Exercise 4.4

Q. 1 Find the square root of the following polynomials by factorization method:

(i) $x^2 - 8x + 16$

Solution

$$x^2 - 8x + 16$$

By factorization

$$x^2 - 8x + 16 = (x)^2 - 2(x)(4) + (4)^2$$

$$\because (a - b)^2 = a^2 - 2ab + b^2$$

$$= (x - 4)^2$$

Taking square root of both sides.

$$\sqrt{x^2 - 8x + 16} = \pm\sqrt{(x - 4)^2}$$

$$= \pm (x - 4)$$

(ii) $9x^2 + 12x + 4$

Solution

$$9x^2 + 12x + 4$$

By factorization

$$9x^2 + 12x + 4 \quad \because (a + b)^2 = a^2 + 2ab + b^2$$

$$= (3x)^2 + 2(3x)(2) + (2)^2$$

Taking square root of both sides.

$$\sqrt{9x^2 + 12x + 4} = \pm\sqrt{(3x + 2)^2}$$

$$= \pm (3x + 2)$$

(iii) $36a^2 + 84a + 49$

Solution

$$36a^2 + 84a + 49$$

$$= (6a)^2 + 2(6a)(7) + (7)^2$$

$$= (6a + 7)^2$$

By factorization

$$\therefore (a + b)^2 = a^2 + 2ab + b^2$$

Taking square root of both sides

$$\sqrt{36a^2 84a + 49} = \pm\sqrt{(6a + 7)^2}$$

$$= \pm (6a + 7)$$

(iv) $64y^2 - 32y + 4$

Solution

$$64y^2 - 32y + 4 \quad \because (a-b)^2 = a^2 - 2ab + b$$
$$= (8y)^2 - 2(8y)(2) + (2)^2$$
$$= (8y-2)^2$$

Taking square root of both sides

$$\sqrt{64y^2 - 32y + 4} = \pm\sqrt{(8y-2)^2}$$
$$= \pm(8y-2)$$

(v) $200t^2 - 120t + 18$

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Solution

$$200t^2 - 120t + 18$$
$$= 2\left(100t^2 - 60t + 9\right)$$
$$\because (a-b)^2 = a^2 - 2ab + b$$
$$= 2\left[(10t)^2 - 2(10t)(3) + (3)^2\right]$$
$$= 2(10t-3)^2$$
$$= 2(10t-3)^2$$

Taking square root of both sides

$$\sqrt{200t^2 - 120t + 18} = \pm\sqrt{2(10t-3)^2}$$
$$= \pm\sqrt{2}(10t-3)$$
$$= \pm\sqrt{2}(10t-3)$$

(vi) $40x^2 + 120x + 90$

Solution:

$$40x^2 + 120x + 90$$
$$= 10\left(4x^2 + 12x + 9\right)$$
$$\because (a+b)^2 = a^2 + 2ab + b^2$$
$$= 10\left[(2x)^2 + 2(2x)(3) + (3)^2\right]$$
$$= 10(2x+3)^2$$

Taking square root of both sides

$$\sqrt{40x^2 + 12x + 90} = \pm\sqrt{10(2x+3)^2}$$
$$= \pm\sqrt{10}(2x+3)$$
$$= \pm\sqrt{10}(2x+3)$$

Q. 2 Find the square root of the following polynomials by division method:

(i) $4x^4 - 28x^3 + 37x^2 + 42x + 9$

Solution:

$$4x^4 - 28x^3 + 37x^2 + 42x + 9$$

Square root by division method:

$$
\begin{array}{r|l}
 & 2x^2 - 7x - 3 \\
\hline
2x^2 & 4x^4 - 28x^3 - 37x^2 + 42x + 9 \\
 & \oplus 4x^4 \\
 & \underline{-} \\
\hline
4x^2 - 7x & -28x^3 + 37x^2 + 42x + 9 \\
 & \oplus 28x^3 \oplus 49x^2 \\
 & \underline{-} \qquad \underline{-} \\
\hline
4x^2 - 14x - 3 & -12x^2 + 42x + 9 \\
 & \ominus 12x^2 \oplus 42x \oplus 9 \\
 & + \qquad - \qquad - \\
\hline
 & 0
\end{array}
$$

Thus required square root is $\pm\left(2x^2-7x-3\right)$.

(ii) $121x^4-198x^3-183x^2+216x+144$

Solution

$121x^4-198x^3-183x^2+216x+144$

$$
\begin{array}{r|l}
 & \quad 11x^2-9x-12 \\
\hline
11x^2 & 121x^4-198x^3-183x^2+216x+144 \\
 & \underset{-}{\oplus}\,121x^4 \\
\hline
22x^2-9x & \quad -198x^3-183x^2+216x+144 \\
 & \quad \underset{+}{\ominus}\,198x^3 \underset{-}{\oplus}\,81x^2 \\
\hline
22x^2-18x-12 & \qquad -264x^2+216x+144 \\
 & \qquad \underset{+}{\ominus}\,264x^2 \underset{-}{\oplus}\,216x \underset{-}{\oplus}\,144 \\
\hline
 & \qquad\qquad 0
\end{array}
$$

$$\left(11x^2-9x-12\right)$$

Thus, required square root is $\pm\left(11x^2-9x-12\right)$

(iii)

$$x^4-10x^3y+27x^2y^2-10xy^3+y^4$$

Solution:

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Solution:

$x^4 - 10x^3y + 27x^2y^2 - 10xy^3 + y^4$

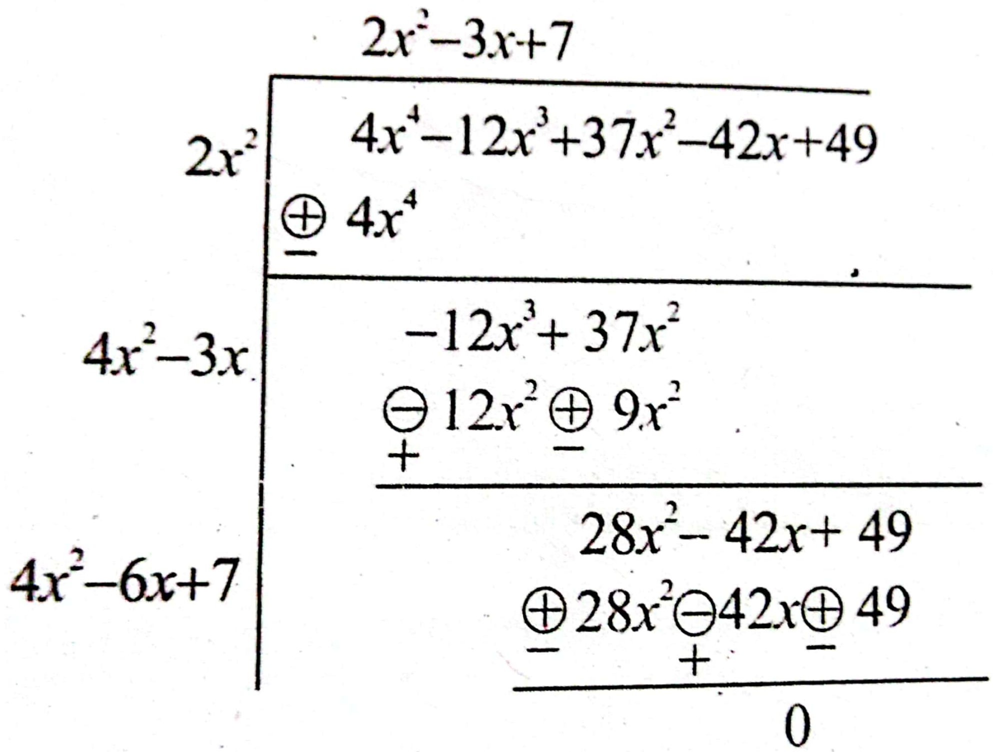
$$
\begin{array}{r|l}
 & x^2 - 5xy + y^2 \\
\hline
x^2 & x^4 - 10x^3y + 27x^2y^2 - 10xy^3 + y^4 \\
 & \underset{-}{\oplus}\, x^4 \\
\hline
2x^2 - 5xy & -10x^3y + 27x^2y^2 - 10xy^3 + y^4 \\
 & \underset{+}{\ominus}\, 10x^3y \underset{-}{\oplus} 25x^2y^2 \\
\hline
2x^2 - 10xy + y^2 & 2x^2y^2 - 10xy^3 + y^4 \\
 & \underset{-}{\oplus} 2x^2y^2 \underset{+}{\ominus} 10xy^3 \underset{-}{\oplus} y^4 \\
\hline
 & 0
\end{array}
$$

Thus, required square root is $\pm\left(x^2 - 5xy + y^2\right)$.

. (iv) $4x^4 - 12x^3 + 37x^2 - 42x + 49$ 00304106

Solution:

$4x^4 - 12x^3 + 37x^2 - 42x + 49$



Thus, required square root is $\pm\left(2x^2-3x+7\right)$.

Q. 3 An investor's return $R(x)$ in rupees after investing x thousand rupees is given by quadratic expression:

$$R(x) = -x^2 + 6x - 8$$

Factor the expression and find the investment levels that result in zero return.

Solution

Investor's return: $\mathrm{R}(x) = -x^2 + 6x - 8$

$$\begin{aligned} \Rightarrow \mathrm{R}(x) &= -\left(x^2 - 6x + 8\right) \\ &= -\left(x^2 - 2x - 4x + 8\right) \\ &= -[x(x-2) - 4(x-2)] \\ \mathrm{R}(x) &= -(x-2)(x-4) \end{aligned}$$

The following form of return function. Shows that return is zero at $x = 2$ or $x = 4$.

Q. 4 A company's profit $f(x)$ in rupees from selling x units of a product is modeled by the cubic expression:

$$P(x) = x^3 - 15x^2 + 75x - 125$$

Find the break-even point(s), where the profit is zero.

Solution

Note: Break-even point means no profit no loss.

$$P(x) = x^3 - 15x^2 + 75x - 125$$

$$P(x) = (x)^3 - 3(x)^2(5) + 3(x) + 3(x)(5)^2 - (5)^3$$

$$P(x) = (x-5)^3$$

$$P(x) = (x-5)(x-5)(x-5)$$

The factorized form of profit function shows that profit is zero at $x = 5$.

Q. 5 The potential energy $V(x)$ in an electric field varies as a cubic function of distance x, given by:

$$V(x) = 2x^3 - 6x^2 + 4x$$

Determine where the potential energy is zero.

Solution

$$\begin{aligned} V(x) &= 2x^3 - 6x^2 + 4x \\ &= 2x\left(x^2 - 3x + 2\right) \\ &= 2x\left(x^2 - x - 2x + 2\right) \\ &= 2x[x(x-1) - 2(x-1)] \\ &= 2x(x-1)(x-2) \end{aligned}$$

The factorized form of electric potential energy function shows that potential energy is zero at.

$x = 0$, or $x = 1$ or $x = 2$

Q. 6 In structural engineering, the deflection $y(x)$ of a beam is given by:

$$Y(x) = 2x^2 - 8x + 6$$

This equation gives the vertical deflection at any point x along the beam. Find the points of zero deflection.

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$$
\begin{aligned}
Y(x) &= 2x^2 - 8x + 6 \\
&= 2\left(x^2 - 4x + 3\right) \\
&= 2\left(x^2 - 3x - x + 3\right) \\
&= 2[x(x-3) - 1(x-3)] \\
&= 2(x-3)(x-1)
\end{aligned}
$$

The factorized form of function shows that deflection is zero at $x = 1$ and $x = 3$