# $9^{th}$ Math Exercise 5.1

Q. 1 Solve and represent the solution set on a real line.

(i) $12x + 30 = -6$

Solution:

$$12x + 30 = -6$$
$$12x = -6 - 30$$
$$12x = -36$$
$$x = \frac{-36}{12}$$
$$x = -3$$

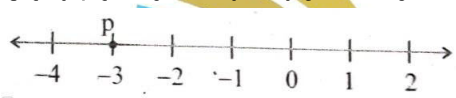
So, $x = -3$ is a solution of given equation because it makes the original equation true.

Check: Substitute $x = -3$ in equation (i)

$$12(-3) + 30 = -6$$
$$-36 + 30 = -6$$
$$-6 = -6 \text{( It is true )}$$

So, $x = -3$ is solution set of given equation.

Solution on Number Line






Point P on the number line represents $x = -3$.

(ii) $\frac{x}{3} + 6 = -12$

Solution:

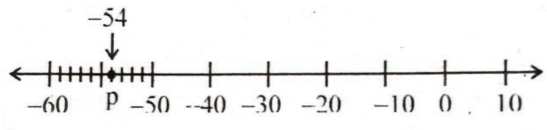
$$\frac{x}{3} + 6 = -12$$
$$\frac{x}{3} = -12 - 6$$
$$\frac{x}{3} = -18$$
$$x = -18 \times 3$$
$$x = -54$$

Check: Substitute $x = -54$ in equation (i)

$$\frac{-54}{3} + 6 = -12$$
$$-18 + 6 = -12$$
$$-12 = -12 \text{ (It is true)}$$

Since, $x = -54$ makes the original equation true so solution of equation is $x = -54$

Solution on number line




Point P on the number line represents -54 .

(iii) $\frac{x}{2}-\frac{3x}{4}=\frac{1}{12}$

Solution

$$\frac{x}{2}-\frac{3x}{4}=\frac{1}{12}$$

$$\frac{2x-3x}{4}=\frac{1}{12} \text{ (L.C.M)}$$

$$\frac{-x}{4}=\frac{1}{12}$$

$$-x=\frac{1}{12}\times 4$$

$$-x=\frac{1}{3}\Rightarrow x=\frac{-1}{3}$$

Check: Substitute $x=\frac{-1}{3}$ in equation (i)

$$(x)\times\frac{1}{2}-\frac{3}{4}x=\frac{1}{12}$$

$$\left(-\frac{1}{3}\times\frac{1}{2}\right)-\left(\frac{3}{4}\times\frac{-1}{3}\right)=\frac{1}{12}$$

$$\frac{-1}{6}+\frac{1}{4}=\frac{1}{12}$$

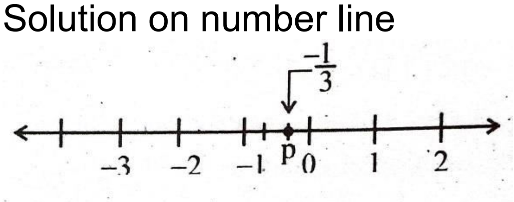
$$\frac{-2+3}{12}=\frac{1}{12}$$



$\frac{1}{12}=\frac{1}{12}$ (It is true).

So, $x=\frac{-1}{3}$ is a solution of given equation.

Solution on number line




Point P represents $x=\frac{-1}{3}$ on the number line.

(iv) $2=7(2x+4)+12x$

Solution

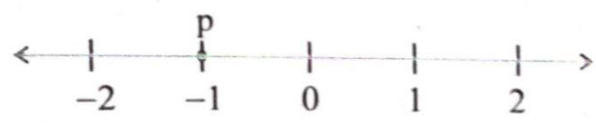
$$2 = 7(2x + 4) + 12x$$
$$2 = 14x + 28 + 12x$$
$$2 = 26x + 28$$
$$2 - 28 = 26x$$
$$-26 = 26x$$
$$\frac{-26}{26} = x$$
$$-1 = x$$
$$\Rightarrow x = -1$$

Check: Substitute $x = -1$ in equation (i)

$$2 = 7[2(-1) + 4] + 12(-1)$$
$$2 = 7[-2 + 4] - 12$$
$$2 = 7[2] - 12$$
$$2 = 14 - 12$$
$$2 = 2(\text{ It is true })$$

So, $x = -1$ is a solution of given equation.

Solution on number line






Point P on the number line represents the solution.

(v) $\frac{2x-1}{3} - \frac{3x}{4} = \frac{5}{6}$

Solution:

$$\frac{2x-1}{3} - \frac{3x}{4} = \frac{5}{6}$$
$$\frac{4(2x-1) - 3(3x)}{12} = \frac{5}{6} \text{( L.C.M )}$$
$$\frac{8x - 4 - 9x}{12} = \frac{5}{6}$$
$$-x - 4 = \frac{5}{6} \times 12$$
$$-x - 4 = \frac{60}{6}$$
$$-x - 4 = 10$$
$$-x = 10 + 4$$
$$-x = 14$$
$$\Rightarrow x = -14$$

Check: Put $x = -14$ in equation (i)

$$\frac{2(-14)-1}{3} - \frac{3(-14)}{4} = \frac{5}{6}$$

$$\frac{-28-1}{3} + \frac{42}{4} = \frac{5}{6}$$

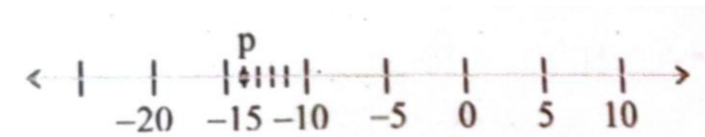
$$\frac{-29}{3} + \frac{21}{2} = \frac{5}{6}$$

$$\frac{-58+63}{6} = \frac{5}{6}$$

$\frac{5}{6} = \frac{5}{6}$ (It is true)

So, $x = -14$ is a solution of given equation.

Solution on number line




Point P on the number line represents $x = -14$.

(vi) $\frac{-5x}{10} = 9 - \frac{10}{5}x$

Solution:

$$\frac{-5}{10}x = 9 - \frac{10}{5}x$$

$$-\frac{1}{2}x = 9 - 2x$$

$$-\frac{1}{2}x + \frac{2x}{1} = 9$$

$$\frac{-x+4x}{2} = 9$$

$$3x = 9 \times 2$$

$$3x = 18$$

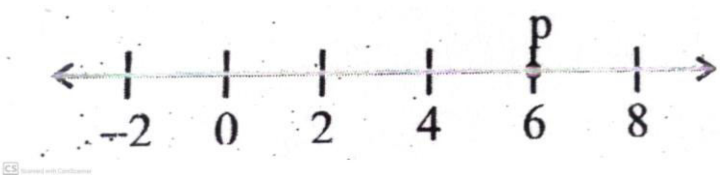
$$x = \frac{18}{3}$$

$$x = 6$$

Check: Substitute $x = 6$ in equation (i)

$$\frac{-5(6)}{10} = 9 - \frac{10}{5}(6)$$
$$\frac{-30}{10} = 9 - 2(6)$$
$$-3 = 9 - 12$$
$$-3 = -3 (\text{ It is true })$$

Solution on number line:




Point P represent the solution $x = 6$ on the number line.

Q. 2 Solve each inequality and represent the solution on a real line.

(i) $x - 6 \leq -2$

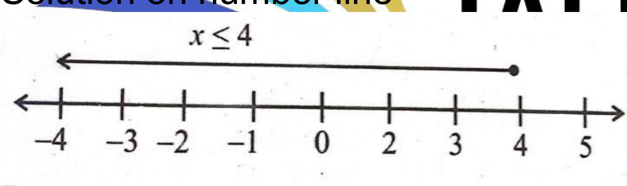
Solution:

$$x - 6 \leq -2$$
$$x \leq -2 + 6$$
$$x \leq 4,$$

The solution of inequality is $(-\infty, 4]$



Solution on number line




Filled circle on 4 means 4 is included in the solution

(ii) $-9 > -16 + x$

Solution

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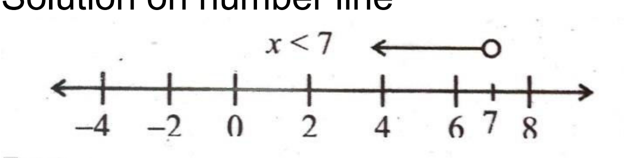
$$-9 > -16 + x$$
$$-9 + 16 > x$$
$$7 > x$$
$$\Rightarrow x < 7$$

The solution of given inequality is $(-\infty, 7)$ or $-\infty < \mathrm{x} < 7$.

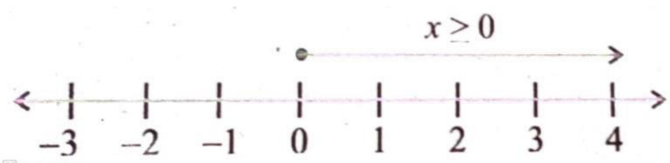
Solution on number line



An empty circle on 7 , means 7 is excluded form the solution.
(iii) $3+2x \geq 3$
Solution:

$$3+2x \geq 3$$
$$2x \geq 3-3$$
$$2x \geq 0$$
$$x \geq \frac{0}{2}$$
$$x \geq 0$$

Thus the solution of given inequality is $[0, \infty)$ or $0 \leq x < \infty$.
Solution on number line

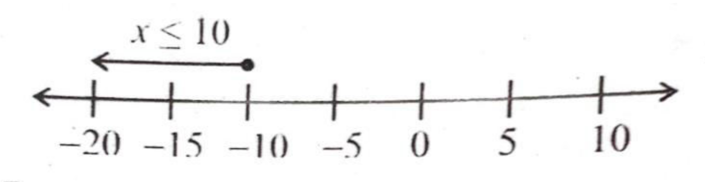



Filled circle on 0 , means 0 in included is the solution.
(iv) $6(x+10) \leq 0$
Solution:



$$6(x+10) \leq 0$$
$$x+10 \leq \frac{0}{6}$$
$$x+10 \leq 0$$
$$x \leq -10$$

The solution of inequality is $(-\infty, -10]$ or $-\infty < x \leq -10$.
Solution on number line

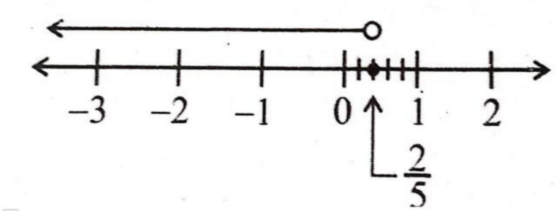



Filled circle on -10 means -10 is included in the solution.
(v) $\frac{5}{3}x - \frac{3}{4} < \frac{-1}{12}$
Solution:

$$\frac{5}{3}x - \frac{3}{4} < \frac{-1}{12}$$
$$\frac{5}{3}x < \frac{3}{4} - \frac{1}{12}$$
$$\frac{5}{3}x < \frac{9-1}{12}$$
$$\frac{5}{3}x < \frac{8}{12}$$
$$\frac{5}{3}x < \frac{2}{3}$$
$$x < \frac{2}{3} \times \frac{3}{5}$$
$$x < \frac{2}{5}$$

The solution of inequality is $\left(-\infty, \frac{2}{5}\right)$ or $-\infty < x \frac{2}{5}$.

Solution on number line






An empty circle on $\frac{2}{5}$ shows that $\frac{2}{5}$ is not included in the solution.

(vi) $\frac{1}{4}x - \frac{1}{2} \leq -1 + \frac{1}{2}x$

Solution:

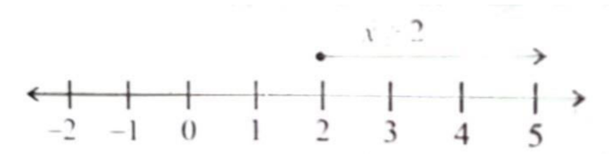
$$\frac{1}{4}x - \frac{1}{2} \leq -1 + \frac{1}{2}x$$

Multiplying both sides by " 4 "

$$4\left(\frac{1}{4}x - \frac{1}{2}\right) \leq 4\left(-1 + \frac{1}{2}x\right)$$
$$4 \times \frac{1}{4}x - 4 \times \frac{1}{2} \leq 4 \times (-1) + 4 \times \left(\frac{1}{2}x\right)$$
$$x - 2 \leq -4 + 2x$$
$$-2 + 4 \leq 2x - x$$
$$2 \leq x$$
$$\Rightarrow x \geq 2$$

The solution of inequality is $[2, \infty)$ or $2 \leq r < \infty$.

Solution on number line




A filled circle on the 2 shows that 2 is included in the solution.

Q. 3 Shade the solution region for the following linear inequalities in $xy$-plane:

(i) $2x + y < 6$

Solution:

$$2x + y < 6$$

i. Associated Eq. of inequality is

$$2x + y = 6$$

ii. Getting two points of line:

For y intercept, put $x = 0$ in equation (ii)

$$2(0) + y = 6 \Rightarrow y = 6 \text{ so, } (0,6)$$

For $x$ intercept, put $y = 0$ in equation (ii)

$$2x + 0 = 6$$

$$2x = 6 \Rightarrow x = 3 \text{ so, } (3,0)$$

iii. Plotting and joining the points $(0,6)$ and $(3,0)$ on coordinate axes draw a solid straight line.

iv. Since the line does not pass through the origin $0(0,0)$, so we can take it as test point.

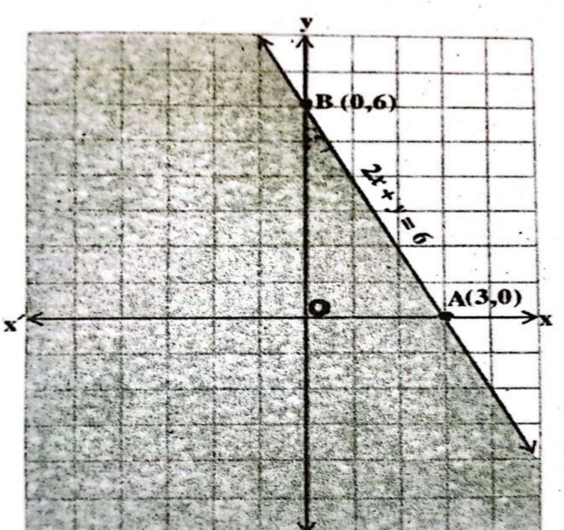



Fig. 3(i)

put $x = 0, y = 0$ in inequality (i) $2(0) + 0 \le 6 \Rightarrow 0 \le 6$ (which is true)
Since the point $(0,0)$ satisfy the inequality $2 + y \le 6$ therefore the graph of its solution is closed lower half plane on the origin side of line of equation (ii).
A portion of closed lower half plane is shown by the shaded region in he figure3(i).
(ii) $3x + 7y \ge 21$
Solution

$$3x + 7y \ge 21$$

i. Associated equation of inequality is

$$3x + 7y = 21$$

ii. Getting two points of line:
Put $x = 0$ in equation (ii),

$$3(0) + 7y = 21 \Rightarrow 7y = 21 \Rightarrow y = 3 \text{ so, } (0,3)$$

Put $y = 0$ in equation(ii),

$$3x + 7(0) = 21$$
$$\Rightarrow 3x = 21$$
$$x = \frac{21}{3}$$
$$x = 7$$



iii. Plotting and joining the points $(0,3)$ and $(7,0)$ on coordinate axes draw a solid straight line.
iv. Since origin $0(0,0)$, does not lie on the line so we can take it as test point.

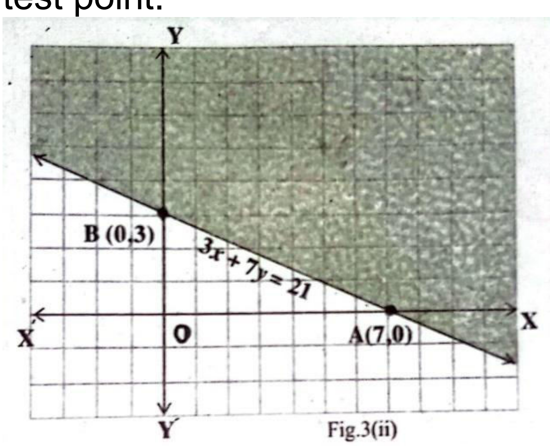



Fig 3 (ii)
Put $x = 0, y = 0$ in incquality (i)

$$3(0) + 7(0) \ge 21 \Rightarrow 0 \ge 21 \text{ (not true)}$$

Since the point $(0,0)$ not satisfy the inequality(i) So the graph of its solution of in equality $3x + 7y \geq 21$ is the region on the opposite side of origin including the line of eq. (ii)

A portion of required closed half plane is shown by shaded region in the figure 3(ii).

(iii) $3x - 2y \geq 6$

Solution:

$$3x - 2y \geq 6$$

i. Associated equation of inequality

$$3x - 2y = 6$$

ii. Getting two points of line:

Put $x = 0$, in equation (ii)

$$3(0) - 2y = 6 \Rightarrow -2y = 6 \Rightarrow y = -3 \text{ so } (0, -3)$$

Put $\mathbf{y} = 0$ in equation (ii)

$$3x - 2(0) = 6 \Rightarrow 3x = 6 \Rightarrow x = 2, \text{ so } (2,0)$$

iii. Plotting and joining the points $(0,3)$ and $(2,0)$ on coordinate axes draw a solid straight line.

iv. Since origin $(0,0)$, does not lie on the line of equation (ii), so we take it as test point.

Put $x = 0, y = 0$ in inequality (i)

$3(0) + 2(0) \geq 6 \Rightarrow 0 \geq 6$ (not true)

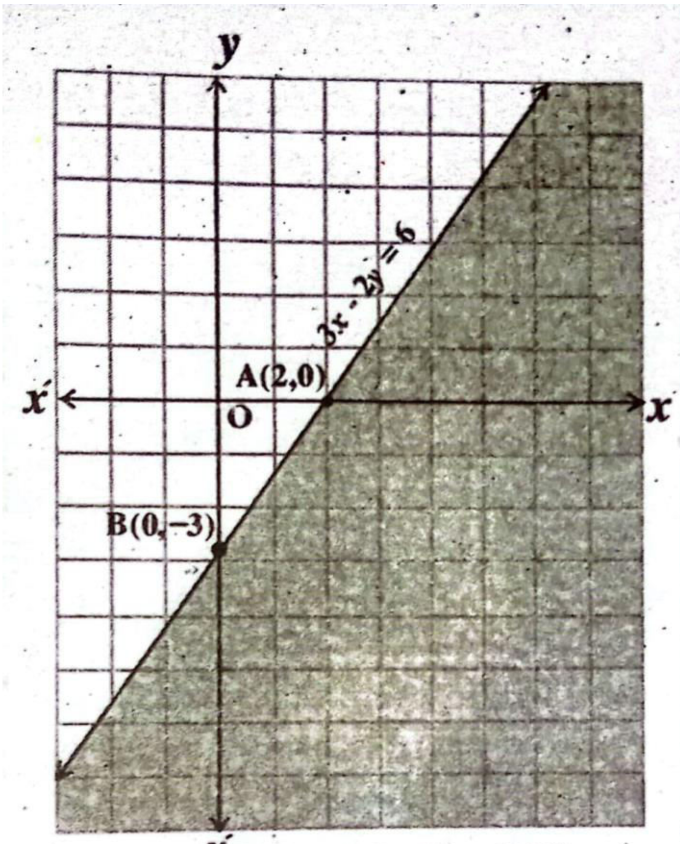


Fig. 3(iii)



Fig. 3(iii)
Since the point $(0,0)$ not satisfy the inequality(i), therefore the graph of solution of in equality $3x - 2y \geq 6$ is the closed lower half plane on opposite to origin side the line of equation (ii) including the line of eq. (ii) A portion of the closed lower half plane is shown as shaded region in the figure 3 (iii).

(iv) $5x - 4y \leq 20$

Solution:

$$5x - 4y \leq 20$$

(i)

i. Associated equation of inequality is

$$5x - 4y = 20$$

(ii)

ii. Getting two points of line:

Put $x = 0$, in (ii)

$$S(0) - 5y = 20 \Rightarrow$$
$$-4y = 20 \Rightarrow y = -5 \Rightarrow (0, -5)$$

Put $y=0$ in equation (ii)

$$5x-4(0)=20$$

$$\Rightarrow 5x=20 \Rightarrow x=4 \text{ so } (4,0)$$

iii. Plotting and joining the points $(0,-5)$ and $(4,0)$ on coordinate axes draw a solid straight line.

iv. Since, the origin i.e. $(0,0)$, does not lie on the line, so we take it as test point.

Put $x=0, y=0$ in inequality (i)

$$5(0)-4(0) \leq 20 \Rightarrow 0 \leq 20 \text{ (True).}$$

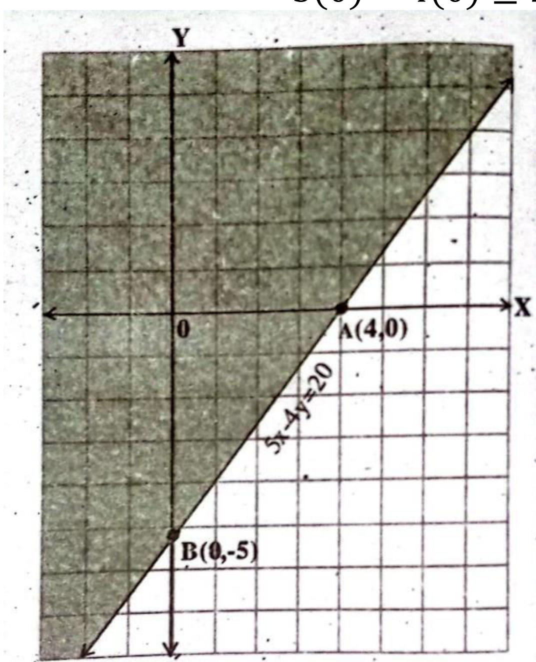



Fig. 3(iv)



Fig. M(iv)

Since the point $(\boldsymbol{n\Omega})$ satisfy the incquality(i), therefore the graph of solution of in equality $5x-4y \leq 20$ is the closed upper half plane on the of origin side of the line of equation (ii).

Including the line of eq. (ii)

A portion of the closed lower half plane is shown as shaded region in the figure 3 (iv).

(v) $2x+1>0$

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Solution:

$$2x+1>0$$

$$2x+1>0$$

i. The corresponding (associated) equation of the inequality (i) is $2x+1=0$ (ii)

$$\Rightarrow 2x = -1 \Rightarrow x = -\frac{1}{2} \text{ i.e; } A\left(-\frac{1}{2}, 0\right)$$

Which is a vertical line (Parallel to $y$-axis). The graph of equation (ii) is drawn in the figure 3 (v).

ii. Taking origin i.e., $0(0,0)$ as a test point.

Clearly origin satisfies the inequality, (i) because

$$2(0) + 1 > 0 \Rightarrow 1 > 0 \text{ (True)}$$

Thus the graph of inequality (i) is the closed right half-plane on the origin-side of the eq.(ii). A portion of the closed half-plane on right side of the line (ii) is shown as shaded region in the figure $3(\mathrm{v})$.

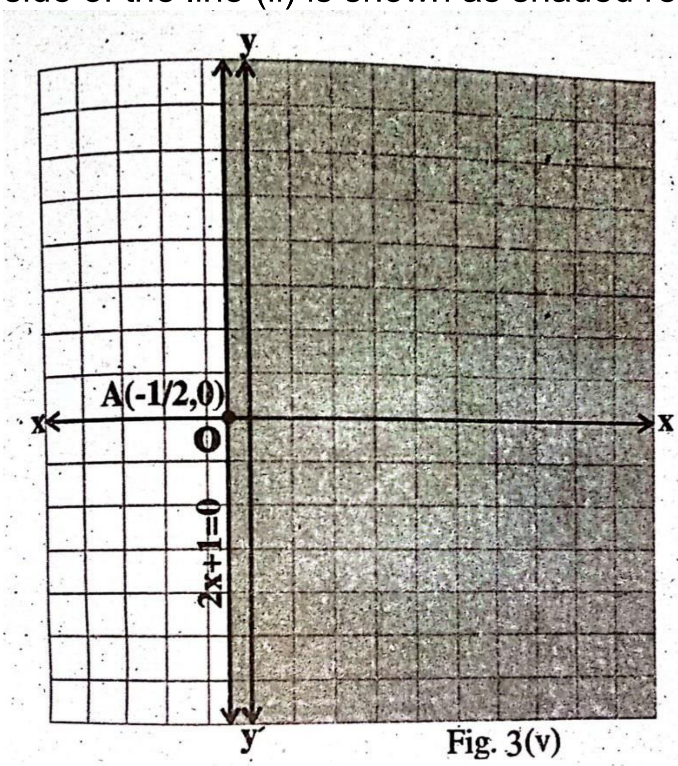



Fig. 3(v)

(vi) $3y - 4 \leq 0$

Solution:

$$3y - 4 \leq 0$$
$$3y - 4 \leq 0$$

i. The associated equation of in equality is:

$$3y - 4$$

ii. By solving equation we get

$$3y - 4 = 0$$
$$3y = 4$$
$$y = \frac{4}{3} = 1\frac{1}{3}$$

iii. Draw the solid line of equation (ii) which is a horizontal line (parallel to $x$-axis) as shown in figure.
iv. Since, origin $(0,0)$ does not lie on the line so, we take it as a test point.
Put $x = 0, y = 0$ in inequality (i),

$$3(0) - 4 \leq 0 \Rightarrow -4 \leq 0 \text{ (true).}$$

Since, point $(0,0)$ satisfy the inequality so its solution graph is on origin-side of line of equation (ii).
So the graph of solution of in equality $3y - 4 \leq 0$ is the closed lower half-plane which is shown as shaded region in the figure 3(vi).

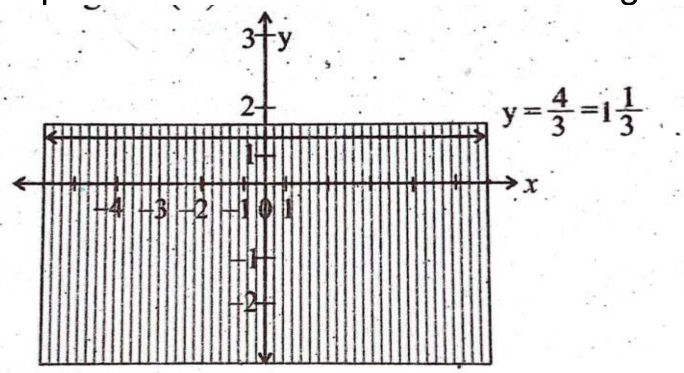


Fig. 3(vi)



Q. 4 Indicate the solution region of the following linear inequalities by shading:
(i)
$2x - 3y \leq 6$
$2x + 3y \leq 12$
region of the following linear
Solution:

$$2x - 3y \leq 6$$
$$2x + 3y \leq 12$$

For $2x - 3y \leq 6$
(a) The corresponding equation of the inequality (i) is $2x - 3y = 6$ (iii)
For $x$-intercept, put $y = 0$ in eq. (iii), we have $2x - 3(0) = 6 \Rightarrow 2x = 6 \Rightarrow x = 3$. So $x$-intercept is $A(3,0)$ For y -intercept, put $x = 0$ in eq.

(iii), we have $2(0)-3y=6 \Rightarrow -3y=6 \Rightarrow y=-2$. So $y$-intercept is $\mathrm{B}(0,-2)$.
The graph of equation (iii) is drawn by joining the points $\mathrm{A}(3,0)$ and $(0,-2)$ in the figure $4(\mathrm{i})(a)$
(b) Taking origin i.e., $0(0,0)$ as a test point. Clearly

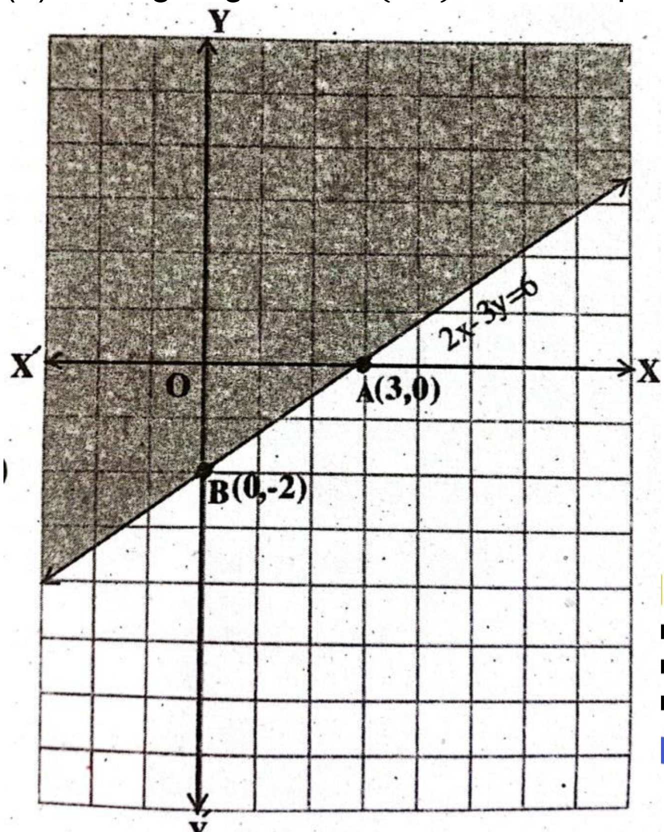


Fig. 4(i)(a)

origin satisfies the inequality (i) because $2(0)-3(0)<6 \Rightarrow 0<6$ (True)
Thus the graph of inequality (i) is the closed upper half plane on the origin side of the equation (iii) A portion of the closed lower half-plane above the line (iii) is shown as shaded region in the figure $4(1)(a)$.
For $2x+3y \leq 12$
(a) The corresponding equation of the inequality
(ii) is $2x+3y=12$. (iv)
For $x$-intercept, put $y=0$ in eq.(iv), we have

$$x+3(0)=12 \Rightarrow 2x=12 \Rightarrow x=6$$

So $x$-intirespl is $\mathrm{C}(6,0)$.
For y -inicrecpt, put $\mathrm{x}=0$ in eq.(iv), we have
$2(0)+3y-12 \Rightarrow 3y=12 \Rightarrow y=4$

So y-intercept is $D(0,4)$.
The graph of equation (iv) is drawn by joining the points $C(6,0)$ and $D(0,4)$ in the figure 4(i)(b).

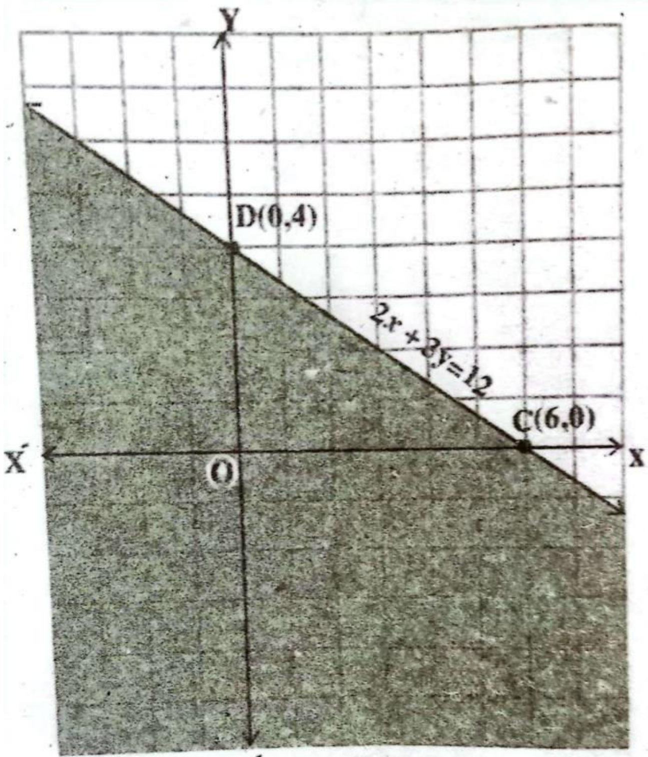


Fig. 4(i)(b)



Thus the graph of inequality (ii) is the closed lower half-plane on the origin-side of the equation (iv) A portion of the closed lower half-plane below the line
(iv) is shown as shaded region in the figure 4(i)(b).
Now the solution region of the given systems of inequalities (i) and (ii) is the intersection of the graphs indicated in the figures $4(1)(a)$ and 4(1)(b) and is shown as shaded region in the figure 4(1)(c).

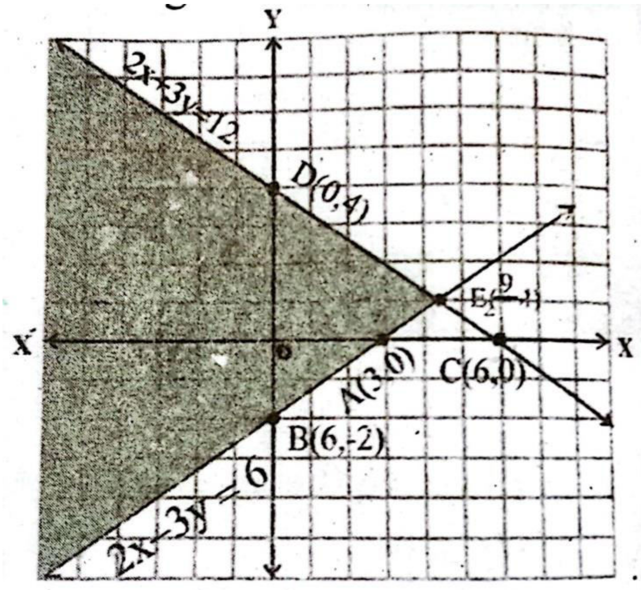


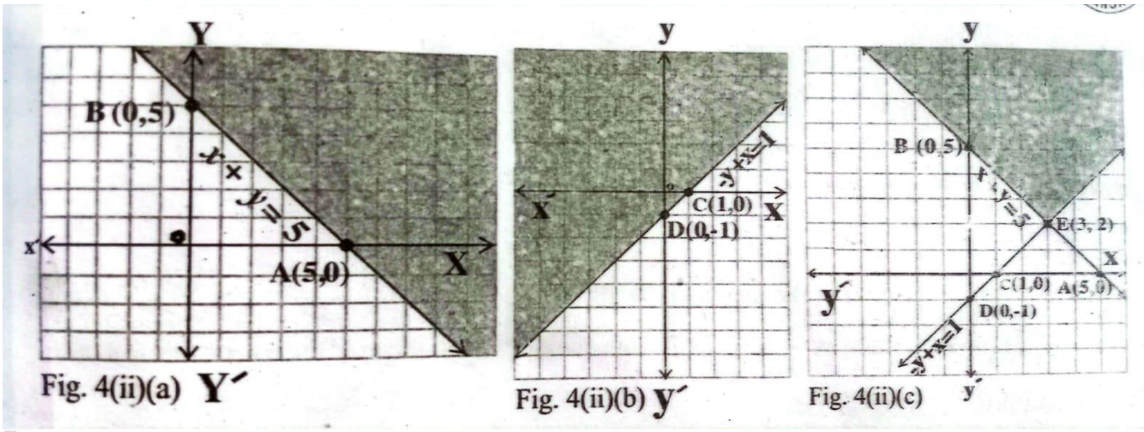
Fig. 4(i)(c)

(ii) $x+y\geq 5, -y+x\leq 1$
Solution:

$$x+y\geq 5$$
$$-y+x\leq 1$$



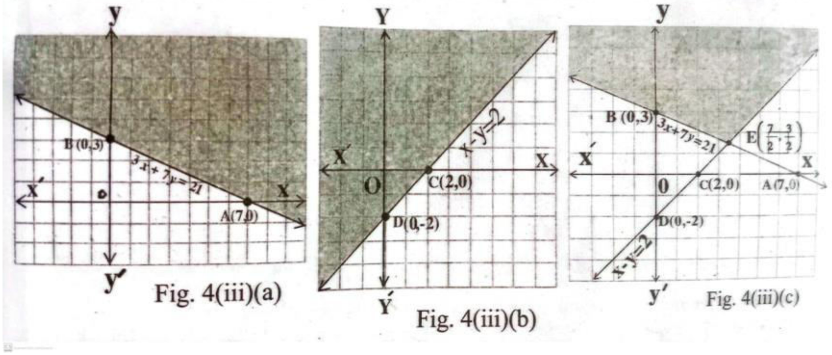
Fig. $4(\mathrm{i})(\mathrm{c})$
The corresponding equation of the inequality (i) is
$$x+y=5$$
Requation (iii) can be written as $\frac{x}{5}+\frac{y}{5}=1$. Thus the line
(iii) intersects the $x$-axis and $y$-axis at $A(5,0)$ and $B(0,5)$ respectively.
The sketch of incquality
(i) is shown as shaded region in the figure 4(ii)(a).
The corresponding equation of the inequality (ii) is $x-y=I$ (iv)
Equation (iv) can be wrimen as $\frac{x}{1}+\frac{y}{-1}=1$. Thus the line (iv) intereets the $x$-axis and $y$-axis at
$C(1,0)$ and $D(0,-1)$ repeatively. The sketch of inequality (ii) is shown as shaded region in the figure 4 (in) (b)
Now the solution region of the given systems of inequalinis (i) and (ii) is the intereection of the graphs (indiated in the figures $4(\mathrm{i})(\mathrm{a})$ and $4(\mathrm{i})(\mathrm{b})$ and is shown as shaded region in the figure $4(\mathrm{ii})(\mathrm{c})$.




(iii)
$3x+7y\geq 21$
$x-y\leq 2$
Solution:

$$3x+7y\geq 21$$
$$x-y\leq 2$$

The corresponding equation of the inequality (i) is $3x+7y=21$ (iii)
Equation (ui) can be written as $\frac{x}{7}+\frac{y}{3}=1$. Thus line (iii) intersects the $x$-axis und y ares at A(7,0) and $\mathrm{B}(0,3)$ respectively. The sketch of inequality (i) is shown as shaded region in the figure 4(iii)(a).
The corresponding equation of the inequality (ii) is $x-y=2$ (iv)
Equation (iv) can be written as $\frac{x}{2}+\frac{y}{-2}=1$. Thus the line (iv) intersects the $x$-axis and $y$-axis at $C(2,0)$ and $D(0,-2)$ respectively. The sketch of inequality (ii) is shown as shaded region in the figure 4(iii)(b).
Now the solution region of the given systems of inequalities (i) and (ii) is the intersection of the graphs indicated in the figures $4(\mathrm{iii})(\mathrm{a})$ and $4(\mathrm{iii})(\mathrm{b})$ and is shown as shaded region in the figure 4(iii)(c).




(iv)

$4x - 3y \leq 12$
$x \geq \frac{-3}{2}$
Solution:

$$4x - 3y \leq 12$$
$$x \geq \frac{-3}{2}$$

The corresponding equation of the inequality (i) is $4x - 3y = 12$ (iii)
Equation (iii) can be written as $\frac{x}{3} + \frac{y}{-4} = 1$. Thus the line (iii) intersects the $x$-axis and $y$-axis at $A(3,0)$ and $B(0,-4)$ respectively.
The sketch of inequality (i) is shown as shaded region in the figure 4(iv)(a).
The sketch of inequality (ii) is shown as shaded region in the figure 4 (iv)(b). The boundary line is drawn at $\left(\frac{-3}{2}, 0\right)$.
Now the solution region of the given systems of inequalities is the intersection of the graphs indicated in the figure 4(iv)(a) and 4(iv)(b) and is shown as shaded region in the figure 4(iv)(c).

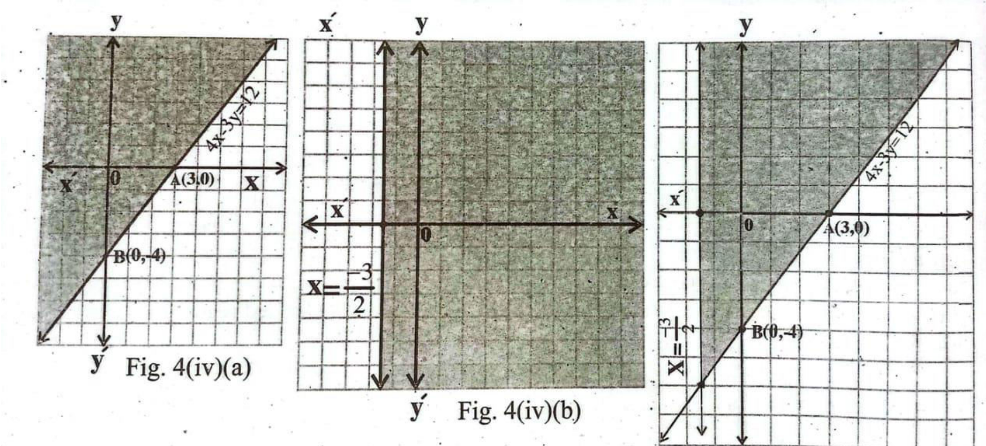


Fig. 4(iv)(a) Fig. 4(iv)(b) Fig. 4(iv)(c)

(v) $3x + 4y \geq 21$
$y \leq 4$
Solution:

$$3x + 7y \geq 21$$
$$y \leq 4$$

The corresponding equation of the inequality (i) is $3x + 7y = 21$ and can be written as $\frac{x}{7} + \frac{y}{3} = 1$ The line $(\mathrm{i})$ intersects the x -axis and y -axis at $\mathrm{A}(7,0)$ and $\mathrm{B}(0,3)$ respectively. The shath of inequality (i) is shown as shaded region in the figure $4(\mathrm{v})(\mathrm{a})$. The sketch of

inequality (ii) is shown as shaded region in the figure $4(\mathrm{v})(\mathrm{b})$. The boundary line is drawn at $c(0,4)$.
Now the solution region of the given systems of inequalities is the intersection of the graphs indicated in the figure $4(\mathrm{v})(\mathrm{a})$ and $4(\mathrm{v})(\mathrm{b})$ and is shown as shaded region in the figure $4(\mathrm{v})(\mathrm{c})$.

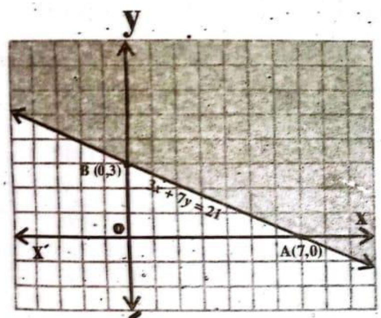


Fig. 4(v)(a)

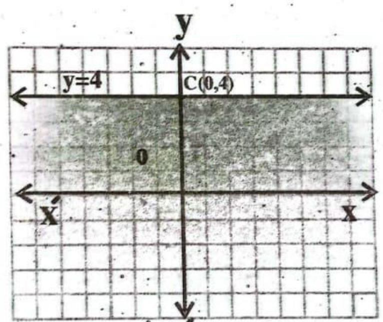


Fig. 4(v)(b)

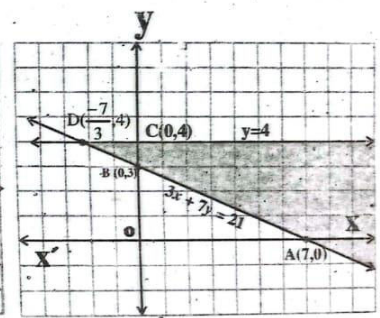


Fig. 4(v)(c)

(vi) $5x + 7y \leq 35$

$$x - 2y \leq 2$$

The associated equation of inequality (i) is $5x + 7y = 35$ and it can be written in the form $\frac{x}{7} + \frac{y}{5} = 1$. This line intersects the x axis and y -axis at point $\mathrm{A}(7,0)$ and $\mathrm{B}(0,5)$ respectively. The sketch of inequality (i) is shown as shaded region in the Fig 4(vi) (a).
The associated equation of inequality (ii) is $x - 2y = 2$ and it can be written in the form $\frac{x}{2} + \frac{y}{-1} = 1$. This line intersects the $x$-axis and $y$-axis at point $C(2,0)$ and $D(0,-1)$ respectively. The sketch of inequality (ii) is shown as shaded region in the fig.4(vi) (b).

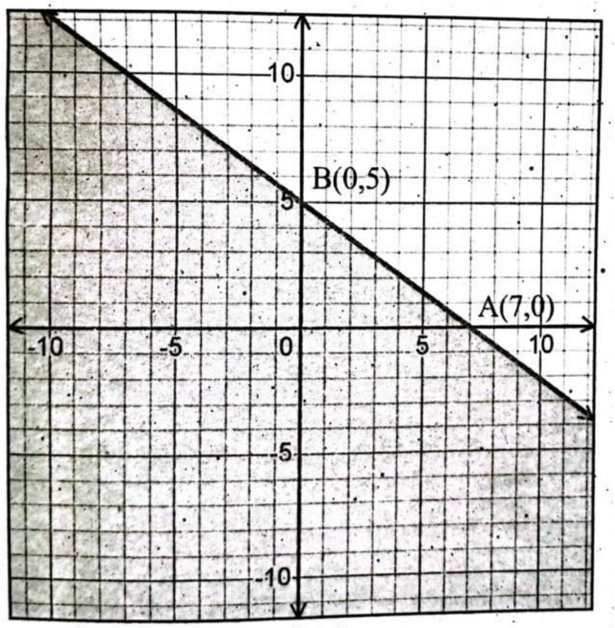


Fig. 4(vi)(a)

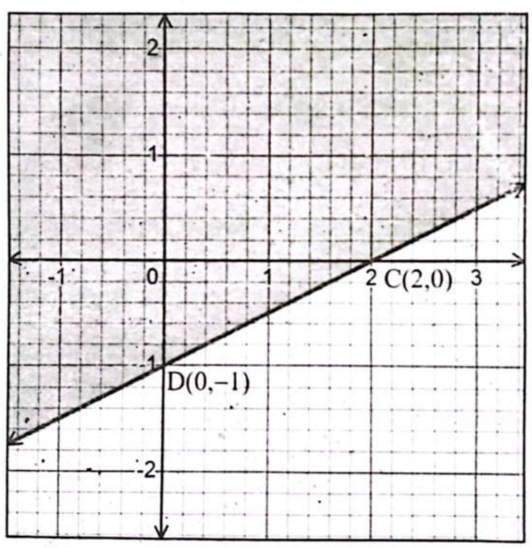


Fig. 4(vi)(b)

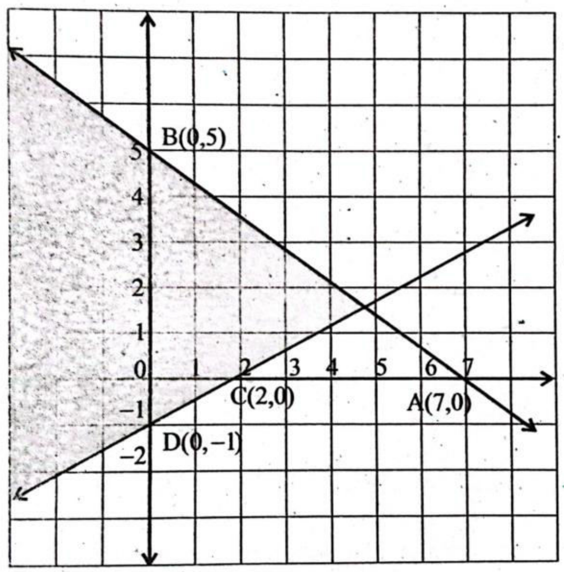


Fig.4(vi) (c)