm{LM}| = \sqrt{(-2-0)^2 + (-4-3)^2}$

$|\mathrm{LM}| = \sqrt{(-2)^2 + (-7)^2}$

$|\mathrm{LM}| = \sqrt{4+49}$

$|\mathrm{LM}| = \sqrt{53}$

(iv) $\mathrm{P}(-8,-7), \mathrm{Q}(0,0)$

Solution:

$\mathrm{P}(-8,-7), \mathrm{Q}(0,0)$

We know that

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

putting values

$|\mathrm{PQ}| = \sqrt{[0-(-8)]^2 + [0-(-7)]^2}$

(b) $\mathrm{A}(-8,3), \mathrm{B}(2,-1)$

Solution

$\mathrm{A}(-8,3), \mathrm{B}(2,-1)$

we know that:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|AB| = \sqrt{[2-(-8)]^2 + (-1-3)^2}$$

$$= \sqrt{(2+8)^2 + (-4)^2}$$

$$= \sqrt{(10)^2 + (-4)^2}$$

$$= \sqrt{100 + 16}$$

$$= \sqrt{116}$$

$= \sqrt{4 \times 29}$ units

$= 2\sqrt{29}$ units

(c) $\mathrm{A}\left(-\sqrt{5}, -\frac{1}{3}\right), \mathrm{B}(-3\sqrt{5}, 5)$

Solution:

$\mathrm{A}\left(-\sqrt{5}, -\frac{1}{3}\right), \mathrm{B}(-3\sqrt{5}, 5)$

We know that

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|AB| = \sqrt{[-3\sqrt{5} - (-\sqrt{5})]^2 + \left[5 - \left(-\frac{1}{3}\right)\right]^2}$$

$$= \sqrt{[-3\sqrt{5} + \sqrt{5}]^2 + \left[5 + \frac{1}{3}\right]^2}$$

$$= \sqrt{(-2\sqrt{5})^2 + \left(\frac{15+1}{3}\right)^2}$$

$$= \sqrt{(-2)^2(\sqrt{5})^2 + \left(\frac{16}{3}\right)^2}$$

$$= \sqrt{4(5) + \frac{256}{9}}$$

$$= \sqrt{20 + \frac{256}{9}}$$

$$= \sqrt{\frac{180 + 256}{9}}$$

$$= \sqrt{\frac{436}{9}}$$

$$= \sqrt{\frac{4 \times 109}{9}}$$

$$|AB| = \frac{2\sqrt{109}}{3}$$

By midpoint formula,

$$\mathrm{M}\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$
$$=\mathrm{M}\left(\frac{-8+2}{2},\frac{3+(-1)}{2}\right)$$
$$=\mathrm{M}\left(\frac{-6}{2},\frac{3-1}{2}\right)$$
$$=\mathrm{M}\left(-3,\frac{2}{2}\right)$$
$$=\mathrm{M}(-3,1)$$

(c) $\mathrm{A}\left(-\sqrt{5},-\frac{1}{3}\right),\mathrm{B}(-3\sqrt{5},5)$

Solution:

$$\mathrm{A}\left(-\sqrt{5},-\frac{1}{3}\right),\mathrm{B}(-3\sqrt{5},5)$$

By midpoint formula,

$$=\mathrm{M}\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$
$$=\mathrm{M}\left(\frac{-\sqrt{5}+(-3\sqrt{5})}{2},\frac{-\frac{1}{3}+5}{2}\right)$$
$$=M\left(\frac{-\sqrt{5}-3\sqrt{5}}{2},\frac{-1+15}{3}\right)$$
$$=M\left(\frac{-4\sqrt{5}}{2},\frac{14}{3\times 2}\right)$$
$$=M\left(-2\sqrt{5},\frac{7}{3}\right)$$

Q. 4 Which of the following points are at a distance of $\mathbf{15}$ units from the origin?

(i) $(\sqrt{176},7)$

Solution:

Given point

$(\sqrt{176},7)$, origin $\mathrm{O}(0,0)$

We know that.

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

putting values,

$|\text{OA}| = \sqrt{(\sqrt{176} - 0)^2 + (7 - 0)^2}$

$= \sqrt{(\sqrt{176})^2 + (7 - 0)^2}$

$= \sqrt{176 + 49}$

$= \sqrt{225}$

$= 15$ unit

Thus the point $(\sqrt{176}, 7)$ is at 15 units from the origin.

(ii) $(10, -10)$

Solution:

Given point, $(10, -10)$

origin, $\text{O}(0,0)$

We know that

$d = \sqrt{(x_1 - x_1)^2 + (y_1 - y_1)^2}$

putting values,

$|OA| = \sqrt{(10 - 0)^2 + (-10 - 0)^2}$

$= \sqrt{(10)^2 + (-10)^2}$

$= \sqrt{100 + 100} = \sqrt{200}$

$= \sqrt{100 \times 2} = 10\sqrt{2}$ units

The point $(10, -10)$ is not at distance of 15 units from origin.

(iii) $(1,15)$

Solution:

Given point $(1,15)$

origin

$O(0,0)$

$\text{d} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$|\text{OA}| = \sqrt{(1 - 0)^2 + (15 - 0)^2} = \sqrt{(1)^2 + (15)^2}$

$= \sqrt{1 + 225} = \sqrt{226}$

Thus distance of $(1,15)$ from origin is not 15 units.

Q. 5 Show that

(i) The points $\text{A}(0,2), \text{B}(\sqrt{3}, 1)$ and $\text{C}(0, -2)$ are vertices of a right triangle

Solution:

$\text{A}(0,2), \text{B}(\sqrt{3}, 1)$ and $\text{C}(0, -2)$

using distance formula we find the square of lengths of sides of Δ.

$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$AB|^2 = (\sqrt{3} - 0)^2 + (1 - 2)^2$$
$$= (\sqrt{3})^2 + (-1)^2$$
$$= 3 + 1 = 4$$

$$BC^2 = (0 - \sqrt{3})^2 + (-2 - 1)^2$$
$$= (\sqrt{3})^2 + (-3)^2$$
$$= 3 + 9 = 12 \text{ (ii)}$$

$$AC^2 = (0 - 0)^2 + \overline{(-2 - 2)^2}$$
$$= (0)^2 + (-4)^2$$
$$= 0 + 16 = 16 \text{ (iii)}$$

From (i), (ii) and (iii) we know that $16 = 12 + 4$

$|AC|^2 = |BC|^2 + |AB|^2$

Since, converse Pythagoras theorem is satisfied, so points A, B and C are vertices of a right-angled triangle.

(ii) The points $A(3,1), B(-2,-3)$ and $C(2,2)$ are vertices of an isosceles triangle.

Solution:

$A(3,1), B(-2,-3)$ and $C(2,2)$

By using distance formula,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|AB| = \sqrt{(-2 - 3)^2 + (-3 - 1)^2}$$
$$= \sqrt{(-5)^2 + (-4)^2}$$
$$= \sqrt{25 + 16} = \sqrt{41} - - - - - - - - -(i)$$

Now,

$$|Bc| = \sqrt{[2 - (-2)]^2 + [2 - (-3)]^2}$$
$$= \sqrt{(2 + 2)^2 + (2 + 3)^2}$$

$$= \sqrt{(4)^2 + (5)^2}$$
$$= \sqrt{16 + 25}$$
$$= \sqrt{41} - - - - - - - - - - - - - -(ii)$$

Now,

$$|AC| = \sqrt{(2-3)^2 + (2-1)^2}$$
$$= \sqrt{(-1) + (1)^2}$$
$$= \sqrt{(1) + (1)} = \sqrt{2}$$

From (i) (ii) and (iii) we observe that $|\mathrm{AB}| = |\mathrm{BC}|$
Which shows that $\triangle \mathrm{ABC}$ with given vertices is an isosceles triangle.
(iii) The points $A(5,2), B(-2,3), C(-3,-4)$ and $D(4,-5)$ are vertices of a parallelogram.
Solution:
$\mathrm{A}(5,2), \mathrm{B}(2,3), (1,4)$ and $\mathrm{D}(4,-5)$
By distance formula.
$$\mathrm{d} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$|AB| = \sqrt{(-2-5) + (3-2)}$$
$$= \sqrt{(-7)^2 + (1)}$$
$$= \sqrt{49+1} = \sqrt{50} - - - - - - -(i)$$
$$|BC| = \sqrt{(-3+2) + (-4-3)}$$
$$= \sqrt{(-1) + (-7)}$$
$$= \sqrt{1+49} = \sqrt{50} - - - - - - -(i)$$
$$|CD| = \sqrt{(4+3) + (-5+4)}$$
$$= \sqrt{(7) + (-1)}$$
$$= \sqrt{49+1} = \sqrt{50} - - - - - - -(i)$$
$$|AD| = \sqrt{(4-5) + (-5-2)}$$
$$= \sqrt{(-1) + (-7)}$$
$$= \sqrt{1+49} = \sqrt{50} - - - - - - -(i)$$

From (i), (ii), (iii) and (iv)

$|\overrightarrow{\mathrm{AB}}| = |\overrightarrow{\mathrm{CD}}|$ and $|\overrightarrow{\mathrm{BC}}| = |\overrightarrow{\mathrm{AD}}|$

Opposite sides are equal in length. Now, we find the midpoints of diagonals.

Midpoint point of diagonal $\overline{\mathrm{AC}}$:

$$\begin{aligned}
\mathrm{M}_1(x,\mathrm{y}) &= \mathrm{M}_1\left(\frac{x_1+x_2}{2},\frac{\mathrm{y}_1+\mathrm{y}_2}{2}\right)\\
&= \mathrm{M}_1\left(\frac{5+(-3)}{2},\frac{2+(-4)}{2}\right)\\
&= \mathrm{M}_1\left(\frac{5-3}{2},\frac{2-4}{2}\right)\\
&= \mathrm{M}_1\left(\frac{2}{2},\frac{-2}{2}\right)\\
&= \mathrm{M}_1(1,-1)
\end{aligned}$$

Midpoint of diagonal $\overline{\mathrm{BD}}$:

$$\begin{aligned}
M(x,y) &= M_2\left(\frac{-2+4}{2},\frac{3+(-5)}{2}\right)\\
&= M_2\left(1,\frac{-2}{2}\right)\\
&= M_2(1,-1)
\end{aligned}$$

Since, midpoints M_1 and M_2 of diagonals $\overline{\mathrm{AC}}$ and $\overline{\mathrm{BD}}$ are same. So diagonals bisects each other.

This, given points are vertices of a parallelogram.

Q. 6 Find h such that the points $A\ (\sqrt{3},-1), B(0,2)$ and $C(h,-2)$ are vertices of a right triangle with right angle at the vertex A.

Solution:

$\mathrm{A}(\sqrt{3},-1),\mathrm{B}(0,2),\mathrm{C}(\ \mathrm{h},-2)$

We know that

$$d^2=(x_2-x_1)^2+(y_2-y_1)^2$$

$$|AB|^2=(0-\sqrt{3})+(2+1)^2=(-\sqrt{3})^2+(3)^2$$

$= 3 + 9 = 12$

Now, $|\overline{\mathrm{BC}}|^2 = (\mathrm{h} - 0)^2 + (-2 - 2)^2$

$= \mathrm{h}^2 + (-4)^2$

$|\overrightarrow{\mathrm{BC}}|^2 = \mathrm{h}^2 + 16$

$|\mathrm{AC}|^2 = (\mathrm{h} - \sqrt{3})^2 + (-2 + 1)^2$

$\because (a - b)^2 = a^2 + b^2 - 2ab$

$= (\mathrm{h})^2 + (\sqrt{3})^2 - 2(\ \mathrm{h})(\sqrt{3}) + (-1)^2$

$= (\mathrm{h})^2 + 3 - 2\sqrt{3}\ \mathrm{h} + 1$

We know that right -triangle with right. angle at vertex A has side $\overline{BC}$ hypotenuse, so, by Pythagoras theorem

$|Bc|^2 = |AB|^2 + |AC|^2$

$(h^2 + 16) = (12) + \left(h^2 - \sqrt{3}h + 4\right)$

$h^2 + 16 = 16 + h^2 - 2\sqrt{3} + h$

$= h^2 + 4 - 2\sqrt{3}h$

$h^2 + 16 - 16 - \cancel{h^2} = -2\sqrt{3}h$

$0 = -2\sqrt{3}h$

$\dfrac{0}{-2\sqrt{3}} = h$

$0\ = h \Rightarrow h = 0$

Q. 7 Find h such that $A(-1, h), B(3,2)$ and $C(7,3)$ are collinear.

Solution:

$A(-1, h), B(3,2) C(7,3)$

The points A, B and C are collinear if

$$\begin{vmatrix} -1 & h & 1 \\ 3 & 2 & 1 \\ 7 & 3 & 1 \end{vmatrix} = 0$$

Expanding by 1^{st} row, -(-1)

$[2(1) - 3(1)] - \mathrm{h}[(3(1) - 7(1)] + \mathrm{I}[3(3) - 7(2] = 0$

$-1(2 - 3) - \mathrm{h}(3 - 7) + [9 - 14] = 0$

$-1(-1) - \mathrm{h}(-4) + 1(-5) = 0$

$1 + 4\ \mathrm{h} - 5 = 0$

$4\ \mathrm{h} - 4 = 0$

$$4h-4$$
$$4h=4$$
$$h=\frac{4}{4}$$
$$h=1$$

Q. 8 The points $A(-5,-2)$ and $B(5,-4)$ are ends of a diameter of a circle. Find the centre and radius of the circle.

Solution:

$A(-5,2)$ and $B(5,-4)$

The midpoint of diameter is centre of circle by midpoint formula.

Let $\mathrm{M}(x_{\mathrm{m}}, y_{\mathrm{m}})$ be midpoint of diameter AB .

$$x_{\mathrm{m}}=\frac{x_1+x_2}{2}=\frac{-5+5}{2}=\frac{0}{2}$$
$$x_{\mathrm{m}}=0$$
$$y_{\mathrm{m}}=\frac{y_1+y_2}{2}$$
$$y_{\mathrm{m}}=\frac{-2+(-4)}{2}=\frac{-2-4}{2}=\frac{-6}{2}$$
$$y_{\mathrm{m}}=-3$$

Thus midpoint of diameter $\overline{\mathrm{AB}}$ or centre of circle is $\mathrm{M}(0,-3)$

(ii) Finding radius:

$\mathrm{A}(-5,-2), \mathrm{M}(0,-3)$

Since, radius of a circle is distance between centre and any point of circle, so we use distance formula.

$$\mathrm{d}=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

putting values

$$|\mathrm{AM}|=\sqrt{[0-(-5)]^2+[-3-(-2)]^2}$$
$$=\sqrt{(0+5)^2+(-3+2)^2}$$
$$=\sqrt{(5)^2+(-1)^2}$$
$$=\sqrt{25+1}$$

So, the radius of circle is $=\sqrt{26}$ units

Q. 9 Find h such that the points $A(h, 1)$, $B(2,7)$ and $C(-6,-7)$ are vertices of a right triangle with right angle at the vertex A.

Solution:

$\mathrm{A}(\mathrm{h}, 1), \mathrm{B}(2,7), \mathrm{c}(-6,-7)$

We know that

$d^2 = (x_1 - x_2)^2 + (y2 - y_1)^2$

$|AB|^2 = (2-h)^2 + (7-1)^2$

$= (2)^2 + (h)^2 - 2(2)(h) + (6)^2$

$= 4 + h^2 - 4h + 36$

$= h^2 - 4h + 40$

Now,

$|\, BC\, |^2 = (-6-2)^2 + (-7-7)^2$

$= (-8)^2 + (-14)^2$

$= 64 + 196$

$= 260$ (ii)

Now,

$|\mathrm{AC}|^2 = (-6-h)^2 + (-7-1)^2$

$= (-1)^2(6+h)^2 + 64$

$= 1[6^2 + h^2 + 2(6)(h)] + 64$

$= 36 + h^2 + 12h + 64$

$= h^2 + 12h + 100$

We know that right-angled triangle with right angle at vertex A has side $\overline{\mathrm{BC}}$ as its hypotenuse.

By using Pythagoras theorem.

$|BC\,|^2 = |AB|^2 + |AC|^2$

From (i), (ii) and (iii)

$260 = (h^2 - 4h + 40) + (h^2 + 12h + 100)$

$260 = 2h^2 + 8h + 140$

$0 = 2h^2 + 8h + 140 - 260$

$\Rightarrow 2h^2 + 8h - 120 = 0$

$2(h^2 + 4h - 60) = 0$

$\therefore h^2 + 4h - 60 = 0$

$h^2 + 10h - 6h - 60 = 0$

$h(h+10) - 6(h+10) = 0$

$(h+10)(h-6) = 0$

$\Rightarrow h + 10 = 0$ or $h - 6 = 0$

$\Rightarrow h = -10$ or $h = 6$

Thus the value of h is either -10 or 6 .

Q. 10 A quadrilateral has the points $A(9,3)$, $B(-7,7), C(-3,-7)$ and $D(5,-5)$ ms its vertices. Find the midpoints of its sides. Show that the figure formed by joining the midpoints consecutively is parallelogram.

Solution:

$\mathrm{A}(9,3), \mathrm{B}(-7,7), \mathrm{C}(-3,-7)$ and $\mathrm{D}(5,-5)$

First we find midpoints of all the sides of quadrilateral $ABCD$.

Finding midpoint of side $\overline{\mathrm{AB}}$:

Let $\mathrm{P}(x, y)$ be midpoint of $\overline{\mathrm{AB}}$.

$$\mathrm{p}\left(\frac{x_1+x_2}{2}, \frac{\mathrm{y}_1+\mathrm{y}_2}{2}\right)$$

$$=\mathbb{P}\left(\frac{9+(-7)}{2}, \frac{3+7}{2}\right)=\mathrm{P}\left(\frac{9-7}{2}, \frac{10}{2}\right)=\left(\frac{2}{2}, 5\right)$$

$$=P(1,5)$$

Let $\mathrm{Q}(\mathrm{x}, \mathrm{y})$ be midpoint of side $\overline{\mathrm{BC}}$.

$$\mathrm{Q}\left(\frac{x_1+x}{2}, \frac{y_1+y_2}{2}\right)$$

$$=\left(\frac{(-7)+(-3)}{2}, \frac{7+(-7)}{2}\right)$$

$$=Q\left(\frac{-10}{2}, \frac{0}{2}\right)$$

$$Q(-5,0)$$

Let $\mathrm{R}(x, y)$ be midpoint of $\overline{\mathrm{CD}}$

$$\mathbb{R}\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$=\mathbb{R}\left(\frac{-3+5}{2}, \frac{(-7)+(-5)}{2}\right)$$

$$=\mathbb{R}\left(\frac{2}{2}, \frac{-12}{2}\right)$$

$$=\mathbb{R}(1,-6)$$

Let $\mathrm{S}(x, y)$ be midpoint of DA.

$$S\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$
$$=S\left(\frac{5+9}{2},\frac{-5+3}{2}\right)$$
$$=S\left(\frac{14}{2},\frac{-2}{2}\right)=S(7,-1)$$

$$=Q\left(\frac{-10}{2},\frac{0}{2}\right)$$
$$Q(-5,0)$$

Let $\mathrm{R}(x,y)$ be midpoint of $\overline{\mathrm{CD}}$

$$\mathbb{R}\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$
$$=\mathbb{R}\left(\frac{-3+5}{2},\frac{(-7)+(-5)}{2}\right)$$
$$=\mathbb{R}\left(\frac{2}{2},\frac{-12}{2}\right)$$
$$=\mathbb{R}(1,-6)$$

Let $\mathrm{S}(x,y)$ be midpoint of DA.

$$S\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$
$$=S\left(\frac{5+9}{2},\frac{-5+3}{2}\right)$$
$$=S\left(\frac{14}{2},\frac{-2}{2}\right)=S(7,-1)$$

Finding he midpoints of diagonals of PQRS

Let $\mathrm{L}(x,y)$ be midpoint of diagonal $\overline{\mathrm{PR}}$:

$$\mathrm{P}(1,5),\mathrm{R}(1,-6)$$
$$\mathrm{L}\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$
$$=\mathrm{L}\left(\frac{1+2}{2},\frac{5-6}{2}\right)=\mathrm{L}\left(\frac{2}{2},-\frac{1}{2}\right)=\mathrm{L}(1,-0.5)$$

Let $\mathrm{M}(x,y)$ be midpoint of diagonal $\overline{\mathrm{QS}}$:

$Q(-5,0), S(7,-1)$

$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= M\left(\frac{-5+7}{2}, \frac{0-1}{2}\right) = \left(\frac{2}{2}, \frac{-1}{2}\right) = M(1,-0.5)$$

Since, midpoints of diagonals coincide which proves that quadrilateral formed by joining the midpoints is a parallelogram.