

Review Exercise 4

Q. 1 Choose the correct option.

(i) The factorization of $12x + 36$ is:

(a) $12(x + 3)$ (b) $12(3x)$ (c) $12(3x + 1)$ (d) $x(12 + 36x)$

(ii) The factors of $4x^2 - 12xy + 9y^2$ are:

(a) $(2x + 3)^2$ (b) $(2x - 3)^2$ (c) $(2x - 3)(2x + 3)$ (d) $(2 + 3x)(2 - 3x)^2$

iii. The HCF of a^3b^3 and ab^2 is:

(a) a^3b^3 (b) ab^2 (c) a^4b^5 (d) a^2b

iv. The LCM of $16x^2$, $4x$ and $30xy$ is:

(a) $480x^3y$ (b) $240xy$ (c) $240x^2y$ (d) $120x^4y$

v. Product of LCM and HCF = ----- of two polynomials.

(a) sum (b) difference (c) product (d) quotient

vi. The square root of $x^2 - 6x + 9$ is:

(a) $\pm(x - 3)$ (b) $\pm(x + 3)$ (c) $x - 3$ (d) $x + 3$

vii. The LCM of $(a - b)^2$ and $(a - b)^4$ is:

(a) $(a \cdot b)^2$ (b) $(a - b)^3$ (c) $(a - b)^4$ (d) $(a - b)^6$

(viii) Factorization of $x^3 + 3x^2 + 3x + 1$ is:

(a) $(x + 1)^3$ (b) $(x - 1)^3$ (c) $(x + 1)(x^2 + x + 1)$ (d) $(x - 1)(x^2 - x + 1)$

ix. Cubic polynomial has degree:

(a) 1 (b) 2 (c) 3 (d) 4

x. One of the factors of $x^3 - 27$ is:

(a) $x - 3$ (b) $x + 3$ (c) $x^2 - 3x + 9$ (d) Both a and c

i	a	ii	b	iii	b	iv	c	v	c
vi	a	vii	c	viii	a	ix	c	x	a

Multiple Choice Questions (Additional)

1. The degree of quadratic polynomial is:

(a) 1 (b) 2 (c) 3 (d) -2

2. The factors of $x^2 - 5x + 6$ are:

(a) $x + 1, x - 6$ (b) $x - 2, x - 3$ (c) $x + 6, x - 1$ (d) $x + 2, x + 3$

3. Factors of $3x^2 - x - 2$ are:

(a) $(x + 1)(3x - 2)$ (c) $(x - 1)(3x - 2)$ (b) $(x + 1)(3x + 2)$ (d) $(x - 1)(3x + 2)$

4. Factors of $x^4 - y^4$ are:

- (a) $(x - y)(x + y)(x^2 + y^2)$ (b) $(x - y)(x^2 + y^2)$
(c) $(x - y)(x + y)(x^2 - y^2)$ (d) $(x + y)(x^2 + y^2)$

5. What will be added to complete the square of $9a^2 - 12ab$?

- (a) $-16b^2$ (b) $16b^2$ (c) $4b^2$ (d) $-4b^2$

6. Find m so that $x^2 + 8x + m$ is a complete square:

- (a) 8 (b) -8 (c) 4 (d) 16

7. What should be added to complete the square of $y^4 + 81$?

- (a) $18y^2$ (b) $-18y^2$ (c) $9y^2$ (d) $18Y$

8. Let $5x^2 - 17xy - 12y^2 = A \times B$ if $A = (x - 4y)$ then B is:

- (a) $(5x + 3y)$ (b) $(5x - 3y)$ (c) $(5x + 3y)$ (d) $(5x - 4y)$

9. Factors of $8x^3 - y^3$ are:

- (a) $(2x + y)(4x^2 + 2xy - y^2)$ (b) $(2x + y)(4x^2 + 2xy + y^2)$ (c) $(2x - y)(4x^2 - 2xy + y^2)$ (d) $(2x - y)(4x^2 + 2xy + y^2)$
 $(x + y)(x^2 - xy + y^2) =$

- (a) $x^3 - y^3$ (b) $\overline{x^3} + y^3$ (c) $(x + y)^3$ (d) $(x - y)^3$

Factors of $x^4 - 16$ is:

- (a) $(x - 2)^2$ (b) $(x - 2)(x + 2)(x^2 + 4)$ (c) $(x - 2)(x + 2)$ (d) $(x + 2)^2$

12. HCF of $x^3y - xy^3$ and $x^3y^2 - x^2y^3$ is

- (a) $xy(x^2 - y^2)$ (b) $xy(x - y)$ (c) $x^2y^2(x - y)$ (d) $xy(x^3 - y^3)$

13. H C.F of $35a^2b^2$ and $20a^3b^3$ is

- (a) $5a^2b^2$ (b) $20a^3b^3$ (c) $35a^5b^5$ (d) $5ab$

14. H C F of $m - 2$ and $m^2 + m - 6$ is

- (a) $m^2 + m - 6$ (b) $m + 2$ (c) $m - 2$ (d) $m + 3$

15. H C.F of $a^3 + b^3$ and $a^2 - ab + b^2$ is

- (a) $a + b$ (b) $a^2 - ab + b^2$ (c) $(a - b)^2$ (d) $a^2 + b^2$

16. H C F of $a^2 - b^2$ and $a^3 - b^3$ is:

- (a) $a - b$ (b) $a + b$ (c) $a^2 + ab + b^2$ (d) $a^2 - ab + b^2$

17. L.C.M of $15x^2z$, $45xy^2$ and $30y^2$ is:

- (a) $90xyz$ (b) $90x^2y^2z^2$ (c) $90x^3y^3z^3$ (d) $15x^2yz$

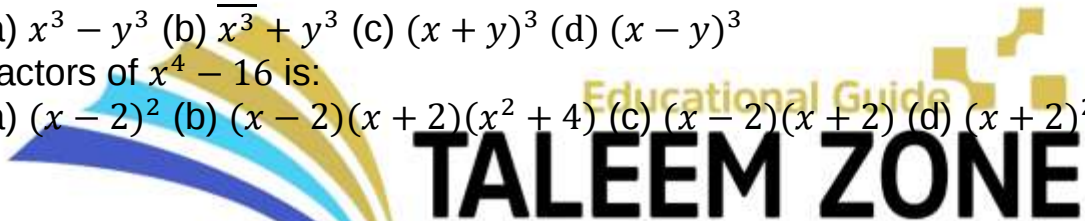
18. L.C.M of $a^2 - b^2$ and $a^4 - b^4$ is:

- (a) $a^2 + b^2$ (b) $a^2 - b^2$ (c) $a^4 - b^4$ (d) $a - b$

19. The square root of $x^2 - 6x + 9$ is:

- (a) $\pm(x + 3)$ (b) $\pm(x - 3)$ (c) $x - 3$ (d) $x + 3$

20. The product of two polynomials is equal to the _____ of their H.C.F and _____



L.C.M.

(a) Sum (b) Difference (c) Product (d) Quotient

Answer Key

1	b	2	b	3	d	4	a	5	c	6	d	7	a	8	c	9	d	$\frac{1}{0}$	b
$\frac{1}{1}$	b	$\frac{1}{2}$	b	$\frac{1}{3}$	a	$\frac{1}{4}$	c	$\frac{1}{5}$	b	$\frac{1}{6}$	a	$\frac{1}{7}$	b	$\frac{1}{8}$	c	$\frac{1}{9}$	b	$\frac{2}{0}$	c

Q. 2 Factorize the following expressions:

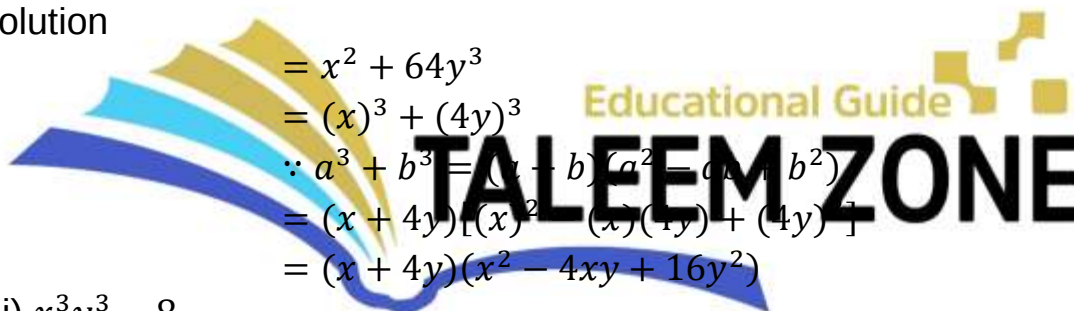
(i) $4x^3 + 18x^2 - 12x$

Solution

$$\begin{aligned}
 &4x^3 + 18x^2 - 12x \\
 &= 2x(2x^2 + 9x - 6) \\
 &= 2x(2x^2 + 9x - 6)
 \end{aligned}$$

(ii) $x^3 + 64y^3$

Solution



$$\begin{aligned}
 &= x^3 + 64y^3 \\
 &= (x)^3 + (4y)^3 \\
 &\because a^3 + b^3 = (a + b)(a^2 - ab + b^2) \\
 &= (x + 4y)[(x)^2 - (x)(4y) + (4y)^2] \\
 &= (x + 4y)(x^2 - 4xy + 16y^2)
 \end{aligned}$$

(iii) $x^3y^3 - 8$

Solution

$$\begin{aligned}
 &= (xy)^3 - (2)^3 \\
 &\because a^3 - b^3 = (a - b)(a^2 + ab + b^2) \\
 &= (xy - 2)[(xy)^2 + (xy)(2) + (2)^2] \\
 &= (xy - 2)(x^2y^2 + 2xy + 4)
 \end{aligned}$$

(iv) $-x^2 - 23x - 60$

Solution

$$\begin{aligned}
 &-x^2 - 23x - 60 \\
 &= -1(x^2 + 23x + 60) \\
 &= -1[x^2 + 20x + 3x + 60] \\
 &= -1[x(x + 20) + 3(x + 20)] \\
 &= -1(x + 20)(x + 3)
 \end{aligned}$$

(v) $2x^2 + 7x + 3$

Solution

$$\begin{aligned} & 2x^2 + 7x + 3 \\ &= 2x^2 + x + 6x + 3 \\ &= x(2x + 1) + 3(2x + 1) \\ &= (2x + 1)(x + 3) \end{aligned}$$

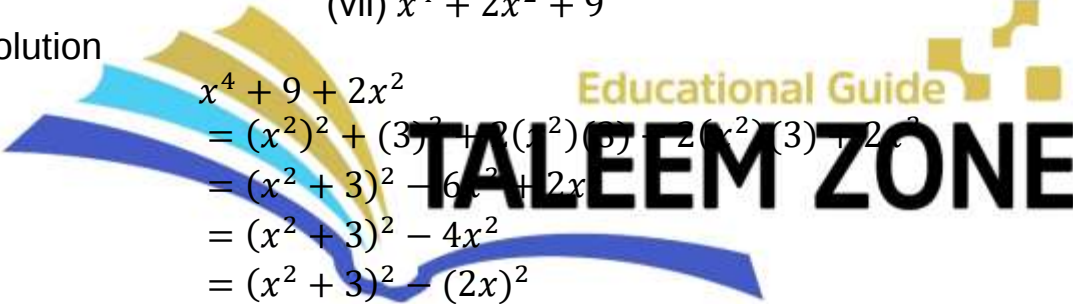
(vi) $x^4 + 64$

Solution

$$\begin{aligned} & x^4 + 64 \\ &= (x^2)^2 + (8)^2 + 2(x^2)(8) - 2(x^2)(8) \\ &\therefore (a + b)^2 = a^2 + 2ab + b^2 \\ &= (x^2 + 8)^2 - 16x^2 \\ &= (x^2 + 8)^2 - (4x)^2 \\ &\quad \therefore a^2 - b^2 = (a + b)(a - b) \\ &= (x^2 + 8 + 4x)(x^2 + 8 - 4x) \\ &= (x^2 + 4x + 8)(x^2 - 4x + 8) \end{aligned}$$

(vii) $x^4 + 2x^2 + 9$

Solution


$$\begin{aligned} & x^4 + 9 + 2x^2 \\ &= (x^2)^2 + (3)^2 + 2(x^2)(3) - 2(x^2)(3) + 2x^2 \\ &= (x^2 + 3)^2 - 6x^2 + 2x^2 \\ &= (x^2 + 3)^2 - 4x^2 \\ &= (x^2 + 3)^2 - (2x)^2 \\ &= (x^2 + 3 + 2x)(x^2 + 3 - 2x) \\ &= (x^2 + 2x + 3)(x^2 - 2x + 3) \end{aligned}$$

(viii) $(x + 3)(x + 4)(x + 5)(x + 6) - 360$

Solution:

$$(x + 3)(x + 4)(x + 5)(x + 6) - 360$$

By rearranging the terms,

$$= (x + 3)(x + 6)(x + 4)(x + 5) - 360$$

$$(\because 3 + 6 = 4 + 5)$$

$$= (x^2 + 6x + 3x + 18)(x^2 + 5x + 4x + 20) - 360$$

$$= (x^2 + 9x + 18)(x^2 + 9x + 20) - 360 \dots - (i)$$

Let $x^2 + 9x = y$

(ii)

Now, expression can be written as:

$$\begin{aligned}
&= (y + 18)(y + 20) - 360 \\
&= y^2 + 20y + 18y + 360 - 360 \\
&= y^2 + 38y \\
&= y(y + 38) \quad (\because y = x^2 + 9x) \\
\Rightarrow &= (x^2 + 9x)(x^2 + 9x + 38) \\
&= x(x + 9)(x^2 + 9x + 38)
\end{aligned}$$

(ix) $(x^2 + 6x + 3)(x^2 + 6x - 9) + 36$

Solution:

$$(x^2 + 6x + 3)(x^2 + 6x - 9) + 36$$

Let $x^2 + 6x = y$

Now, expression (i) can be written as:

$$\begin{aligned}
&= (y + 3)(y - 9) + 36 \\
&= y^2 - 9y + 3y - 27 + 36 \\
&= y^2 - 6y + 9 \\
&= (y)^2 - 2(y)(3) + (3)^2 \\
&= (y - 3)^2
\end{aligned}$$

$$\because x^2 + 6x = y$$

$$= (x^2 + 6x - 3)^2$$

Q. 3 Find LCM and HCF by prime factorization method:

(i) $4x^3 + 12x^2, 8x^2 + 16x$

Solution:

$$4x^3 + 12x^2, 8x^2 + 16x$$

Factorization

$$4x^3 + 12x^2 = 4x^2(x + 3) = (2) \times (2) \cdot (x) x(x + 3)$$

$$8x^2 + 16x = 8x(x + 2) = 2 \times (2) \times (2) \cdot (x) (x + 2)$$

$$\text{H.C.F} = 2 \times 2 \times x = 4x$$

L.C.M

$$\text{Product of common factors} = 2 \times 2 \times x = 4x$$

Product of non-common factors

$$= x(x + 3) \times 2(x + 2)$$

$$= 2x(x + 3)(x + 2)$$

$$\text{L.C.M} = (4x)[(2x(x + 3)(x + 2))]$$

$$= 8x^2(x + 3)(x + 2)$$

(ii) $x^3 + 3x^2 - 4x \cdot x^2 - 4x + 3$

Solution:

$$\begin{aligned}
 x^3 + 3x^2 - 4x &= x(x^2 + 3x - 4) \\
 &= x(x^2 + 4x - x - 4) \\
 &= x[x(x + 4) - 1(x + 4)] \\
 &= x(x + 4)(x - 1) \\
 x^4 - 4x + 3 &= x^2 - 3x - x + 3 \\
 &= x(x - 3) - 1(x - 3) \\
 &= (x - 3)(x - 1)
 \end{aligned}$$

$$\text{H.C. } F = (x - 1)$$

$$\begin{aligned}
 &= (x - 1) \times x(x - 3)(x + 4) \\
 &= x(x - 1)(x - 3)(x + 4)
 \end{aligned}$$

$$\text{(iii) } x^2 + 8x + 16, x^2 - 16$$

Solution:

$$x^2 + 8x + 16 \cdot x^2 - 16$$

Solution:

$$\begin{aligned}
 x^2 + 8x + 16 &= (x)^2 + 2(x)(4) + (4)^2 \\
 &= (x + 4)^2 \\
 &= (x + 4)(x + 4)
 \end{aligned}$$

$$\begin{aligned}
 x^2 - 16 &= (x)^2 - (4)^2 \\
 &= (x + 4)(x - 4)
 \end{aligned}$$

$$\text{H.C. } F = (x + 4)$$

$$\text{L.C.M} = [\text{common factor}] \times [\text{non common factors}]$$

$$\begin{aligned}
 &= x[(x + 4)] \times [(x + 4)(x - 4)] \\
 &= (x + 4)^2(x - 4)
 \end{aligned}$$

$$\text{(iv) } x^3 - 9x, x^2 - x - 6$$

Solution:

$$\begin{aligned}
 x^3 - 9x &= x(x^2 - 9) \\
 &= x[(x)^2 - (3)^2] \\
 &= x(x + 3)(x - 3)
 \end{aligned}$$

$$\begin{aligned}
 x^2 - x - 6 &= x^2 - 3x + 2x - 6 \\
 &= x(x - 3) + 2(x - 3) \\
 &= (x - 3)(x + 2)
 \end{aligned}$$

$$\text{H.C.F} = (x - 3)$$

$$\text{L.C.M} = \text{common factors} \times \text{non common factors}$$

$$\begin{aligned}
 &= (x - 3) \times x(x + 2)(x + 3) \\
 &= x(x + 2)(x - 3)(x + 3) \\
 &= x(x + 2)[(x)^2 - (3)^2] \\
 &= x(x + 2)(x^2 - 9)
 \end{aligned}$$

Q. 4 Find square root by factorization and division method of the expression $16x^4 + 8x^2 + 1$.

Solution

$$16x^4 + 8x^2 + 1$$

Square root by factorization

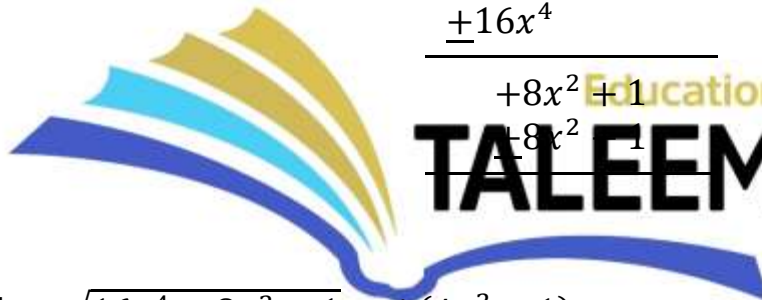
$$\begin{aligned} 16x^4 + 8x^2 + 1 \\ &= (4x^2)^2 + 2(4x^2)(1) + (1)^2 \\ &= (4x^2 + 1)^2 \end{aligned}$$

Taking square of both sides

$$\begin{aligned} \sqrt{16x^4 + 8x^2 + 1} &= \pm\sqrt{(4x^2 + 1)^2} \\ &= \pm(4x^2 + 1) \end{aligned}$$

Square root by division method:

$$8x^2 + 1$$



$$\begin{array}{r} 4x^2 + 1 \\ \hline 16x^4 + 8x^2 + 1 \\ \underline{+16x^4} \\ +8x^2 + 1 \\ \underline{+8x^2 + 1} \\ 0 \end{array}$$

Thus $\sqrt{16x^4 + 8x^2 + 1} = \pm(4x^2 + 1)$

Q. 5 Huria is analyzing the total cost of her loan, modeled by the expression $C(x) = x^2 - 8x + 15$, where x represents the number of years. What is the optimal repayment period for Huria's loan?

Solution

$$\begin{aligned} C(x) &= x^2 - 8x + 15 \\ C(x) &= x^2 - 5x - 3x + 15 \\ C(x) &= x(x - 5) - 3(x - 5) \\ &= (x - 5)(x - 3) \end{aligned}$$

The factorized form of expression shows that cost of loan is zero at $x = 5$ and $x = 3$.

So, the optional repayment period is 3 years or 5 years.