Review Exercise 3

Q. 1 Choose the correct option.

i. The set builder form of the set $\left\{1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots\right\}$ is :

(a) $\left\{x \,\middle|\, x = \frac{1}{n}, n \in W\right\}$ (b) $\left\{x \,\middle|\, x = \frac{1}{2n+1}, n \in W\right\}$ (c) $\left\{x \,\middle|\, x = \frac{1}{\mathrm{n}+1}, \mathrm{n} \in \mathrm{W}\right\}$ (d) $\{x \mid x = 2\mathrm{n}+1, \mathrm{n} \in \mathrm{W}\}$

ii. If $\mathrm{A} = \{\ \}$, then $\mathrm{P(A)}$ is:

(a)$\{\ \}$ (b) $\{1\}$ (c) $\{\{\}\}$ (d) ϕ

iii. If $\mathrm{U} = \{1,2,3,4,5\}, \mathrm{A} = \{1,2,3\}$ and $\mathrm{B} = \{3,4,5\}$, then $U - (A \cap B)$ is:

(a) $\{1,2,4,5\}$ (b) $\{2,3\}$ (c) $\{1,3,4,5\}$ (d) $\{1,2,3\}$

iv. If A and B are overlapping sets, then $\mathrm{n(A - B)}$ is equal to:

(a) $n(A)$ (b) $n(B)$ (c) $A \cap B$ (d) $n(A) - n(A \cap B)$

v. If $A \subseteq B$ and $B - A \neq \phi$, then $n(B - A)$ is equal to:

(a) 0 (b) $n(B)$ (c) $n(\wedge)$ (d) $n(B) \rightarrow n(A)$

vi. If $n(A \cup B) = 50, n(A) = 30 n(B) = 35$, then $n(A \cap B) =$:

(a) 23 (b) 15 (c) 9 (d) 40

vii. If $\mathrm{A} = \{1,2,3,4\}$ and $\mathrm{B} = \{x,y,z\}$, then Cartesian product of A and B contains exactly ....elements

(a) 13 (b) 12 (c) 10 (d) 6

viii. If $f(x) = x^2 - 3x + 2$, then the value of $f(a+1)$ is equal to:

(a) $a+1$ (b) a^2+1 (c) a^2+2a+1 (d) a^2-a

ix. Given that $f(x) = 3x+1$, if $f(x) = 28$, then the value of x is:

(a) 9 (b) 27 (c) 3 (d) 18

x. Let $\mathrm{A} = \{1,2,3\}$ and $\mathrm{B} = \{a, \mathrm{b}\}$ two non-empty sets and $f\mathrm{:A} \rightarrow \mathrm{B}$ be a function defined as $f = \{(1,a),(2,b), (3, \mathrm{b})\}$, then which of the following statement is true?

(a) f is injective (b) f is surjective (c) f is bijective (d) f is into only

Answer Key

i	b	ii	c	iii.	a	iv	d	v	d
vi	b	vii	b	viii	d	ix	a	x	b

MCQS (Additional)

1.A collection of well-defined distinct objects is called:
(a) subset (b) power set (c) set (d) Venn diagram

2. Which of the following is the set of first hundred whole number?
(a) $\{1,2,3,\ldots,100\}$ (b) $\{1,2,3,\ldots,99\}$ (c) $\{0,1,2;3,\ldots,100\}$ (d) $\{0,1,2,3,\ldots,99\}$

3.The different number of ways to describe a set are:
(a) 1 (b) 2 (c) 3 (d) 4

4.A set with no element is called:
(a) subset (b) Null set (c) singleton set (d) super set

5.The set $\{x/x\in\mathbb{Y} \wedge x \geq 106\}$ is:
(a) infinite set (b) subset (c) supper set (d) finite set

6.The set having only one element is called:
(a) Null set (b) power set (c) singleton set (d) subset

7. The number of elements in power set $\{a,b,c,d\}$ is:
(a) 4 (b) 8 (c) 16 (d) 32

8. Power set of an empty set is:
(a) ϕ (b) $\{a\}$ (c) $\{\phi,\{a\}\}$ (d) $\{d\}$

9. If $P \subseteq Q$ then $P \cup Q$ is equal to
(a) P (b) Q (c) ϕ (d) U

10. If $X \subseteq Y$ then $X \cap Y$ is equal to:
(a) X (b) Y (c) ϕ (d) U

11. If $A \subseteq B$ then $A - B$ is equal to:
(a) A (b) B (c) ϕ (d) U

12. $(A \cup B) \cup C$ is equal to:
(a) $A \cap (B \cup C)$ (b) $(A \cup B) \cap C)$ (c) $A \cup (B \cup C)$ (d) $A \cap (B \cap C)$

13. $A \cup (B \cap C)$ is equal to:
(a) $A \cup (B \cup C)$ (b) $A \cap (B \cap C)$ (c) $(A \cap B) \cup (A \cap C)$ (d) $(A \cup D) \cap (A \cup C)$

14. If A and B are disjoint sets, then $A \cup A$ is equal to: masm.
(a) A (b) B (c) ϕ (d) $B \cup A$

15. If $A \sim B = \phi$, then set A and B are ....sets.
(a) sub (b) over lapping (c) disjoint (d) power
16. The complement of U is:
(a) U (b) ϕ (c) impossible (d) union
17. The complement of ϕ is:
(a) U (b) 4 (c) impossible (d) union
18. $\mathrm{A} \cap \mathrm{A}' =$
(a) U (b) A (c) A' (d) { }
19. $A \cup \mathrm{A}' =$
(a) U (b) A (c) A^{c} (d) { }
20. The set $\{x \mid x \in B$ and $x \in A\}$ is:
(a) $A \cup B$ (b) $A \cap B$ (c) $\mathrm{A} - \mathrm{B}$ (d) $B - A$
21. The set $\{x \mid x \in A$ and $x \in B\}$ is:
(a) $A \cup B$ (b) $A \cap B$ (c) $\mathrm{A} - \mathrm{B}$ (d) $\mathrm{B} - \mathrm{A}$
22. $\mathrm{N} \cup \mathrm{W} = \ldots\ldots..$
(a) ϕ (b) $\{0\}$ (c) N (d) W
23. $\mathrm{N} - \mathrm{W} = \ldots\ldots...$
(a) ϕ (b) $\{0\}$ (c) N (d) W

24. The domain of $\mathrm{R} = \{([illegible],2),(2,3),(3,3)([illegible],4)\}$ is:
(a) $\{1,3,4\}$ (b) $\{1,2,3\}$ (c) $\{[illegible],2,4\}$ (d) $\{2,3,4\}$
25. The Range of R = $\{(1,3),(2,2),(3,1)(4,4)\}$ is:
(a) $\{1,2,4\}$ (b) $\{3,2,4\}$ (c) $\{1,2,3,4\}$ (d) $\{1,3,4\}$
26. Point $(-3,4)$ lies in the quadrant:
(a) I (b) II (c) III (d) IV
27. The point $(-4,-5)$ lies in ... quadrant
(a) I (b) II (c) III (d) IV

28. If $\mathrm{g}(x) = 7x - 2$ then $\mathrm{g}(-1) = \ldots$
(a) -2 (b) -7 (c) -9 (d)-1
29. If Range $(f) \subset B$, then function is:
(a) into (b) onto (c) bijective (d) injective
30. If $f(x) = 5x + 12$ and $f(x) = 32$ then $x =$
(a) -4 (b) 4 (c) 20 (d) 32

Answer Key

1	c	2	d	3	c	4	b	5	a	6	c	7	c	8	d	9	b	10	a
11	c	12	c	13	d	14	d	15	c	16	b	17	a	18	d	19	a	20	d

21	b	22	d	23	a	24	b	25	c	26	b	27	c	28	c	29	a	30	b

Q. 2 Write each of the tabular forms:

(i) $\{x \mid x = 2n, n \in N\}$

Solution

$$\{x \mid x = 2n, n \in N\}$$

Tabular Form

$$\{2,4,6,8,...\}$$

(ii) $\{x \mid x = 2m + 1, m \in N\}$

Solution:

$$\{x \mid x = 2m + 1, m \in N\}$$

Tabular Form

$$\{3,5,7,9,...\}$$

(iii) $\{x \mid x = 11n, n \in W \wedge n < 11\}$

Solution:

$$\{x \mid x = 11n, n \in W \wedge n < 11\}$$

Tabular Form

$$\{0,11,22,33,44,55,66,77,88,99,110\}$$

(iv) $\{x \mid x \in \mathbb{E} \wedge 4 < x < 6\}$

Solution:

Tabular Form

ϕ or { }

(v) $\{x \mid x \in O \wedge 5 \leq x < 7\}$

Solution:

$\{x \mid x \in O \wedge 5 \leq x < 7\}$

Tabular Form

{5}

(vi) $\left\{x \mid x \in Q \wedge x^2 = 2\right\}$

Solution:

$\left\{x \mid x \in Q \wedge x^2 = 2\right\}$

{ } or ϕ

(vii) $\{x \mid x \in \mathbb{Q} \wedge x = -x\}$

Solution:

$\{x \mid x \in \mathrm{Q} \wedge x = -x\}$

Tabular Form

$$(\because x + x = 0)$$
$$0 + 0 = 0$$

{0}

(viii) $\{x \mid x \in \mathbb{R} \wedge x \notin Q'\}$

Solution:

$\{x \mid x \in \mathrm{R} \wedge x \notin Q'\}$

$\mathrm{Q} = \mathrm{R} - \mathrm{Q}'$

Q.3 Let $\mathrm{U} = \{1,2,3,4,5,6,7,8,9,10\}$,

$\mathrm{A} = \{2,4,6,8,10\}, \mathrm{B} = \{1,2,3,4,5\}$ and

$\mathrm{C} = \{1,3,5,7,9\}$

List the members of each the following sets:

Solution:

$$U = \{1,2,3,4,5,6,7,8,9,10\}$$
$$A = \{2,4,6,8,10\}$$
$$B = \{1,2,3,4,5\}$$
$$C = \{1,3,5,7,9\}$$

(i) A'

Solution:

$$A' = U - A$$
$$A' = \{1,2,3,4,5,6,7,8,9,10\} - \{2,4,6,8,10\}$$
$$A' = \{1,3,5,7,9\}$$

(ii) B'

Solution:

$$B' = U - B$$
$$B' = \{1,2,3,4,5,6,7,8,9,10\} - \{1,2,3,4,5\}$$
$$B' = \{6,7,8,9,10\}$$

(iii) $\mathrm{A} \cup \mathrm{B}$

Solution:

$A \cup B$

$A \cup B = \{2,4,6,8,10\} \cup \{1,2,3,4,5\}$

$A \cup B = \{1,2,3,4,5,6,8,10\}$

(iv) $\mathbf{A} - \mathbf{B}$

Solution:

A-B

$A - B = \{2,4,6,8,10\} - \{1,2,3,4,5\}$

$A - B = \{6,8,10\}$

(v) $\mathrm{A} \cap \mathrm{C}$

Solution

$$A \cap C = \{2,4,6,8,10\} \cap \{1,3,5,7,9\}$$
$$A \cap C = \phi$$

(vi) $A' \cup C'$

Solution:

$$A' \cup C'$$
$$A' = U - A$$
$$A' = \{1,2,3,4,5,6,7,8,9,$$
$$10\} - \{2,4,6,8,10\}$$
$$A' = \{1,3,5,7,9\}$$

Now, $C' = U - C$

$$C' = \{1,2,3,4,5,6,7,8,9,10\} - \{1,3,5,7,9\}$$
$$C' = \{2,4,6,8,10\}$$

Now, $A' \cup C' = \{1,3,5,7,9\} \cup \{2,4,6,8,10\}$

$A' \cup C' = \{1,2,3,4,\ldots,10\}$

(vii) $A' \cup C$

Solution:

$$A' = U - A$$
$$A' = \{1,2,3,4,5,67,8,9,10\} - \{2,4,6,8,10\}$$
$$A' = \{1,3,5,7,9\}$$

Now, $A' \cup C = \{1,3,5,7,9\} \cup \{1,3,5,7,9\}$

$$A' \cup C = \{1,3,5,7,9\}$$

(viii) U'

Solution:

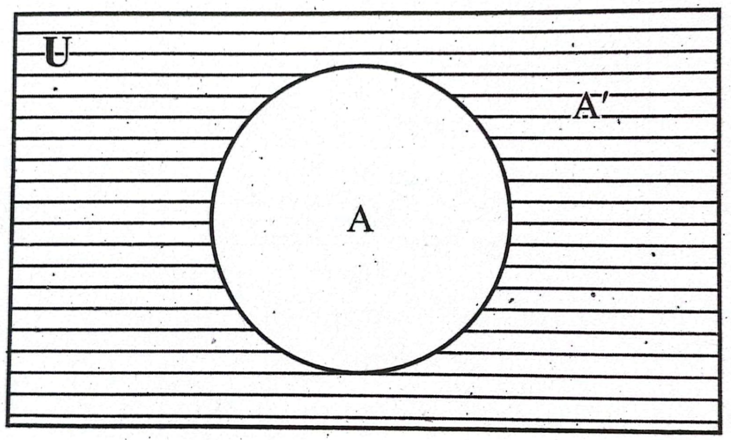
$$\mathbf{U = U - U}$$
$$\mathbf{U'} = \{1,2,3,4,5,67,8,9,10\} - \{1,2,3,4,\ldots,10\}$$
$$\mathbf{U'} = \phi$$

Q. 4 Using the Venn diagrams, if necessary, find the single sets equal to the following:

(i) A'

Solution:

$$\mathbf{A' = U - A}$$



Horizontally lined region shows A'.

(ii) $\mathbf{A} \cap \mathbf{B}$

Solution:

$A \cap B$

$A \cap U = A$

$(\because A \subset U)$

(iii) $A \cup U$

Solution:

$A \cup U = U$

$(\because A \subset U)$

(iv) $A \cup \phi$

Solution:

$A \cup \phi = A$

$A \cup \phi = A$

$(\because \phi \subset A)$

(v) $\phi \cap \phi$

Solution:

$\phi \cap \phi$

$\phi \cap \phi = \phi$

Q. 5 Use Venn diagrams to verify the following:

(i) $A - B = A \cup B'$

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Solution:

$A - B = A \cup B'$

L.H.S $= A - B$

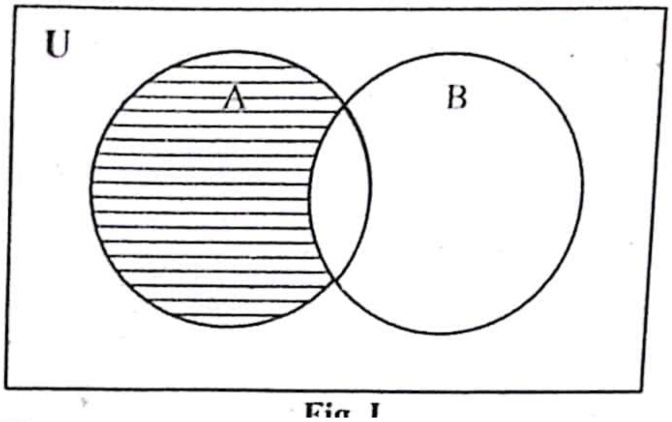

Fig. I

$A - B$ is shown by region of horizontal line segments..

R.H.S $= A \cap B'$

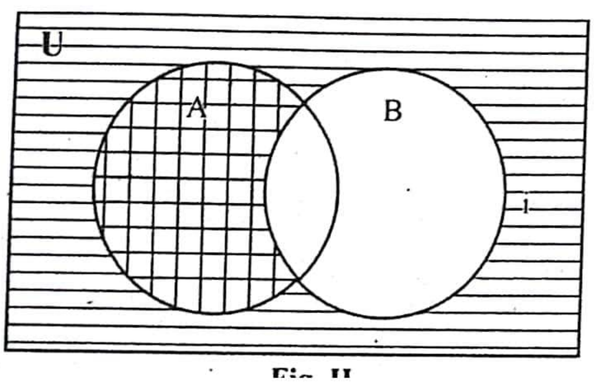

Fig. II

B' is shown by region of squares and horizontal line segments.
$A \cap B'$ is shown by region of squares. From fig. (I) and Fig. (ii) regions showing (A-B) and $A \cap B'$ are same, so

$$\mathrm{A - B = A \cap \ B'}.$$

(ii) $\mathrm{(A - B)' \cap B = B}$

Solution:

L.H.S $(A - B)' \cap B$

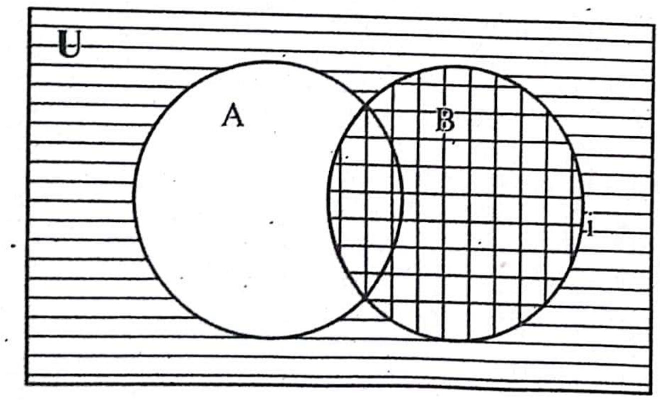

Fig. I

$(A-B)' \cap B$ is shown by regions of

R.H.S = B

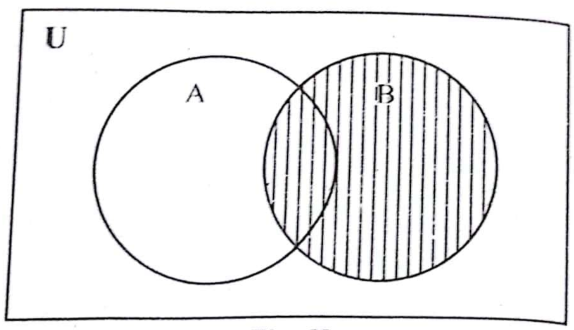

Fig. II

Fig. II

Regions showing $(A-B)' \cap B$ and B are same so,

$$(A-B)' \cap B = B$$

Q. 6 Verify the properties for the sets A, B and C given below:

Solution:

(a)

$$\text{A} = \{1,2,3,4\}, \text{B } \{3,4,5,6,7,8\}.$$
$$\text{C} = \{5,6,7,9,10\}$$

(i) Associativity of Union

Solution:

$$(A \cup B) \cup C = A \cup (B \cup C)$$
$$\text{L.}H.S = (A \cup B) \cup C$$
$$= (\{1,2,3,4\} \cup \{3,4,5,6,7,8\}) \cup C$$
$$= \{1,2,3,4,5,6,7,8\} \cup \{5,6,7,9,10\}$$
$$= \{1,2,3,4,5,6,7,8,9,10\} \qquad \text{(i)}$$

Now, R.H.S $= A \cup (B \cup C)$

$= A \cup (\{3,4,5,6,7,8\} \cup \{5,6,7,9,10\})$

$= \{1,2,3,4\} \cup \{3,4,5,6,7,8,910\}$

$= \{1,2,3,4,5,6,7,8,9,10\}$ (ii)

From (i) and (ii) L.H.S = R.H.S

$(A \cup B) \cup C = A \cup (B \cup C)$ Hence proved

(ii) Associativity of intersection

Solution:

$$(A \cap B) \cap C = A \cap (B \cap C)$$

L.H.S $= (A \cap B) \cap C$

$= (\{1,2,3,4\} \cap \{3,4,5,6,7,8\}) \cap C$

$= \{3,4\} \cap \{5,6,7,9,10\}$

$= \{\ \}$ (i)

R.H.S $= A \cap (B \cap C)$

$= A \cap (\{3,4,5,6,7,8\} \cap \{5,6,7,9,10\})$

$= \{1,2,3,4\} \cap \{5,6,7\}$

$= \{\ \}$ (ii)

From (i) and (ii) L.H.S = R.H.S

$(A \cap B) \cap C = A \cap (B \cap C)$ Hence proved.

(iii)Distributivity of Union over intersection

Solution:

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

$L.H.S$ $A \cup (B \cap C)$

$A \cup (\{3,4,5,6,7,8\} \cap \{5,6,7,9,10\})$

$= \{1,2,3,4\} \cup \{5,6,7\}$

$= \{1,2,3,4,5,6,7\}$ (i)

$R.H.S$ $(A \cup B) \cap (A \cup C)$

$(A \cup B) = \{1,2,3,4\} \cup \{3,4,5,6,7,8\}$

$(A \cup B) = \{1,2,3,4,5,6,7,8\}$

Now, $A \cup C = \{1,2,3,4\} \cup \{5,6,7,9,10\}$

$= \{1,2,3,4,5,6,7,9,10\}$

Now,

$(A \cup B) \cap (A \cup C)$

$= \{1,2,3,4,5,6,7,8\} \cap 1,2,3,4,5,6,7,9,10\}$

$= \{1,2,3,4,5,6,7\}$ (ii)

From (i) and (ii), L.H.S = R.H.S

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ Hence proved

(iv)Distributivity of intersection over union
(a)

$$A = \{1,2,3,4\}, B = \{3,4,5,6,7,8\},$$
$$C = \{5,6,7,9,10\}$$

Solution:

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

L.H.S $= A \cap (B \cup C)$

$= A \cap (\{3,4,5,6,7,8\} \cup \{5,6,7,9,10\})$

$= \{1,2,3,4\} \cap \{3,4,5,6,7,8,9,10\}$

$= \{3,4\}$ (i)

Now, R.H.S $= (A \cap B) \cup (A \cap C)$

$(A \cap B) = \{1,2,3,4\} \cap \{3,4,5,6,7,8\}$

$(A \cap B) = \{3,4\}$

Now, $(A \cap C) = \{1,2,3,4\} \cap \{5,6,7,9,10\}$

$= \{\}$

Now, $(A \cap B) \cup (A \cap C) = \{3,4\} \cup \{\}$

$= \{3,4\}$

From (i) and (ii)

L.H.S = R.H.S

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

(b) $A = \phi, B = \{0\}, C = \{0,1,2\}$

(i) Associvity of union

$(A \cup B) \cup C = A \cup (B \cup C)$

L.H.S $= (A \cup B) \cup C$

$= (\{\ \} \cup \{0\}) \cup C$

$= \{0\} \cup \{0,1,2\}$

$= \{0,1,2\} \qquad (i)$

R.H.S $= A \cup (B \cup C)$

$= A \cup (\{0\} \cup \{0,1,2\})$

$= \{\ \} \cup \{0,1,2\}$

$= \{0,1,2\} \qquad (ii)$

From (i) and (ii) L.H.S = R.H.S

$$(A \cup B) \cup C = A \cup (B \cup C)$$
$$= (0,4,2) \cup (0,1,2)$$

Hence proved.
(ii) Associativity of intersection

$$(A \cap B) \cap C = A \cap (B \cap C)$$

$$\begin{aligned}
\text{L.H.S} &= (A \cap B) \cap C \\
&= (\{\ \} \cap \{0\}) \cap C \\
&= \{\ \} \cap \{0,1,2)\} \\
&= \{\ \} \qquad \text{(i)} \\
\text{R.H.S} &= A \cap (B \cap C) \\
&= A \cap (\{0\} \cap \{0,1,2\}) \\
&= \{\ \} \cap \{0\} \\
&= \{\ \} \qquad \text{(ii)}
\end{aligned}$$

From (i) and (ii) L.H.S = R.H.S

$$(A \cap B) \cap C = A \cap (B \cap C)$$

Hence proved.
(iii)Distributivity of Union over intersection
Solution:

$$\begin{aligned}
A \cup (B \cap C) &= (A \cup B) \cap (A \cup C) \\
\text{L.H.S} &= A \cup (B \cap C) \\
&= A \cup \{(0\} \cap \{0,1,2\}) \\
&= \{\ \} \cup \{0\} \\
&= \{0\} \qquad \text{(i)} \\
\text{R.H.S} &= (A \cup B) \cap (A \cup C) \\
&= (\{\ \} \cup \{0\}) \cap (\{1\} \cup \{0,1,2\}) \\
&= \{0\} \cap \{0,1,2\} \\
&= \{0\} \qquad \text{(ii)}
\end{aligned}$$

From (i) and (ii), L.H.S = R.H.S
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
Hence proved.

(iv) Distributivity of Intersection over Union
Solution:

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$$\begin{aligned}
\text{LH.S} &= A \cap (B \cup C) \\
&= \{\,\} \cap (\{\,\} \cup \{0,1,2\}) \\
&= \{\,\} \cap \{0,1,2\} \\
&= \{\,\} \qquad \text{(i)}
\end{aligned}$$

$$\begin{aligned}
\text{R.H.S} &= (A \cap B) \cup (A \cap C) \\
&= (\{\,\} \cap \{0\}) \cup (\{\,] \cap \{0,1,2\}) \\
&= \{\,\} \cup \{\,\} \\
&= \{\,\} \qquad \text{(ii)}
\end{aligned}$$

From (i) and (ii) L.H.S = R.H.S

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

(c) $A = N, B = Z, C = Q$

Solution:

$$N \subset Z \subset Q$$

(i) Associativity of union

Solution

$$(A \cup B) \cup C = A \cup (B \cup C)$$

$$\begin{aligned}
\text{L.H.S} &= (A \cup B) \cup C \\
&= (N \cup Z) \cup Q \\
&= Z \cup Q \\
&= Q
\end{aligned}$$

$$\begin{aligned}
\text{R.H.S} &= A \cup (B \cup C) \\
&= N \cup (Z \cup Q) \\
&= N \cup Q \\
&= Q
\end{aligned}$$

From (i) and (ii) L.H.S = R.H.S
$(A \cup B) \cup C = A \cup (B \cup C)$

Hence proved.
(ii) Associativity of intersection enesion

Solution:

$$
\begin{aligned}
(A \cap B) \cap C &= A \cap (B \cap C) \\
\text{L.H.S} &= (A \cap B) \cap C \\
&= (N \cap Z) \cap Q \\
&= N \cap Q \\
&= N \qquad (i)
\end{aligned}
$$

$$
\begin{aligned}
\text{R.H.S} &= A \cap (B \cap C) \\
&= N \cap (Z \cap Q) \\
&= N \cap Z \\
&= N \qquad \text{(ii)}
\end{aligned}
$$

From (i) and (ii) L.H.S = R.H.S
$(A \cap B) \cap C = A \cap (B \cap C)$

Hence proved
(iii) Distributivity of union over intersection

Solution

$$
\begin{aligned}
A \cup (B \cap C) &= (A \cup B) \cap (A \cup C) \\
\text{L.H.S} &= A \cup (B \cap C) \\
&= N \cup (Z \cap Q) \\
&= N \cup Z \\
&= Z \qquad (i) \\
\text{R.H.S} &= (A \cup B) \cap (A \cup C) \\
&= (N \cup Z) \cap (N \cup Q) \\
&= Z \cap Q \\
&= Z \qquad (ii)
\end{aligned}
$$

From (i) and (ii), L.H.S = R.H.S
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
Hence proved.
(iv) Distributivity of intersection over union
Solution:

$$
\begin{aligned}
A \cap (B \cup C) &= (A \cap B) \cup (A \cap C) \\
\text{L.H.S} &= A \cap (B \cup C) \\
&= N \cap (Z \cup Q) \\
&= N \cap Q \\
&= N \qquad (i) \\
\text{R.H.S} &= (A \cap B) \cup (A \cap C \\
&= (N \cap Z) \cup (N \cap Q) \\
&= N \cup N \\
&= N \qquad (ii)
\end{aligned}
$$

From (i) and (ii) L.H.S = R.H.S

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Hence proved.

Q. 7 Verify De Morgan's Laws for the following sets:

$\mathrm{U} = \{1,2,3,\ldots,20\}, \mathrm{A} = \{2,4,6,\ldots,20\}$ and
$\mathrm{B} = \{1,3,5,\ldots 19\}$.

Solution:

$$
\begin{aligned}
U &= \{1,2,3,4,\ldots,20\} \\
A &= \{2,4,6,8,\ldots,20\} \\
B &= \{1,3,5,\ldots,19\}
\end{aligned}
$$

(i) $(\mathrm{A} \cup \mathrm{B})' = \mathrm{A}' \cap \mathrm{B}'$

Solution:

$$
\begin{aligned}
(A \cup B)' &= A' \cap B' \\
\text{L.H.S} &= (A \cup \mathrm{B})' \\
(A \cup \mathrm{B}) &= \{2,4,6,8,\ldots,20\} \cup \{1,3,5,7,\ldots,19\} \\
&= \{1,2,3,4,5,6,\ldots,19,20\}
\end{aligned}
$$

Now, $(A \cup B)' = U - (A \cup B)$

$$
\begin{aligned}
&= \{1,2,3,4,\ldots,20\} - \{1,2,3,4,\ldots,20\} \\
&= \{\ \} \qquad (i)
\end{aligned}
$$

Now, R.H.S $= \mathrm{A}' \cap \mathrm{B}'$

$$
\begin{aligned}
A' &= U - A \\
A' &= \{1,2,3,4,5,\ldots,20\} - \{2,4,6,\ldots,20\} \\
A' &= \{1,3,5,7,\ldots,19\}
\end{aligned}
$$

Now, $B' = U - B$

$$
\begin{aligned}
B' &= \{1,2,3,4,\ldots,20\} - \{1,3,5,\ldots,19\} \\
B' &= \{2,4,6,8,\ldots,20\}
\end{aligned}
$$

R.H.S $= A' \cap B'$.

$= \{1,3,5,\ldots,19\} \cap \{2,4,6,\ldots,20\}$

$= \{\ \}$ (ii)

From (i) and (ii), L.H.S =R.H.S

$(A \cup B)' = A' \cap B'$ Hence proved.

(ii) $(A \cap B)' = A' \cup B'$

LH.S $= (A \cap B)'$

$(A \cap B) = \{2,4,6,\ldots,20\} \cap \{1,3,5,\ldots,19\}$

$(A \cap B) = \{\ \}$

Now, $(A \cap B)' = U - (A \cap B)$

$= \{1,2,3,4,\ldots,20\} - \{\ \}$

$= \{1,2,3,4,\ldots,20\}$ (i)

R.H.S $= A' \mathrm{U} B'$

$$
\begin{aligned}
A' &= U - A \\
A' &= \{1,2,3,4,\ldots,20\} - \{2,4,6,\ldots,20\} \\
A' &= \{1,3,5,\ldots,19\}
\end{aligned}
$$

Now, $\mathrm{B}' = \mathrm{U} - \mathrm{B}$

$$
\begin{aligned}
B' &= \{1,2,3,4,\ldots,20\} - \{1,3,5,\ldots,19\} \\
B' &= \{2,4,6,\ldots,20\}
\end{aligned}
$$

Now,

$A' \cup B' = \{1,3,5,\ldots,19\} \cup \{2,4,6,\ldots,20\}$

$= \{1,2,3,4,5,\ldots,20\}$ (ii)

From (i) and (ii) L.H.S = R.H.S

$$(A \cap B)' = A' \cup B'$$

Q. 8 Consider the set $\mathbb{P} = \{x \mid x = 5\text{ m},\ \text{m} \in \mathbb{N}\}$ and $\mathbb{Q} = \{x \mid x = 2\text{ m},\ \text{m} \in \mathbb{N}\}$. Find $P \cap Q$

Solution:

$$P = \{x \mid x = 5\text{ m},\ \text{m} \in \text{N}\}$$
$$Q = \{x \mid x = 2\text{ m},\ \text{m} \in \text{N}\}$$

In tabular form:

$$P = \{5,10,15,20,25,\ldots\}$$
$$Q = \{2,4,6,8,10,\ldots\}$$

Finding $\text{P} \cap \text{Q}$:

$$P \cap Q = \{5,10,15,20,25,\ldots\} \cap \{2,4,6,8,10,\ldots\}$$
$$P \cap Q = \{10,20,30,40,\ldots\}$$

Q. 9 From suitable properties of union and intersection, deduce the following results:

(i) $A \cap (A \cup B) = A \cup (A \cap B)$

Solution:

$$A \cap (A \cup B) = A \cup (A \cap B)$$

L.H.S $= A \cap (A \cup B)$

L.H.S $= (A \cap A) \cup (A \cap B)$ (by distributive property of intersection over union)

L.H.S $= A \cup (A \cap B)$

L.H.S = R.H.S

(ii) $A \cup (A \cap B) = A \cap (A \cup B)$

L.H.S $= A \cup (A \cap B)$

$= (A \cup A) \cap (A \cup B)$ (by distributive property of union over intersection)

$= A \cap (A \cup B)$

L.H.S = R.H.S

Q. 10 If $g(x)\ 7x - 2$ and $s(x) = 8x^2 - 3$ find

Solution:

$$g(x) = 7x - 2$$
$$s(x) = 8x^2 - 3$$

(i) $g(0)$

Solution

Put $x = 0$ in $g(x)$

$$g(0) = 7(0) - 2 = 0 - 2 = -2$$

(ii) $g(-1)$

Solution:

Put $x = -1$ in $g(x)$

$$g(-1) = 7(-1) - 2 = -7 - 2 = -9$$

(iii) $g\left(-\frac{5}{3}\right)$

Solution:

Finding $g\left(-\frac{5}{3}\right)$

Put $x = \frac{-5}{3}$ in $g(x)$

$$\begin{aligned} g\left(\frac{-5}{3}\right) &= 7\left(\frac{-5}{3}\right) - 2 \\ &= \frac{-35}{3} - 2 \\ &= \frac{-35-6}{3} \\ &= \frac{-41}{3} \end{aligned}$$

(iv) $s(1)$

Solution:

Put $x = 1$ in $s(x)$

$$s(1) = 8(1)^2 - 3 = 8(1) - 3 = 8 - 3 = 5$$

(v) $s(-9)$

Solution:

put $x = -9$ in $s(x)$

$$\begin{aligned} s(-9) &= 8(-9)^2 - 3 \\ &= 8(81) - 3 \\ &= 648 - 3 \\ &= 645 \end{aligned}$$

(vi) $s\left(-\frac{7}{2}\right)$

Solution:

Finding $s\left(-\frac{7}{2}\right)$

Put $x = -\frac{7}{2}$ in $s(x)$

$$\begin{aligned} s\left(-\frac{7}{2}\right) &= 8\left(-\frac{7}{2}\right)^2 - 3 \\ &= 8\left(\frac{49}{4}\right) - 3 \\ &= 2(49) - 3 \\ &= 98 - 3 \\ &= 95 \end{aligned}$$

Q. 11 Given that $f(x) = ax + b$, where a and b are constant numbers. If $f(-2) = 3$ and $f(4) = 10$, then find the values of a and b.
Solution:

$$f(-2) = 3 - - - - (\text{i})$$

$$f(4) = 10 - - - - (\text{ii})$$

$$f(x) = ax + b$$

To find $\text{f}(-2)$, put $x = -2$ in $\text{f}(x)$.

$$f(-2) = a(-2) + b$$

$$f(-2) = -2a + b$$

Putting value from (i)

$$2a - b = -3 - - - - \quad \text{(iii)}$$

To find $\text{f}(4)$, put $x = 4$ in $\text{f}(x)$

$$f(4) = a(4) + b$$

$$f(4) = 4a + b$$

from (ii), putting value.

$$10 = 4a + b$$

$$\Rightarrow 4a + b = 10 - - - - (\text{iv})$$

Adding eq. (iii) (iv)

$$2a - \cancel{b} = -3$$

$$4a + \cancel{b} = 10$$

$$6a = 7$$

$$a = \frac{7}{6}$$

Put ii in eq. (iv)

$$4\left(\frac{7}{6}\right) + b = 10$$

$$\frac{2 \times 7}{3} + b = 10$$

$$\frac{14}{3} + b = 10$$

$$b = 10\frac{14}{3}$$

$$b = \frac{30 - 14}{3}$$

$$b = \frac{16}{3}$$

Q. 12 Consider the function defined by $k(x) = 7x - 5$. If $k(x) = 100$, find the value of x.

Solution:

$$k(x) = 7x - 5$$
$$k(x) = 100$$

By comparing, we get

$$7x - 5 = 100$$
$$7x = 100 + 5$$
$$7x = 105$$
$$x = \frac{105}{7}$$
$$x = 15$$

Q. 13 Consider the function $g(x) = mx^2 + n$, where m and n are constant numbers. If $g(4) = 20$ and $g(0) = 5$, find the values of m and n.

Solution:

$$g(x) = mx^2 + n$$
$$g(4) = 20 - - - - - - (1)$$
$$g(0) = 5 - - - - - - - - (2)$$

Put $x = 4$ in $g(x)$

$$g(4) = m(4)^2 + n$$
$$g(4) = m(16) + n$$
$$g(4) = 16m + n$$

From (1) Put $g(4) = 20$ i.e.

$$20 = 16m + n$$
$$\Rightarrow 16m + n = 20 - - - - - - (3)$$

Now, put $x = 0$ in $g(x)$

$$g(0) = m(0)^2 + n$$
$$g(0) = m(0) + n$$
$$g(0) = n$$

From (2), $g(0) = 5$, so

$$5 = n$$
$$\Rightarrow n = 5$$

Put it in eq. (3)

$$16m + 5 = 20$$
$$16m = 20 - 5$$
$$16m = 15$$
$$m = \frac{15}{16}$$

Q. 14 A shopping mall has 100 products from various categories labeled 1 to 100 , representing the universal set U.The products are categorized as follows:
Set A. Electronics, consisting of 30 products labeled from 1 to 30.
Set B: Clothing comprises 25 products labeled from 31 to 55.
Set C: Beauty Products, comprising 25 products labeled from 76 to 100. Write each set in tabular form,and find the union of all three sets.
Solution:
Tubular form:

$$\mathrm{U} = \{1,2,3,4,\ldots,100\}, \mathrm{n(U)} = 100$$

$$\mathrm{A} = \{1,2,3,4,\ldots,30\}, \mathrm{n(A)} = 30$$

$$B = \{31,32,33,\ldots.. \quad 55\}, n(B) = 28$$

$$C = \{76,77,78,\ldots\ldots \quad 100\}, n(C) = 25$$

Finding union of all three sets

$$(\mathrm{A} \cup \mathrm{B}) \cup \mathrm{C}$$

$=(\{1,2,3,4,\ldots,30\} \cup \{31,32,33,\ldots..55\}) \cup \{76,77,78,\ldots\ldots100\}$
$=\{1,2,3,4,5,\ldots\ldots\ldots,55\} \cup \{76,77,78,79,\ldots\ldots.\ .100\}$
$=\{1,2,3,4,\ldots\ldots.,55,75,76,77,78,79,\ldots\ldots..100\}$

Q. 15 Out of the 180 students who appeared in the annual examination, 120 passed the math test, 90 passed the science test, and 60 passed both the math and science tests.
(a) How many passed either the math or science test?
(b) How many did not pass either of the two test?
(c) How many passed the science test but not the math test?
(d) How many failed the science test?
Solution:
Total students = 180
Let A and B are the sets representing the students who passed Math test and Science test respectively.
120 passed Math test, i.e. $\mathrm{n(A)} = 120$
90 passed Science test Le. $n(B) = 90$
60 passed both Math and Science test

$$n(A \cap B) = 90$$

(a) Students who passed either the math or science test is $n(A \cup B)$.
We know that

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$= 120 + 90 - 60$$
$$= 120 + 30$$
$$= 150$$

(b) Students who did not pass either of two test is $n(A \cup B)'$.

$$n(A \cup B)' = n(U) - n(A \cup B)$$
$$= 180 - 150$$
$$= 30$$

Thus 30 students could not pass any subject.

(c) Students only passed in science test.

$$= n(B) - n(A \cap B)$$
$$= 90 - 60$$
$$= 30$$

Thus 30 students passed only science test.

(d) Failed in science test

$$N(B)' = n(U) - n(B)$$
$$= 180 - 90$$
$$= 90$$

Thus 90 students failed in science test.

Q. 16 In a software house of a city with 300 software developers, a survey was conducted to determine which programming languages are liked more. The survey revealed the following statistics:

I50 developers like Python.
130 developers like Java.
120 developers like PHP.
70 developers like both Python and Java.
60 developers like both Python and PHP.
40 developers like all three languages: Python, Java and PHP.
(a) How many developers use at least one of these languages?
(b) How many developers use only of these languages?
(c) How many developers do not use any of these languages?
(d) How many developers use only PHP?

40 developers like all three languages: Python, Java and PHP.
(a) How many developers use at least one of these languages?
(b) How many developers use only of these languages?
(c) How many developers do not use any of these languages?
(d) How many developers use only PHP?

Solution:

Let A,B and C be three sets representing the software developers who like python, Java and PHP respectively.

Total soft ware developers $= n(U) = 300$

150 like python, $n(A) = 150$

130 like java, $n(B) = 130$

120 like PHP, $n(C) = 120$

70 like both python and Java $= n(A \cap B) = 70$

50 like both Java and PHP $= n(A \cap C) = 50$

60 like both python and PHP $= n(A \cap C) = 60$

40 like all the three languages $= n(A \cap B \cap C) = 40$

(a) Using principle of inclusion and exclusion.

$$n(A \cup B \cup C) = n(A) + n(B) + n(c) - n(A \cap B)$$
$$- n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$
$$= 150 + 130 + 120 - 70 - 50 - 60 + 40$$
$$= 440 - 180$$

=260

(b) Developers who like python only:

$$= n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C)$$
$$= 150 - 70 - 60 + 40$$
$$= 190 - 130 = 60$$

Developers who like Java only:

$$= n(B) - n(A \cap B) - n(B \cap C) + n(A \cap B \cap C)$$
$$= 130 - 70 - 50 + 40$$
$$= 170 - 120$$
$$= 50$$

Developers who like PHP only:

$$= n(C) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$
$$= 120 - 50 - 60 + 40$$
$$= 160 - 110$$
$$= 50$$

Developers who use only one of these.

Languages $= 60 + 50 + 50 = 160$

(c)

$$n(A \cup B \cup C)' = n(U) - n(A \cup B \cup C)$$
$$= 300 - 260$$
$$= 40$$

(d) developers who use only PHP only:

$$= n(C) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C$$
$$= 120 - 50 - 60 + 40$$
$$= 160 - 110$$
$$= 50$$