Class 9th Math Exercise 2.4

Q. 1 Without using calculator, evaluate the following:

(i) $\log_2 18 - \log_2 9$

Solution:

$=\log_2 18 - \log_2 9$

$= \log_2 \left(\frac{18}{9}\right)$

$= \log_2 2$

$= 1$

(ii) $\log_2 64 + \log_2 2$

Solution:

$\log_2 64 + \log_2 2$

$= \log_2 (64 \times 2)$

$= \log_2 (128)$

$= \log_2 (2^7)$

$= 7\log_2 2 \because \log_2 2 = 1$

$= 7(1) = 7$

(iii) $\frac{1}{3}\log_3 8 - \log_3 18$

Solution:

$=\frac{1}{3}\log_3 8 - \log_3 18$

$=\log_3 8^{\frac{1}{3}} - \log_3 18$

$=\log_3 \left(\frac{8^{\frac{1}{3}}}{18}\right)$

$$= \log_3 \frac{(2^3)^{\frac{1}{3}}}{18}$$
$$= \log_3 \frac{2}{18}$$
$$= \log_3 \frac{1}{9}$$
$$= \log_3 \frac{1}{3^2}$$
$$= \log_3 3^{-2}$$
$$= -2\log_3 3$$
$$= -2(1)\ (\because \log_a a = 1)$$
$$= -2$$
(iv) $2\log 2 + \log 25$

Solution:
$$= 2\log 2 + \log 25$$
$$= \log 2^2 + \log 25$$
$$= \log 4 + \log 25$$
$$= \log(4 \times 25)$$
$$= \log 100$$
$$= \log 10^2$$
$$= 2\log 10$$
$$= 2(1) = 2 \ \because \log 10 = 1$$
(v) $\frac{1}{3}\log_4 64 + 2\log_5 25$

Solution:
$$\frac{1}{3}\log_4 64 + 2\log_5 25$$
$$= \frac{1}{3}\log_4 (4^3) + 2\log_5 5^2$$
$$= \frac{1}{3} \times 3\log_4 4 + 2 \times 2\log_5 5$$
$$= 1(1) + 4(1)\ (\because \log_a a = 1)$$
$$= 1 + 4$$
$$= 5$$
vi) $\log_3 12 + \log_3 0.25$

Solution:

$= \log_3 12 + \log_3 0.25$
$= \log_3 (12 \times 0.25)$
$= \log_3(3) = 1$

Q. 2 Write the following as a single logarithm.

(i) $\frac{1}{2}\log 25 + 2\log 3$

Solution:

$= \frac{1}{2}\log 25 + 2\log 3$

$=\log (25)^{\frac{1}{2}} + \log 3^2$

$=\log (5^2)^{\frac{1}{2}} + \log 9$

$=\log 5 + \log 9$

$=\log (5 \times 9)$

$=\log 45$

(ii) $\log 9 - \log \frac{1}{3}$

Solution:

$= \log 9 - \log \frac{1}{3}$

$= \log \left(\frac{9}{\frac{1}{3}}\right)$

$= \log (9 \times 3)$

$= \log 27$

(iii) $\log_5 b^2 \cdot \log_3 5^3$

Solution:

$= \log_{\mathrm{b}} \mathrm{b}^2 \cdot \log_{\mathrm{a}} 5^3$

$= 2\log_{\mathrm{s}} \mathrm{b} \cdot 3\log_{\mathrm{a}} 5$

$= \frac{6\log b}{\log 5} \times \frac{\log 5}{\log a}$ (by change of base law)

$= \frac{6\log b}{\log a}$

$= 6\log_{\mathrm{a}} \mathrm{b}$

(iv) $2\log_3 x + \log_3 y$

Solution:

$= 2\log_3 x + \log_3 y$
$= \log_3 x^2 + \log_3 y$
$= \log_3 (x^2 y)$
(v) $4\log_5 x - \log_5 y + \log_5 z$
Solution:
$= 4\log_5 x - \log_5 y + \log_5 z$
$= \log_5 x^4 - \log_5 y + \log_5 z$
$$= \log_5 \left(\frac{x^4}{y}\right) + \log_5 z$$
$$= \log_5 \left(\frac{x^4}{y} \times z\right)$$
$$= \log_5 \left(\frac{x^4 z}{y}\right)$$
(vi) $2\ln a + 3\ln b - 4\ell nc$
Solution:
$= 2\ln a + 3\ln b - 4\ln c$
$= \ln a^2 + \ln b^3 - \ln c^4$
$= \ln (a^2 \times b^3) - \ln c^4$
$$= \ln \left(\frac{a^2 b^3}{c^4}\right)$$
Q. 3 Expand the following using laws of logarithms:
(i) $\log \left(\frac{11}{5}\right)$
Solution:
$$\log \left(\frac{11}{5}\right) = \log 11 - \log 5$$
(ii) $\log_5 \sqrt{8a^6}$
Solution:

$\log_5 \sqrt{8a^6}$

$= \log_5 (8a^6)^{\frac{1}{2}}$

$= \dfrac{1}{2}\log_5 (8a^6)$

$= \dfrac{1}{2}(\log_5 8 + \log_5 a^6)$

$= \dfrac{1}{2}[\log_5 2^3 + \log_5 a^6]$

$= \dfrac{1}{2}[3\log_5 2 + 6\log_5 a]$

$= \dfrac{1}{2} \times 3\log_5 2 + \dfrac{1}{2} \times 6\log_5 a\Big]$

$= \dfrac{3}{2}\log_5 2 + 3\log_5 a$

(iii) $\ln\left(\frac{a^2b}{c}\right)$

Solution:

$= \ell n\left(\dfrac{a^2\text{ b}}{\text{c}}\right)$

$= \ell\text{n}a^2\text{ b} - \ell\text{nc}$

$= \ell\text{n}a^2 + \ell\text{nb} - \ell\text{nc}$

$= 2\ell\text{n}a + \ell\text{nb} - \ell\text{nc}$

(iv) $\log\left(\frac{xy}{z}\right)^{\frac{1}{9}}$

Solution:

$= \log\left(\dfrac{xy}{z}\right)^{\frac{1}{9}}$

$= \dfrac{1}{9}\log\left(\dfrac{xy}{z}\right)$

$= \dfrac{1}{9}(\log xy - \log z)$

$= \dfrac{1}{9}(\log x + \log y - \log z)$

$= \dfrac{1}{9}\log x + \dfrac{1}{9}\log y - \dfrac{1}{9}\log z$

(v) $\ln \sqrt[3]{16x^3}$

Solution:

$$= \ln \sqrt[3]{16x^3}$$

$$= \ln (16x^3)^{\frac{1}{3}}$$

$$= \frac{1}{3}[\ln 16x^3]$$

$$= \frac{1}{3}[\ln 16 + \ln x^3]$$

$$= \frac{1}{3}[\ln 16 + 3\ln x]$$

$$= \frac{1}{3}\ln 16 + \frac{1}{3} \times 3\ln x$$

$$= \frac{1}{3}\ln 2^4 + \ln x$$

$$= \frac{1}{3} \times 4\ln 2 + \ln x$$

$$= \frac{4}{3}\ln 2 + \ln x$$

(vi) $\log_2 \left(\frac{1-a}{b}\right)^3$

Solution:

$$= \log_2 \left(\frac{1-a}{b}\right)^3$$

$$= 5\log_2 \left(\frac{1-a}{b}\right)$$

$$= 5[\log_2 (1-a) - \log_2 b]$$

$$= 5\log_2 (1-a) - 5\log_2 b$$

Q. 4 Find the value of x in the following equations:

(i) $\log 2 + \log x = 1$

Solution:

$$\log 2 + \log x = 1$$

$$\log_{10} 2x = 1$$

In exponential form

$$10^1 = 2x$$

$$\frac{10}{2} = x$$

$$5 = x$$

$\Rightarrow x = 5$

(ii) $\log_2 x + \log_2 8 = 5$

Solution:

$\log_2 x + \log_2 8 = 5$

$\log_2 (x \times 8) = 5$

$\log_2 8x = 5$

2	8
2	4
2	2
	1

In exponential form, we get

$2^5 = 8x$

$32 = 8x$

$\frac{32}{8} = x$

$4 = x$

$\Rightarrow x = 4$

(iii) $(81)^x = (243)^{x+2}$

Solution:

$(81)^x = (243)^{x+2}$

3	243
3	81
3	27
3	9
3	3
	1

$(3^4)^x = (3^5)^{x+2}$

$3^{4x} = 3^{5x+10}$

By comparing exponents, we get

$4x = 5x + 10$

$-10 = 5x - 4x$

$-10 = x$

$\Rightarrow x = -10$

(iv) $\left(\frac{1}{27}\right)^{x-6} = 27$ Solution:

$$\left(\frac{1}{27}\right)^{x-6} = 27$$
$$(27^{-1})^{x-6} = (27)^1$$
$$(27)^{-x+6} = (27)^1$$

By comparing exponents, we get

3	27
3	9
3	3
	1

$$-x + 6 = 1$$
$$-x = 1 - 6$$
$$-x = -5$$
$$\Rightarrow x = 5$$

(v) $\log(5x - 10) = 2$

Solution:

$\log_{10}(5x - 10) = 2$

In exponential form, we get

$$10^2 = 5x - 10$$
$$100 = 5x - 10$$
$$100 + 10 = 5x$$
$$110 = 5x$$
$$\frac{110}{5} = x$$
$$22 = x$$
$$\Rightarrow x = 22$$

(vi) $\log_2(x + 1) - \log_2(x - 4) = 2$

Solution:

$\log_2(x + 1) - \log_2(x - 4) = 2$

By quotient law

$$\Rightarrow \log_2\left(\frac{x + 1}{x - 4}\right) = 2$$

In exponential form, we get

$$2^2 = \frac{x + 1}{x - 4}$$

$$4=\frac{x+1}{x-4}$$
$$\Rightarrow 4(x-4)=x+1$$
$$4x-16=x+1$$
$$4x-x=1+16$$
$$3x=17$$
$$x=\frac{17}{3}=5\frac{2}{3}$$

Q. 5 Find the values of the following with the help of logarithm table:

(i) $\frac{3.68\times4.21}{5.234}$

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Solution:

Let $x=\frac{3.68\times4.21}{5.234}$

Taking log of both sides.

$$\log\ x=\log\left(\frac{3.68\times4.21}{5.234}\right)$$

By applying laws of logarithm.

$$\log\ x=\log\ 3.68+\log\ 4.21-\log\ 5.234$$
$$\log\ x=0.5658+0.6243-0.7188$$
$$\log\ x=1.1901-0.7188$$
$$\log\ x=0.4713$$

Characteristic $=0$

Mantissa $=.4713$

$x=$ antilog (0.4713)

$x=2.960$

Thus $\frac{3.68\times4.21}{5.234}\approx2.960$

(ii) $4.67\times2.11\times2.397$

Solution:

Let $x=4.67\times2.11\times2.397$

Taking log of both sides.

$\log\ x=\log\,(4.67\times2.11\times2.397)$

By applying law of logarithm.

$$\log\ x=\log\ 4.67+\log\ 2.11+\log\ 2.397$$
$$\log\ x=0.6693+0.3243+0.3796$$
$$\log\ x=1.3732$$

Characteristic $= 1$
Mantissa $= .3732$

$x =$ antilog (1.3732)

$x \approx 23.62$

Thus $4.67 \times 2.11 \times 2.397 \approx 23.62$

(iii) $\frac{(20.46)^2 \times (2.4122)}{754.3}$

Solution:

Let $x = \frac{(20.46)^2 \times (2.412)}{754.3}$

($\because 2.4122 \approx 2.412$) (Round off)

Taking $\log$ of both sides.

$$\log\ x = \log \frac{(20.46)^2 \times (2.412)}{754.3}$$

By applying the laws of logarithm.

$$\log\ x = \log (20.46)^2 + \log (2.412) - \log (754.3)$$
$$\log\ x = 2\log (20.46) + \log (2.412) - \log (754.3)$$
$$\log\ x = 2(1.3109) + (0.3824) - (2.8776)$$
$$\log\ x = 2.6218 + 0.3824 - 2.8776$$
$$\log\ x = 3.0042 - 2.8776$$
$$\log\ x = 0.1266$$

Characteristic $= 0$
Mantissa = .1266

$x = \text{antilog}(0.1266)$

$x \approx 1.339$

Thus $\frac{(20.46)^2 \times (2.412)}{754.3} \approx 1.339$

(iv) $\frac{\sqrt[3]{9.364} \times 21.64}{3.21}$

Solution:

Let $x = \frac{\sqrt[3]{9.364} \times 21.64}{3.21}$

Taking $\log$ of both sides,

$$\log\ x = \frac{\sqrt[3]{9.364} \times 21.64}{3.21}$$

By applying the laws of logarithm.

$$\log x = \log \sqrt[3]{9.364} + \log 21.64 - \log 3.21$$
$$\log x = \log (9.364)^{\frac{1}{3}} + \log 21.64 - \log 3.21$$
$$\log x = \frac{1}{3}\log (9.364) + \log 21.64 - \log 3.21$$
$$\log x = \frac{1}{3}(0.9715) + (1.3353) - (0.5065)$$
$$\log x = 0.3238 + 1.3353 - 0.5065$$
$$\log x = 1.6591 - 0.5065$$
$$\log x = 1.1526$$

Characteristic $= 1$

Mantissa $= .1526$

$x =$ antilog (1.1526)

$x = 14.21$

Thus $\frac{\sqrt[3]{9.364}\times 21.64}{3.21} = 14.21$

Q. 6 The formula to measure the magnitude of earthquakes is given by $M = \log_{10}\left(\frac{A}{A_o}\right)$. If amplitude (A) is $\mathbf{10,000}$ and reference amplitude (A_o) is 10 . What is the magnitude of the earthquake?

Solution:

Given that $M = \log_{10}\left(\frac{A}{A_0}\right)$, $A = 10{,}000$,

$\mathrm{A}_0 = 10$

Putting the values

$$M = \log_{10}\left(\frac{10{,}000}{10}\right)$$
$$M = \log_{10} 10^3$$
$$M = 3\log_{10} 10$$
$$M = 3(1)$$
$$M = 3$$

Thus the magnitude of earthquake is 3.

Q. 7 Abdullah invested Rs. $\mathbf{100,000}$ in a saving scheme and gains interest at the rate of 5% per annum so that the total value of this investment after t years is Rs y. This is modelled by an equation $y = 100{,}000(1.05)^t, t \geq 0$.

Find after how many years the investment will be double.

Solution:

Given that $\mathrm{y} = 100{,}000(1.05)^{\mathrm{t}}$

Single investment = Rs. $100{,}000/-$

Double investment $= $ Rs. $100{,}000 \times 2$
$= $ Rs. $200{,}000/-$
Let investment after t years is y .
$\mathrm{y} =$ Rs. 200,000 putting the value in eq.(i).

$$200{,}000 = 100{,}000(1.05)^t$$

$$\frac{200{,}000}{100{,}000} = (1.05)^t$$

$$2 = (1.05)^t$$

Taking log of both sides

$$\log\ 2 = \log\ (1.05)^t$$

$$\log\ 2 = t \times 0.0212$$

$$\frac{0.3010}{0.0212} = t$$

$$14.19 = t$$

$\Rightarrow = 14.19$ years
On rounding off we get
$t \approx 14$ years
The investment will be double after 14 years.
Q. 8 Huria is hiking up a mountain where the temperature decreases by 3% (or a factor of 0.97) for every 100 metres gained in altitude. The initial temperature at sea level is $20^\circ C$. Using the formula $T = T_i \times (0.97)^{100}$, calculate the temperature at an altitude of $\mathbf{500}$ metres.
Solution:

Given that $\mathrm{T} = \mathrm{T_0} \times (0.97)^{\frac{\mathrm{h}}{100}}$ (i)
Initial temperature at sea level $= \mathrm{T}/= 20\mathrm{c}^\circ$
Altitude or height $= \mathrm{h} = 500\ \mathrm{m}.$
Putting the values in eq.(i)

$$\mathrm{T} = 20 \times (0.97)^{\frac{500}{100}}$$

$$\mathrm{T} = 20 \times (0.97)^5$$

Taking log of both sides

$\log\ T = \log\ 20 \times (0.97)^5$

$\log\ T = \log\ 20 + \log\ (0.97)^5$

$\log\ T = \log\ 20 + 5\log\ 0.97$

$\log\ T = 1.3010 + 5(\bar{1}.9867)$

$\log\ T = 1.3010 + 5(-1 + .9867)$

$\log\ T = 1.3010 + 5(-0.0133)$

$\log\ T = 1.3010 - 0.0665$

$\log\ T = 1.2345$

Characteristic $= 1$

Mantissa $= .2345$

$\mathrm{T} =$ antilog (1.2345)

$\mathrm{T} \approx 17.16$

Thus temperature at an attitude of 500 m will be $17.16°\mathrm{C}$ approximately.