Review Exercise 4

Q. 1 Choose the correct option.

(i)The factorization of $12x + 36$ is:

(a) $12(x + 3)$ (b) $12(3x)$ (c) $12(3x + 1)$ (d) $x(12 + 36x)$

(ii) The factors of $4x^2\ 12y + 9$ are:

(a) $(2x + 3)^2$ (b) $(2x - 3)^2$ (c) $(2x - 3)(2x + 3)$ (d) $(2 + 3x)(2 - 3x)^2$

iii. The HCF of a^3b^3 and ab^2 is:

(a) a^3b^3 (b) ab^2 (c) a^4b^5 (d) a^2b

iv. The LCM of $16x^2, 4x$ and $30xy$ is:

(a) $480x^3y$ (b) $240xy$ (c) $240x^2y$ (d) $120x^4y$

v. Product of LCM and HCF =---------- of two polynomials.

(a) sum (b) difference (c) product (d) quotient

vi. The square root of $x^2 - 6x + 9$ is:

(a) $\pm(x - 3)$ (b) $\pm(x + 3)$ (c) $x - 3$ (d) $x + 3$

vii. The LCM of $(a - b)^2$ and $(a - b)^4$ is:

(a) $(a \cdot b)^2$ (b) $(a - b)^3$ (c) $(a - b)^4$ (d) $(a - b)^6$

(viii) Factorization of $x^3 + 3x^2 + 3x + 1$ is:

(a) $(x + 1)^3$ (b) $(x - 1)^3$ (c) $(x + 1)(x^2 + x + 1)$ (d) $(x - 1)(x^2 - x + 1)$

ix. Cubic polynomial has degree:

(a) 1 (b) 2 (c) 3 (d) 4

x. One of the factors of $x^3 - 27$ is:

(a) $x - 3$ (b) $x + 3$ (c) $x^2 - 3x + 9$ (d) Both a and c

i	a	ii	b	iii	b	iv	c	v	c
vi	a	vii	c	viii	a	ix	c	x	a

Multiple Choice Questions (Additional)

1. The degree of quadratic polynomial is:

(a) 1 (b) 2 (c) 3 (d) -2

2. The factors of $x^2 - 5x + 6$ are:

(a) $x + 1, x - 6$ (b) $x - 2, x - 3$ (c) $x + 6, x - 1$ (d) $x + 2, x + 3$

3. Factors of $3x^2 - x - 2$ are:

(a) $(x + 1)(3x - 2)$ (c) $(x - 1)(3x - 2)$ (b) $(x + 1)(3x + 2)$ (d) $(x - 1)(3x + 2)$

4. Factors of $x^4 - y^4$ are:
(a) $(x-y)(x+y)(x^2+y^2)$ (b) $(x-y)(x^2+y^2)$
(c) $(x-y)(x+y))(x^2-y^2)$ (d) $(x+y)(x^2+y^2)$
5. What will be added to complete the square of $9a^2 - 12ab$?
(a) $-16b^2$ (b) $16b^2$ (c) $4b^2$ (d) $-4b^2$
6. Find m so that $x^2 + 8x + m$ is a complete square:
(a) 8 (b) -8 (c) 4 (d) 16
7. What should be added to complete the square of $y^4 + 81$?
(a) $18y^2$ (b) $-18y^2$ (c) $9y^2$ (d)18Y
8. Let $5x^2 - 17xy - 12y^2 = A \times B$ if $A = (x - 4y)$ then B is:
(a) $(5x + 3y)$ (b) $(5x - 3y)$ (c) $(5x + 3y)$ (d) $(5x - 4y)$
9. Factors of $8x^3 - y^3$ are:
(a) $(2x + y)(4x^2 + 2xy - y^2)$ (b) $(2x + y)(4x^2 + 2xy + y^2)$ (c) $(2x - y)(4x^2 - 2xy + y^2)$ (d) $(2x - y)(4x^2 + 2xy + y^2)$
$(x + y)(x^2 - xy + y^2) =$
(a) $x^3 - y^3$ (b) $x^3 + y^3$ (c) $(x + y)^3$ (d) $(x - y)^3$
Factors of $x^4 - 16$ is:
(a) $(x-2)^2$ (b) $(x-2)(x+2)(x^2+4)$ (c) $(x-2)(x+2)$ (d) $(x+2)^2$

12. HCF of $x^3y - xy^3$ and $x^3y^2 - x^2y^3$ is
(a) $xy(x^2 - y^2)$ (b) $xy(x - y)$ (c) $x^2y^2(x - y)$ (d) $xy(x^3 - y^3)$
13. H C.F of $35a^2b^2$ and $20a^3b^3$ is
(a) $5a^2b^2$ (b) $20a^3b^3$ (c) $35a^5b^5$ (d) 5 ab
14. H C F of $m - 2$ and $m^2 + m - 6$ is
(a) $m^2 + m - 6$ (b) $m + 2$ (c) $m - 2$ (d) $m + 3$
15. H C.F of $a^3 + b^3$ and $a^2 - ab + b^2$ is
(a) $a + b$ (b) $a^2 - ab + b^2$ (c) $(a - b)^2$ (d) $a^2 + b^2$
16. H C F of $a^2 - b^2$ and $a^3 - b^3$ is:
(a) $a - b$ (b) $a + b$ (c) $a^2 + ab + b^2$ (d) $a^2 - ab + b^2$

17. L.C.M of $15x^2z, 45xy^2$ and $30y^2$ is:
(a) $90xyz$ (b) $90x^2y^2z^2$ (c) $90x^3y^3z^3$ (d) $15x^2yz$
18. L.C.M of $a^2 - b^2$ and $a^4 - b^4$ is:
(a) $a^2 + b^2$ (b) $a^2 - b^2$ (c) $a^4 - b^4$ (d) $a - b$
19. The square root of $x^2 - 6x + 9$ is:
(a) $\pm(x + 3)$ (b) $\pm(x - 3)$ (c) $x - 3$ (d) $x + 3$
20. The product of two polynomials is equal to the of their H.C.F and

L.C.M.
(a) Sum (b) Difference (c) Product (d) Quotient
Answer Key

1	b	2	b	3	d	4	a	5	c	6	d	7	a	8	c	9	d	10	b
11	b	12	b	13	a	14	c	15	b	16	a	17	b	18	c	19	b	20	c

Q. 2 Factorize the following expressions:
(i) $4x^3 + 18x^2 - 12x$
Solution

$$4x^3 + 18x^2 - 12x$$
$$= 2x(2x^2 + 9x - 6)$$
$$= 2x(2x^2 + 9x - 6)$$

(ii) $x^3 + 64y^3$
Solution

$$= x^2 + 64y^3$$
$$= (x)^3 + (4y)^3$$
$$\because a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$
$$= (x + 4y)[(x)^2 - (x)(4y) + (4y)^2]$$
$$= (x + 4y)(x^2 - 4xy + 16y^2)$$

(iii) $x^3y^3 - 8$
Solution

$$= (xy)^3 - (2)^3$$
$$\because a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$
$$= (xy - 2)[(xy)^2 + (xy)(2) + (2)^2]$$
$$= (xy - 2)(x^2y^2 + 2xy + 4)$$

(iv) $-x^2 - 23x - 60$

Solution

$$-x^2 - 23x - 60$$
$$= -1(x^2 + 23x + 60)$$
$$= -1[x^2 + 20x + 3x + 60]$$
$$= -1[x(x + 20) + 3(x + 20)]$$
$$= -1(x + 20)(x + 3)$$

(v) $2x^2 + 7x + 3$

Solution

$$2x^2 + 7x + 3$$
$$= 2x^2 + x + 6x + 3$$
$$= x(2x + 1) + 3(2x + 1)$$
$$= (2x + 1)(x + 3)$$

(vi) $x^4 + 64$

Solution

$$x^4 + 64$$
$$= (x^2)^2 + (8)^2 + 2(x^2)(8) - 2(x^2)(8)$$
$$\therefore (a + b)^2 = a^2 + 2ab + b^2$$
$$= (x^2 + 8)^2 - 16x^2$$
$$= (x^2 + 8)^2 - (4x)^2$$
$$\because a^2 - b^2 = (a + b)(a - b)$$
$$= (x^2 + 8 + 4x)(x^2 + 8 - 4x)$$
$$= (x^2 + 4x + 8)(x^2 - 4x + 8)$$

(vii) $x^4 + 2x^2 + 9$

Solution

$$x^4 + 9 + 2x^2$$
$$= (x^2)^2 + (3)^2 + 2(x^2)(3) - 2(x^2)(3) + 2x^2$$
$$= (x^2 + 3)^2 - 6x^2 + 2x^2$$
$$= (x^2 + 3)^2 - 4x^2$$
$$= (x^2 + 3)^2 - (2x)^2$$
$$= (x^2 + 3 + 2x)(x^2 + 3 - 2x)$$
$$= (x^2 + 2x + 3)(x^2 - 2x + 3)$$

(viii) $(x + 3)(x + 4)(x + 5)(x + 6) - 360$

Solution:

$$(x + 3)(x + 4)(x + 5)(x + 6) - 360$$

By rearranging the terms,

$$= (x + 3)(x + 6)(x + 4)(x + 5) - 360$$
$$(\because 3 + 6 = 4 + 5)$$
$$= (x^2 + 6x + 3x + 18)(x^2 + 5x + 4x + 20) - 360$$
$$= (x^2 + 9x + 18)(x^2 + 9x + 20) - 36 \cdots - (\mathrm{i})$$

Let $x^2 + 9x = y$

(ii)

Now, expression can be written as:

$$= (y + 18)(y + 20) - 360$$
$$= y^2 + 20y + 18y + 360 - 360$$
$$= y^2 + 38y$$
$$= y(y + 38)\ (\because y = x^2 + 9x)$$
$$\Rightarrow = (x^2 + 9x)(x^2 + 9x + 38)$$
$$= x(x + 9)(x^2 + 9x + 38)$$

(ix) $(x^2 + 6x + 3)(x^2 + 6x - 9) + 3609304147$

Solution:

$$(x^2 + 6x + 3)(x^2 + 6x - 9) + 36$$

Let $x^2 + 6x = y$

Now, expression (i) can be written as:

$$= (y + 3)(y - 9) + 36$$
$$= y^2 - 9y + 3y - 27 + 36$$
$$= y^2 - 6y + 9$$
$$= (y)^2 - 2(y)(3) + (3)^2$$
$$= (y - 3)^2$$
$$\because x^2 + 6x = y$$
$$= (x^2 + 6x - 3)^2$$

Q. 3 Find LCM and HCF by prime factorization method:

(i) $4x^3 + 12x^2, 8x^2 + 16x$

Solution:

$$4x^3 + 12x^2 . 8x^2 + 16x$$

Factorization

$$4x^3 + 12x^2 = 4x^2(x + 3) = (2) \times (2) . (x)\ x(x + 3)$$
$$8x^2 + 16x = 8x(x + 2) = 2 \times (2) \times (2). (x)\ (x + 2)$$

H.C.F $= 2 \times 2 \times x = 4x$

L.C.M

Product of common factors $= 2 \times 2 \times x = 4x$

Product of non-common factors

$$= x(x + 3) \times 2(x + 2)$$
$$= 2x(x + 3)(x + 2)$$

L.C.M $= (4x)[(2x(x + 3)(x + 2)]$

$$= 8x^2(x + 3)(x + 2)$$

(ii) $x^3 + 3x^2 - 4x \cdot x^2 - 4x + 3$

Solution:

$$x^3 + 3x^2 - 4x = x(x^2 + 3x - 4)$$
$$= x(x^2 + 4x - x - 4)$$
$$= x[x(x+4) - 1(x+4)]$$
$$= x(x+4)(x-1)$$
$$x^4 - 4x + 3 = x^2 - 3x - x + 3$$
$$= x(x-3) - 1(x-3)$$
$$= (x-3)(x-1)$$

H.C. $F = (x-1)$

$$= (x-1) \times x(x-3)(x+4)$$
$$= x(x-1)(x-3)(x+4)$$

(iii) $x^2 + 8x + 16, x^2 - 16$

Solution:

$$x^2 + 8x + 16 \cdot x^2 - 16$$

Solution:

$$x^2 + 8x + 16 = (x)^2 + 2(x)(4) + (4)^2$$
$$= (x+4)^2$$
$$= (x+4)(x+4)$$
$$x^2 - 16 = (x)^2 - (4)^2$$
$$= (x+4)(x-4)$$

H.C. $F = (x+4)$

L.C.M = [common factor] × [non common factors]

$$= x[(x+4)] \times [(x+4)(x-4)]$$
$$= (x+4)^2(x-4)$$

(iv) $x^3 - 9x, x^2 - x - 6$

Solution:

$$x^3 - 9x = x(x^2 - 9)$$
$$= x[(x)^2 - (3)^2]$$
$$= x(x+3)(x-3)$$
$$x^2 - x - 6 = x^2 - 3x + 2x - 6$$
$$= x(x-3) + 2(x-3)$$
$$= (x-3)(x+2)$$

H.C.F $= (x-3)$

L.C.M = common factors × non common factors

$$= (x-3) \times x(x+2)(x+3)$$
$$= x(x+2)(x-3)(x+3)$$
$$= x(x+2)[(x)^2 - (3)^2]$$
$$= x(x+2)(x^2 - 9)$$

Q. 4 Find square root by factorization and division method of the expression $16x^4 + 8x^2 + 1$.

Solution

$$16x^4 + 8x^2 + 1$$

Square root by factorization

$$\begin{aligned} &16x^4 + 8x^2 + 1 \\ &= (4x^2)^2 + 2(4x^2)(1) + (1)^2 \\ &= (4x^2 + 1)^2 \end{aligned}$$

Taking square of both sides

$$\begin{aligned} \sqrt{16x^4 + 8x^2 + 1} &= \pm\sqrt{(4x^2 + 1)^2} \\ &= \pm(4x^2 + 1) \end{aligned}$$

Square root by division method:

$8x^2 + 1$

$$\begin{array}{r|l} & 4x^2 + 1 \\ \hline 16x^4 + 8x^2 + 1 & \\ \pm 16x^4 & \\ \hline +8x^2 + 1 & \\ \pm 8x^2 \quad 1 & \\ \hline \end{array}$$

Thus $\sqrt{16x^4 + 8x^2 + 1} = \pm(4x^2 + 1)$

Q. 5 Huria is analyzing the total cost of her loan, modeled by the expression $C(x) = x^2 - 8x + 15$, where x represents the number of years. What is the optimal repayment period for Huria's loan?

Solution

$$\begin{aligned} \mathrm{C}(x) &= x^2 - 8x + 15 \\ \mathrm{C}(x) &= x^2 - 5x - 3x + 15 \\ \mathrm{C}(x) &= x(x - 5) - 3(x - 5) \\ &= (x - 5)(x - 3) \end{aligned}$$

The factorized form of expression shows that cost of loan is zero at $x = 5$ and $x = 3$.

So, the optional repayment period is 3 years or 5 years.