9th Math Review Exercise No.6

Q. 1 Choose the correct option.

i. The value of $\tan^{-1}\left(\frac{1}{2}\right)$ in radians is:
(a) $\frac{\pi}{2}$ (b) $\frac{3\pi}{2}$ (c) 0.4636π (d) 0.4636

ii. In a right triangle, the hypotenuse is 13 units and one of the angles is $\theta = 30°$.What is the length of the opposite side?
(a) 6.5 units (b) 7.5 units (c) 6 units (d) 5 units

iii. A person standing 50 m away from a building sees the top of the building at an angle of elevation of $45°$. Height of the building is:
(a) 50 m (b) 25 m (c) 35 m (d) 70 m

iv $\text{Sec}^2\theta - \tan^2\ \theta =$
(a) $\sin^2\ \theta$ (b) 1 (c) $\cos^2\ \theta$ (d) $\cot^2\ \theta$

v If $\sin\ \theta = \frac{3}{5}$, and θ is an acute angle, $\cos^2\ \theta =$
(a) $\frac{7}{25}$ (b) $\frac{24}{25}$ (c) $\frac{16}{25}$ (d) $\frac{4}{25}$

vi. $\frac{5\pi}{24}\text{rad} =$ degrees.
(a) $30°$ (b) $37.5°$ (c) $45°$ (d) $52.5°$

vii. $292.5° =$ rad.
(a) $\frac{17\pi}{6}$ (b) $\frac{17\pi}{4}$ (c) 1.6π (d) 1.625π

viii.Which of the following is a valid identity?
(a) $\cos\left(\frac{\pi}{2} - \theta\right) = \sin\ \theta$ (b) $\cos\left(\frac{\pi}{2} - \theta\right) = \cos\ \theta$
(c) $\cos\left(\frac{\pi}{2} - \theta\right) = \sec\ \theta$ (d) $\cos\left(\frac{\pi}{2} - \theta\right) = \text{cosec}\theta$

ix. $\sin\ 60° =$
(a) 1 (b) $\frac{1}{2}$ (c) $\sqrt{(3)^2}$ (d) $\frac{\sqrt{3}}{2}$

x. $\cos^2\ 100\pi + \sin^2\ 100\pi =$
(a) 1 (b) 2 (c) 3 (d) 4

Answer Key

i	d	ii	a	iii	a	iv	b	v	c
vi	b	vii	d	viii	a	ix	d	x	a

Multiple Choice Questions (Additional)

1.The plane figure formed by two rays sharing a common endpoint is called:

(a)an angle (b) a degree (c) triangle (d) a radian
2. In sexagesimal system of measurement, the angle is measured in:
(a) Radian (b) $°\mathrm{C}$ (c) Gradian. (d) $\mathrm{D}°\mathrm{M}'\mathrm{S}'''$
3 . $25° =$
(a) $360'$ (b) $630'$ (c) $1500'$ (d) $9000'$

4. $\frac{5\pi}{4}$ radians =
(a) $125°$ (b) $135°$ (c) $150°$ (d) $225°$
5. 2π radians =
(a) $0°$(b) $90°$ (c) $180°$ (d) $360°$
6. π radians =
(a) $0°$ (b) $90°$ (c) $180°$ (d) $360°$
7. $\frac{\pi}{2}$ radians =
(a) $30°$ (b) $45°$ (c) $60°$ (d) $90°$
8. $\frac{\pi}{3}$ radians =
(a) $30°$ (b) $45°$ (c) $60°$ (d) $90°$
9. $\frac{\pi}{4}$ radians =
(a) $30°$ (b) $45°$ (c) $60°$ (d) $90°$
10. $\frac{\pi}{6}$ radians =
(a) $30°$ (b) $45°$ (c) $60°$ (d) $90°$
11. $\frac{3\pi}{2}$ radians =
(a) $90°$ (b) $180°$ (c) $270°$ (d) $360°$
12. Fundamental trigonometric ratios are:
(a) 3 (b) 4 (c) 5 (d) 6
13. $\sin\theta \times \cos\theta \times \sec\theta \times \mathrm{cosec}\theta$
(a) $\sin\theta \times \cos\theta$ (b) $\tan\theta + \cot\theta$ (C) $\sec\theta \times \mathrm{cosec}\theta$ (d) $\tan\theta \times \cot\theta$
14. $1 + \tan^2\theta =$
(a) $\sin^2\theta$ (b) $\sec^2\theta =$ (c) $\cos^2\theta$ (d) $\mathrm{cosec}^{-1}\theta$
15. $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta}$
(a) $2\sec^2\theta$ (b) $2\cos^2\theta$ (c) $2\mathrm{cosec}^2\theta$ (d) $2\tan^2\theta$
16. $\frac{1}{2}\mathrm{cosec}45° =$
(a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\sqrt{2}$ (d) $\frac{\sqrt{3}}{2}$
17. $\sec\theta\cot\theta =$
(a) $\sin\theta$ (b) $\mathrm{cosec}\theta$ (c) $\sec\theta$ (d) $\tan\theta$

18. $\cot^2\ \theta - \operatorname{cosec}^2\theta =$
(a) -1 (b) 1 (c) 0 (d) $\tan\ \theta$
19. $\sin\ 45^\circ \cos\ 45^\circ =$ Type equation here.
(a) 1 (b) $\sqrt{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{2}$
20. $\cos\ 30^\circ \tan\ 45^\circ =$
(a) $\frac{2}{\sqrt{3}}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{\sqrt{3}}{2}$
21. $\sec\ 45^\circ \operatorname{cosec} 45^\circ = ---$
(a) 2 (b) $\sqrt{2}$ (c) $\frac{1}{2}$ d) $\frac{1}{\sqrt{2}}$
22. $\mathrm{Cot} 45^\circ + \tan\ 45^\circ = \cdots -$
(a) $\sqrt{2}$ (b) 2 (c) $\frac{1}{\sqrt{2}}$ (d) 1
23. $\mathrm{Sin} 30^\circ \mathrm{Cos} 30^\circ = \cdots$
(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{\sqrt{3}}{4}$ (d) $\frac{2}{\sqrt{3}}$
24. $\tan\ 30^\circ \sin\ 60^\circ = ---$
(a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\sqrt{3}$ (d) $\frac{1}{\sqrt{3}}$
25. If $\tan\ \theta = \sqrt{3}$, then θ is equal to:
(a) 90° (b) 45° (c) 60° (d) 30°

Answer Key

1	a	2	d	3	c	4	d	5	d	6	c	7	d	8	c	9	b	10	a
11	c	12	d	13	d	14	b	15	a	16	b	17	b	18	a	19	d	20	d
21	a	22	b	23	c	24	a	25	c										

Q. 2 Convert the given angles from:
(a) degrees to radians giving answer in terms of π.
(i) 255°
Solution

$$255^\circ$$
$$= 255 = \times \frac{\pi}{180} \mathrm{rad}$$
$$= \frac{51\pi}{36} \mathrm{rad} = \frac{17\pi}{12} \mathrm{rad}$$

(ii) $75^\circ 45'$

Solution:

$$75^\circ 45'$$

$= 75^\circ + \left(\frac{45}{60}\right)^\circ$

$= 75.75^\circ$

$= 75.75 \times \frac{\pi}{180} \text{rad}$

$= \frac{7575}{100} \times \frac{\pi}{180} \text{rad}$

$= \frac{101}{240} \text{rad}$

(iii) 142.5°

Solution:

142.5°

$= 142.5 \times \frac{\pi}{180} \text{rad}$

$= \frac{1425}{10} \times \frac{\pi}{180} \text{rad}$

$= \frac{19\pi}{24} \text{rad.}$

(b) radians to degrees giving answer in degrees and minutes.

(i) $\frac{17\pi}{24}$

solution:

$\frac{17\pi}{24} \text{rad}$

$$= \left(\frac{17t}{24} \times \frac{180}{t}\right)^0 = 127^\circ + 0.5^\circ$$

$$= 127.5^\circ = 127^\circ + (0.5 \times 60)'$$

$$= 127^\circ 30'$$

(ii) $\frac{7\pi}{12}$

Solution

$\frac{7\pi}{12}$

$= \frac{7\pi}{12} \text{rad}$

$= \left(\frac{7\pi}{12} \times \frac{180}{t}\right)^0$

$= (7 \times 15)^\circ$

$= 105^{\circ}$

(iii) $\frac{11\pi}{16}$

(iii) $\frac{11\pi}{16}$

Solution:

$$\frac{11\pi}{16}$$

$$=\frac{11\pi}{16}\text{rad}$$

$$=\left(\frac{11\pi}{16}\times\frac{{}^{45}180}{\pi}\right)^{\circ}$$

$$=123.75^{\circ} = 123^{\circ} + 0.79$$

$$=123^{\circ} + (0.75 \times 60)'$$

$$=123^{\circ} + 45' = 123^{\circ}45'$$

Q. 3 Prove the following trigonometric identities:

(i) $\frac{\sin\ \theta}{1-\cos\ \theta} = \frac{1+\cos\ \theta}{\sin\ \theta}$

Solution

$$\frac{\sin\ \theta}{1-\cos\ \theta} = \frac{1+\cos\ \theta}{\sin\ \theta}$$

L.H.S $= \frac{\sin\ \theta}{1-\cos\ \theta}$

$$= \frac{\sin\ \theta}{1-\cos\ \theta} \times \frac{1+\cos\ \theta}{1+\cos\ \theta}$$

$$= \frac{\sin\ \theta(1+\cos\ \theta)}{(1)^2-(\cos\ \theta)^2}$$

$$= \frac{\sin\ \theta(1+\cos\ \theta)}{1-\cos^2\ \theta}$$

$$= \frac{\sin\ \theta(1+\cos\ \theta)}{\sin^2\ \theta}[\because \sin^2\ \theta + \cos^2\ \theta = 1]$$

$$= \frac{\sin\ \theta(1+\cos\ \theta)}{\sin\ \theta \sin\ \theta}$$

$$= \frac{1+\cos\ \theta}{\sin\ \theta}$$

= R.H.S

(ii) $\sin\ \theta(\text{cosec}\theta - \sin\ \theta) = \frac{1}{\sec^2\ \theta}$09306181

Solution

$$\sin\ \theta(\mathrm{cosec}\theta - \sin\ \theta) = \frac{1}{\sec^2\ \theta}$$

L.H.S $= \sin\ \theta(\mathrm{cosec}\theta - \sin\ \theta)$

$= \sin\ \theta \cdot \mathrm{cosec}\theta - \sin\ \theta \cdot \sin\ \theta$

$= 1 - \sin^2\ \theta\ (\because \sin\ \theta \cdot \mathrm{cosec}\theta = 1)$

$= \cos^2\ \theta$

$= \dfrac{1}{\sec^2\ \theta}$

$=$ R.H.S

Hence proved.

(iii)

$$\frac{\mathrm{cosec}\theta - \sec\ \theta}{\mathrm{cosec}\theta + \sec\ \theta} = \frac{1 - \tan\ \theta}{1 + \tan\ \theta}$$

Solution

$$\frac{\mathrm{cosec}\theta - \sec\ \theta}{\mathrm{cosec}\theta + \sec\ \theta} = \frac{1 - \cos\ \theta}{1 + \tan\ \theta}$$

L.H.S $= \dfrac{\mathrm{cosec}\theta - \sec\ \theta}{\mathrm{cosec}\theta + \sec\ \theta}$

$$= \frac{\frac{1}{\sin\ \theta} - \frac{1}{\cos\ \theta}}{\frac{1}{\sin\ \theta} + \frac{1}{\cos\ \theta}}$$

Multiplying and dividing by $\sin\ \theta$

$$= \frac{\sin\ \theta\left(\frac{1}{\sin\ \theta} - \frac{1}{\cos\ \theta}\right)}{\sin\ \theta\left(\frac{1}{\sin\ \theta} + \frac{1}{\cos\ \theta}\right)}$$

$$= \frac{\frac{\sin\ \theta}{\sin\ \theta} - \frac{\sin\ \theta}{\cos\ \theta}}{\frac{\sin\ \theta}{\sin\ \theta} + \frac{\sin\ \theta}{\cos\ \theta}}$$

$$= \frac{1 - \tan\ \theta}{1 + \tan\ \theta}$$

$=$ R.H.S

Hence proved.

(iv) $\tan\ \theta + \cot\ \theta = \dfrac{1}{\sin\ \theta\cos\ \theta}$

Solution

$$\tan\ \theta+\cot\ \theta=\frac{1}{\sin\ \theta \cos\ \theta}$$

L.H.S $=\tan\ \theta+\cot\ \theta$

$$=\frac{\sin\ \theta}{\cos\ \theta}+\frac{\cos\ \theta}{\sin\ \theta}$$

$$=\frac{\sin^2\ \theta+\cos^2\ \theta}{\sin\ \theta \cos\ \theta}$$

$$=\frac{1}{\sin\ \theta \cos\ \theta}\left[\because \sin^2\ \theta+\cos^2\ \theta=1\right]$$

= R.H.S

Hence proved.

(v) $\frac{\cos\ \theta+\sin\ \theta}{\cos\ \theta-\sin\ \theta}+\frac{\cos\ \theta-\sin\ \theta}{\cos\ \theta-\sin\ \theta}=\frac{2}{1-2\sin^2\ \theta}$

Solution

$$\frac{\cos\ \theta+\sin\ \theta}{\cos\ \theta-\sin\ \theta}+\frac{\cos\ \theta-\sin\ \theta}{\cos\ \theta+\sin\ \theta}=\frac{2}{1-2\sin^2\ \theta}$$

L.H.S $\frac{\cos\ \theta+\sin\ \theta}{\cos\ \theta-\sin\ \theta}+\frac{\cos\ \theta-\sin\ \theta}{\cos\ \theta+\sin\ \theta}$

$$=\frac{(\cos\ \theta+\sin\ \theta)^2+(\cos\ \theta-\sin\ \theta)^2}{(\cos\ \theta-\sin\ \theta)(\cos\ \theta+\sin\ \theta)}$$

$\because (a+b)^2+(a-b)^2=2(a^2+b^2)$

$(a-b)(a+b)=a^2-b^2$

$$=\frac{2[(\cos\ \theta)^2+(\sin\ \theta)^2]}{(\cos\ \theta)^2-(\sin\ \theta)^2}$$

$$=\frac{2(\cos^2\ \theta+\sin^2\ \theta)}{\cos^2\ \theta-\sin^2\ \theta}$$

$$=\frac{2(1)}{(1-\sin^2\ \theta)-\sin^2\ \theta}\ (\because \cos^2\ \theta=1-\sin^2\ \theta)$$

$$=\frac{2}{1-\sin^2\ \theta-\sin^2\ \theta}$$

$$=\frac{2}{1-2\sin^2\ \theta}$$

= R.H.S. Hence proved.

(vi) $\frac{1+\cos\ \theta}{1-\cos\ \theta}=(\operatorname{cosec}\theta+\cot\ \theta)^2$

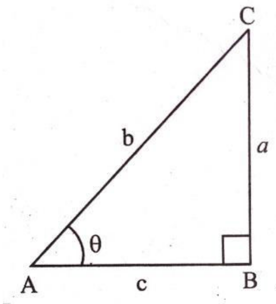
Solution

$$\frac{1+\cos\ \theta}{1-\cos\ \theta}=(\operatorname{cosec}\theta+\cot\ \theta)^2$$

Hence proved.

Q. 4 If $\tan\ \theta = \frac{3}{\sqrt{2}}$, then find the remaining trigonometric ratios when θ lies in first quadrant.

Solution



$\tan\ \theta = \frac{3}{\sqrt{2}}$

Let $\triangle ABC$ is a right angled in which

$$\mathrm{m}\angle\ \mathrm{B} = 90^{\circ}, \mathrm{m}\angle\ \mathrm{A} = 0 = \tan\ \theta = \frac{a}{\mathrm{c}}$$

Comparing (i) and (ii),

$$\frac{a}{\mathrm{c}} = \frac{3}{\sqrt{2}} \Rightarrow a = 3, \mathrm{c} = \sqrt{2}$$

By Pythagoras, theorem

$$b^2 = a^2 + c^2$$
$$b^2 = (\sqrt{2})^2 + (3)^2$$
$$b^2 = 2 + 9$$
$$b^2 = 11$$
$$\sqrt{b^2} = \sqrt{11}$$
$$b = \sqrt{11}\ \mathrm{cm}$$

Remaining ratios

$$\sin\ \theta = \frac{a}{b} = \frac{3}{\sqrt{11}}$$

$$\cos\ \theta = \frac{c}{b} = \frac{\sqrt{2}}{\sqrt{11}} = \sqrt{\frac{2}{11}}$$

$$\mathrm{cosec}\theta = \frac{b}{a} = \frac{\sqrt{11}}{3}$$

L.H.S $= \frac{1 + \cos\ \theta}{1 - \cos\ \theta}$

$$= (1 + \cos\ \theta) \div (1 - \cos\ \theta)$$

dividing by $\sin\theta$

$$= \frac{(1+\cos\theta)}{\sin\theta} \div \frac{(1-\cos\theta)}{\sin\theta}$$

$$= \left(\frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}\right) \div \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)$$

$$\left(\because \frac{\cos\theta}{\sin\theta} = \cot\theta\right)$$

$$= (\operatorname{cosec}\theta + \cot\theta) \div (\operatorname{cosec}\theta - \cot\theta)$$

$$= \frac{\operatorname{cosec}\theta + \cot\theta}{\operatorname{cosec}\theta - \cot\theta}$$

$$= \frac{(\operatorname{cosec}\theta + \cot\theta)}{(\operatorname{cosec}\theta - \cot\theta)} \times \frac{(\operatorname{cosec}\theta + \cot\theta)}{(\operatorname{cosec}\theta + \cot\theta)}$$

$$= \frac{(\operatorname{cosec}\theta + \cot\theta)^2}{\operatorname{cosec}^2\theta - \cot^2\theta}$$

$$= \frac{(\operatorname{cosec}\theta + \cot\theta)^2}{1}$$

$$= (\operatorname{cosec}\theta + \cot\theta)^2$$

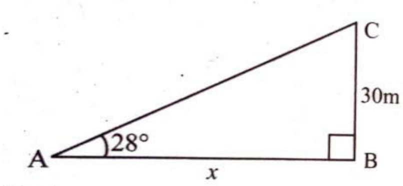
$=$ R.H.S

$$\sec\theta = \frac{b}{c} = \frac{\sqrt{11}}{\sqrt{2}} = \sqrt{\frac{11}{2}}$$

$$\cot\theta = \frac{c}{a} = \frac{\sqrt{2}}{3}$$

Q. 5 From a point on the ground, the angle of elevation to the top of a $30-$ meter-high building is 28°. How far is the point from the base of the building?

Solution:



Height of building $= m\overline{BC} = 30m$

Angle of elevation $= 0 = 28^\circ$

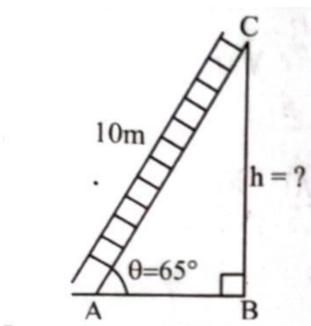
Distance of point from building $= m\overline{AB} = ?$

$$\text{As } \tan\ 28^\circ = \frac{m\overline{BC}}{m\overline{AB}}$$
$$\Rightarrow \tan\ 28^\circ = \frac{30}{x}$$
$$\Rightarrow x \cdot \tan\ 28^\circ = 30$$
$$\Rightarrow x = \frac{30}{\tan\ 28^\circ}$$
$$x = 56.42 \text{ m}$$

So, required distance is 56.42 m

Q. 6 A ladder leaning against a wall forms an angle of 65° with the ground. If the ladder is 10 meters long, how high does it reach on the wall?

Solution:



Length of ladder $= m\overline{AC} = 10m$

Height of wall $= m\overline{BC} = h = ?$

Angle made by ladder

With ground $= \theta = 65^\circ$

As $\sin\ \theta\ = \frac{m\overline{BC}}{m\overline{AC}}$

$$\sin\ 65^\circ\ = \frac{\text{h}}{10}$$
$$\Rightarrow\ \text{h} = 10 \times \sin\ 65^\circ$$
$$\text{h} = 9.06 \text{ m}$$

So, height of wall is 9.06 m