

EXERCISE 12.3

1. Two numbers are randomly chosen from 1 to 10 with replacement. Find the probability that:
 - (i) both are prime
 - (ii) their product is even
2. One letter is chosen from the word PUNJAB and another from LAHORE. What is the probability that:
 - (i) both are vowels
 - (ii) one is consonant and other is vowel
3. A single dice is rolled twice. Find the probability that one roll is a multiple of 3 and the other is a 5.
4. Two dice are rolled. Find the probability of getting an odd number on one and a multiple of 2 on other.
5. From a pack of well shuffled cards, two cards are drawn at random one by one with replacement. Find the probability that the first is heart and second is king.
6. If two cards are selected from a standard deck of 52 cards without replacement, find the probability that
 - (i) Both are black.
 - (ii) Both are queens.
 - (iii) Both are spades.
 - (iv) Both are diamonds.
7. Saleem draws two cards one by one without replacement from a well shuffled pack of 52 playing cards. What is the probability that first card is jack and the second card is queen.

EXERCISE 12.3 - Multiplication Rule of Probability

Key Concept: Independent vs Dependent Events

Type	Definition	Formula
Independent Events	Outcome of one event does not affect the other (with replacement)	$P(A \cap B) = P(A) \times P(B)$
Dependent Events	Outcome of one event affects the other (without replacement)	$(P(A \cap B) = P(A) \times P(B A))$

Question 1

Two numbers randomly chosen from 1 to 10 with replacement.

Solution:

Step 1: Identify prime numbers from 1 to 10:

$$\{2, 3, 5, 7\} \rightarrow 4 \text{ prime numbers}$$

Total numbers = 10

Step 2: Probability of choosing a prime:

$$P(\text{prime}) = \frac{4}{10} = \frac{2}{5}$$

1(i) Both are prime

$$P(\text{both prime}) = \frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$$

Answer: $\frac{4}{25}$

1(ii) Product is even

Product is even if at least one number is even.

Step 1: Even numbers from 1 to 10: $\{2, 4, 6, 8, 10\} \rightarrow 5$ numbers

$$P(\text{even}) = \frac{5}{10} = \frac{1}{2}$$

Step 2: $P(\text{both odd}) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Step 3: Product is even = at least one even:

$$P(\text{product even}) = 1 - P(\text{both odd}) = 1 - \frac{1}{4} = \frac{3}{4}$$

Answer: $\frac{3}{4}$

Question 2

One letter from PUNJAB and one from LAHORE

Solution:

Step 1: Letters in PUNJAB: P, U, N, J, A, B (6 letters)

- Vowels: U, A $\rightarrow 2$ vowels
- Consonants: P, N, J, B $\rightarrow 4$ consonants

Step 2: Letters in LAHORE: L, A, H, O, R, E (6 letters)

- Vowels: A, O, E $\rightarrow 3$ vowels
- Consonants: L, H, R $\rightarrow 3$ consonants

2(i) Both are vowels

$$P(\text{vowel from PUNJAB}) = \frac{2}{6} = \frac{1}{3} \quad P(\text{vowel from LAHORE}) = \frac{3}{6} = \frac{1}{2} \quad P(\text{both vowels}) = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$$

Answer: $\frac{1}{6}$

2(ii) One is consonant and other is vowel

Two cases:

Case 1: Consonant from PUNJAB, Vowel from LAHORE

$$P = \frac{4}{6} \times \frac{3}{6} = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$$

Case 2: Vowel from PUNJAB, Consonant from LAHORE

$$P = \frac{2}{6} \times \frac{3}{6} = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$$

Step 3: Total probability:

$$P = \frac{1}{3} + \frac{1}{6} = \frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$$

Answer: $\frac{1}{2}$

Question 3

A single die is rolled twice. Find probability that one roll is multiple of 3 and the other is 5.

Solution:

Step 1: Multiples of 3 on die: $\{3, 6\} \rightarrow 2$ numbers

$$P(\text{multiple of 3}) = \frac{2}{6} = \frac{1}{3} P(5) = \frac{1}{6}$$

Step 2: Two cases:

Case 1: First roll = multiple of 3, Second roll = 5

$$P = \frac{1}{3} \times \frac{1}{6} = \frac{1}{18}$$

Case 2: First roll = 5, Second roll = multiple of 3

$$P = \frac{1}{6} \times \frac{1}{3} = \frac{1}{18}$$

Step 3: Total probability:

$$P = \frac{1}{18} + \frac{1}{18} = \frac{2}{18} = \frac{1}{9}$$

Answer: $\frac{1}{9}$

Question 4

Two dice are rolled. Find probability of getting odd number on one and multiple of 2 on other.

Solution:

Step 1: Odd numbers on die: $\{1, 3, 5\} \rightarrow 3$ numbers

$$P(\text{odd}) = \frac{3}{6} = \frac{1}{2}$$

Step 2: Multiples of 2: $\{2, 4, 6\} \rightarrow 3$ numbers

$$P(\text{multiple of 2}) = \frac{3}{6} = \frac{1}{2}$$

Step 3: Two cases:

Case 1: First = odd, Second = multiple of 2

$$P = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Case 2: First = multiple of 2, Second = odd

$$P = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Step 4: Total probability:

$$P = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

Answer: $\frac{1}{2}$

Question 5

Two cards drawn with replacement. First is heart and second is king.

Solution:

Step 1: Probability of heart:

$$P(\text{heart}) = \frac{13}{52} = \frac{1}{4}$$

Step 2: Probability of king:

$$P(\text{king}) = \frac{4}{52} = \frac{1}{13}$$

Step 3: With replacement, events are independent:

$$P = \frac{1}{4} \times \frac{1}{13} = \frac{1}{52}$$

Answer: $\frac{1}{52}$

Question 6

Two cards selected without replacement from 52 cards.

6(i) Both are black

Step 1: Total black cards = 26

Step 2: Without replacement:

$$P(\text{both black}) = \frac{26}{52} \times \frac{25}{51} = \frac{26}{52} \times \frac{25}{51} = \frac{1}{2} \times \frac{25}{51} = \frac{25}{102}$$

Answer: $\frac{25}{102}$

6(ii) Both are queens

Step 1: Total queens = 4

$$P(\text{both queens}) = \frac{4}{52} \times \frac{3}{51} = \frac{1}{13} \times \frac{1}{17} = \frac{1}{221}$$

Answer: $\frac{1}{221}$

6(iii) Both are spades

Step 1: Total spades = 13

$$P(\text{both spades}) = \frac{13}{52} \times \frac{12}{51} = \frac{1}{4} \times \frac{12}{51} = \frac{12}{204} = \frac{1}{17}$$

Answer: $\frac{1}{17}$

6(iv) Both are diamonds

Step 1: Total diamonds = 13

$$P(\text{both diamonds}) = \frac{13}{52} \times \frac{12}{51} = \frac{1}{4} \times \frac{12}{51} = \frac{12}{204} = \frac{1}{17}$$

Answer: $\frac{1}{17}$

Question 7

Two cards drawn without replacement. First is jack, second is queen.

Solution:

Step 1: Number of jacks = 4, Number of queens = 4

Step 2: Without replacement:

$$P(\text{jack then queen}) = \frac{4}{52} \times \frac{4}{51} = \frac{1}{13} \times \frac{4}{51} = \frac{4}{663}$$

Answer: $\frac{4}{663}$

Summary Table

1(i)	$\frac{4}{25}$
1(ii)	$\frac{3}{4}$
2(i)	$\frac{1}{6}$
2(ii)	$\frac{1}{2}$
3	$\frac{1}{9}$
4	$\frac{1}{2}$
5	$\frac{1}{52}$
6(i)	$\frac{25}{102}$
6(ii)	$\frac{1}{221}$
6(iii)	$\frac{1}{17}$
6(iv)	$\frac{1}{17}$
7	$\frac{4}{663}$

Key Formulas

Rule	Formula
Independent Events	$P(A \cap B) = P(A) \times P(B)$
Dependent Events	$(P(A \cap B) = P(A) \times P(B A))$
With Replacement	Probability remains constant
Without Replacement	Probability changes after each draw

Important Note: With vs Without Replacement

With Replacement	Without Replacement
Card is returned to deck	Card is not returned
Total cards stay 52	Total cards decrease
Probabilities remain constant	Probabilities change
Events are independent	Events are dependent