EXERCISE 1.2

1. Simplify and write in the form $a + bi$:

(i)	$(2 + 5i) + (3 - zi)$	(ii)	$(16 - 3i) + (9 + 2i)$
(iii)	$(9 - 2i) - (7 - 3i)$	(iv)	$(11 + 9i) - (9 - 7i)$
(v)	$(3 + 4i)\,(2 - 3i)$	(vi)	$(5 - 2i)\,(3 - 4i)$
(vii)	$(3 - 5i) \div (2 - 4i)$	(viii)	$(5 + 2i) \div (6 - 3i)$

2. Write additive inverse for each complex number:
(i) $3 + 2i$ (ii) $4 - 3i$ (iii) $5 - 7i$ (iv) $-\frac{2}{3} + \frac{5}{4}i$

3. Find multiplicative inverse for each complex number:
(i) $4 + 5i$ (ii) $6 + 2i$ (iii) $7 - 3i$ (iv) $\sqrt{5} - 4i$

4. If $z_1 = 2 + 5i$, $z_2 = 1 - 3i$ and $z_3 = 2 + i$, then verify that

(i)	$z_1 + z_2 = z_2 + z_1$	(ii)	$z_1 z_2 = z_2 z_1$
(iii)	$(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)$	(iv)	$(z_1 z_2) z_3 = z_1 (z_2 z_3)$
(v)	$z_1 + (-z_1) = (-z_1) + z_1 = 0$		

5. If $\dfrac{(1+i)^2}{2-i} = x + iy$, then find the values of x and y.

6. If $(2x + iy)(1 - i) = 4 + 2i$, then find the values of x and y.

7. Find the values of a and b, if $(a + bi)(1 + 3i) = -8 + 11i$.

Complex Numbers - Exercise 1.2 Solutions

Question 1: Simplify and write in the form $\mathbf{a + bi}$

(i) $(\mathbf{2 + 5i}) + (\mathbf{3 - 2i})$

Step 1: Add the real parts: $2 + 3 = 5$

Step 2: Add the imaginary parts: $5i + (-2i) = 3i$

Answer: $\boxed{5 + 3i}$

(ii) $(\mathbf{16 - 3i}) + (\mathbf{9 + 2i})$

Step 1: Add the real parts: $16 + 9 = 25$

Step 2: Add the imaginary parts: $-3i + 2i = -i$

Answer: $\boxed{25 - i}$

(iii) $(\mathbf{9 - 2i}) - (\mathbf{7 - 3i})$

Step 1: Subtract the real parts: $9 - 7 = 2$

Step 2: Subtract the imaginary parts: $-2i - (-3i) = -2i + 3i = i$

Answer: $\boxed{2 + i}$

(iv) $(\mathbf{11+9i})-(\mathbf{9-7i})$

Step 1: Subtract the real parts: $11-9=2$

Step 2: Subtract the imaginary parts: $9i-(-7i)=9i+7i=16i$

Answer: $\boxed{2+16i}$

(v) $(\mathbf{3+4i})(\mathbf{2-3i})$

Step 1: Use FOIL method:

- First: $3\cdot 2=6$
- Outer: $3\cdot(-3i)=-9i$
- Inner: $4i\cdot 2=8i$
- Last: $4i\cdot(-3i)=-12i^2$

Step 2: Combine: $6-9i+8i-12i^2$

Step 3: $i^2=-1$, so $-12i^2=-12(-1)=12$

Step 4: Combine like terms: $(6+12)+(-9i+8i)=18-i$

Answer: $\boxed{18-i}$

(vi) $(\mathbf{5-2i})(\mathbf{3-4i})$

Step 1: Use FOIL method:

- First: $5\cdot 3=15$
- Outer: $5\cdot(-4i)=-20i$
- Inner: $(-2i)\cdot 3=-6i$
- Last: $(-2i)\cdot(-4i)=8i^2$

Step 2: Combine: $15-20i-6i+8i^2$

Step 3: $i^2=-1$, so $8i^2=8(-1)=-8$

Step 4: Combine like terms: $(15-8)+(-20i-6i)=7-26i$

Answer: $\boxed{7-26i}$

(vii) $(\mathbf{3-5i})\div(\mathbf{2-4i})$

Step 1: Write as fraction: $\frac{3-5i}{2-4i}$

Step 2: Multiply numerator and denominator by conjugate of denominator: $2+4i$

Step 3: $\frac{3-5i}{2-4i}\cdot\frac{2+4i}{2+4i}=\frac{(3-5i)(2+4i)}{(2-4i)(2+4i)}$

Step 4: Simplify numerator using FOIL:

- $3\cdot 2=6$
- $3\cdot 4i=12i$
- $(-5i)\cdot 2=-10i$
- $(-5i)\cdot 4i=-20i^2=20$

Step 5: Numerator: $6 + 12i - 10i + 20 = 26 + 2i$

Step 6: Denominator: $(2)^2 - (4i)^2 = 4 - 16i^2 = 4 - 16(-1) = 4 + 16 = 20$

Step 7: $\frac{26+2i}{20} = \frac{26}{20} + \frac{2}{20}i = \frac{13}{10} + \frac{1}{10}i$

Answer: $\boxed{\dfrac{13}{10} + \dfrac{1}{10}i}$

(viii) $(\mathbf{5+2i}) \div (\mathbf{6-3i})$

Step 1: Write as fraction: $\frac{5+2i}{6-3i}$

Step 2: Multiply numerator and denominator by conjugate of denominator: $6 + 3i$

Step 3: $\frac{5+2i}{6-3i} \cdot \frac{6+3i}{6+3i} = \frac{(5+2i)(6+3i)}{(6-3i)(6+3i)}$

Step 4: Simplify numerator using FOIL:

- $5 \cdot 6 = 30$
- $5 \cdot 3i = 15i$
- $2i \cdot 6 = 12i$
- $2i \cdot 3i = 6i^2 = -6$

Step 5: Numerator: $30 + 15i + 12i - 6 = 24 + 27i$

Step 6: Denominator: $(6)^2 - (3i)^2 = 36 - 9i^2 = 36 - 9(-1) = 36 + 9 = 45$

Step 7: $\frac{24+27i}{45} = \frac{24}{45} + \frac{27}{45}i = \frac{8}{15} + \frac{3}{5}i$

Answer: $\boxed{\dfrac{8}{15} + \dfrac{3}{5}i}$

Question 2: Write additive inverse for each complex number

Recall: The additive inverse of $a + bi$ is $-a - bi$

(i) $\mathbf{3 + 2i}$

Step 1: Negate both parts: $-(3 + 2i) = -3 - 2i$

Answer: $\boxed{-3 - 2i}$

(ii) $\mathbf{4 - 3i}$

Step 1: Negate both parts: $-(4 - 3i) = -4 + 3i$

Answer: $\boxed{-4 + 3i}$

(iii) $\mathbf{5 - 7i}$

Step 1: Negate both parts: $-(5 - 7i) = -5 + 7i$

Answer: $\boxed{-5 + 7i}$

(iv) $\mathbf{-\frac{2}{3} + \frac{5}{4}i}$

Step 1: Negate both parts: $-\left(-\frac{2}{3} + \frac{5}{4}i\right) = \frac{2}{3} - \frac{5}{4}i$

Answer: $\boxed{\dfrac{2}{3} - \dfrac{5}{4}i}$

Final Answers Summary

1(i)	$5+3i$
1(ii)	$25-i$
1(iii)	$2+i$
1(iv)	$2+16i$
1(v)	$18-i$
1(vi)	$7-26i$
1(vii)	$\frac{13}{10}+\frac{1}{10}i$
1(viii)	$\frac{8}{15}+\frac{3}{5}i$
2(i)	$-3-2i$
2(ii)	$-4+3i$
2(iii)	$-5+7i$
2(iv)	$\frac{2}{3}-\frac{5}{4}i$

Complex Numbers - Exercise 1.2 (Continued)

Question 3: Find multiplicative inverse for each complex number

Recall: The multiplicative inverse of $a+bi$ is $\frac{a-bi}{a^2+b^2}$

(i) $\mathbf{4+5i}$

Step 1: The multiplicative inverse is $\frac{1}{4+5i}$

Step 2: Multiply by conjugate: $\frac{1}{4+5i}\cdot\frac{4-5i}{4-5i}=\frac{4-5i}{16+25}$

Step 3: Simplify: $\frac{4-5i}{41}$

Answer: $\boxed{\frac{4}{41}-\frac{5}{41}i}$

(ii) $\mathbf{6+2i}$

Step 1: The multiplicative inverse is $\frac{1}{6+2i}$

Step 2: Multiply by conjugate: $\frac{1}{6+2i}\cdot\frac{6-2i}{6-2i}=\frac{6-2i}{36+4}$

Step 3: Simplify: $\frac{6-2i}{40}=\frac{3-i}{20}$

Answer: $\boxed{\frac{3}{20}-\frac{1}{20}i}$

(iii) $\mathbf{7-3i}$

Step 1: The multiplicative inverse is $\frac{1}{7-3i}$

Step 2: Multiply by conjugate: $\frac{1}{7-3i}\cdot\frac{7+3i}{7+3i}=\frac{7+3i}{49+9}$

Step 3: Simplify: $\frac{7+3i}{58}$

Answer: $\boxed{\frac{7}{58}+\frac{3}{58}i}$

(iv) $\mathbf{\sqrt{5}-4i}$

Step 1: The multiplicative inverse is $\frac{1}{\sqrt{5}-4i}$

Step 2: Multiply by conjugate: $\frac{1}{\sqrt{5}-4i} \cdot \frac{\sqrt{5}+4i}{\sqrt{5}+4i} = \frac{\sqrt{5}+4i}{5+16}$

Step 3: Simplify: $\frac{\sqrt{5}+4i}{21}$

Answer: $\boxed{\frac{\sqrt{5}}{21} + \frac{4}{21}i}$

Question 4: Verify properties using $\mathbf{z_1 = 2 + 5i}$, $\mathbf{z_2 = 1 - 3i}$, $\mathbf{z_3 = 2 + i}$

(i) $\mathbf{z_1 + z_2 = z_2 + z_1}$

Step 1: LHS $= z_1 + z_2 = (2 + 5i) + (1 - 3i) = 3 + 2i$

Step 2: RHS $= z_2 + z_1 = (1 - 3i) + (2 + 5i) = 3 + 2i$

Step 3: LHS = RHS

Answer: Verified: Commutative property of addition

(ii) $\mathbf{z_1 z_2 = z_2 z_1}$

Step 1: LHS $= z_1 z_2 = (2 + 5i)(1 - 3i)$

Step 2: Expand: $2(1) + 2(-3i) + 5i(1) + 5i(-3i)$

- $= 2 - 6i + 5i - 15i^2$
- $= 2 - i + 15$
- $= 17 - i$

Step 3: RHS $= z_2 z_1 = (1 - 3i)(2 + 5i)$

Step 4: Expand: $1(2) + 1(5i) + (-3i)(2) + (-3i)(5i)$

- $= 2 + 5i - 6i - 15i^2$
- $= 2 - i + 15$
- $= 17 - i$

Step 5: LHS = RHS

Answer: Verified: Commutative property of multiplication

(iii) $\mathbf{(z_1 + z_2) + z_3 = z_1 + (z_2 + z_3)}$

Step 1: LHS $= (z_1 + z_2) + z_3$

- $z_1 + z_2 = 3 + 2i$
- $(3 + 2i) + (2 + i) = 5 + 3i$

Step 2: RHS $= z_1 + (z_2 + z_3)$

- $z_2 + z_3 = (1 - 3i) + (2 + i) = 3 - 2i$
- $(2 + 5i) + (3 - 2i) = 5 + 3i$

Step 3: LHS = RHS

Answer: Verified: Associative property of addition

(iv) $\mathbf{(z_1 z_2) z_3 = z_1 (z_2 z_3)}$

Step 1: LHS $= (z_1 z_2)z_3$

- $z_1 z_2 = 17 - i$ (from part ii)
- $(17 - i)(2 + i)$
- $= 17(2) + 17(i) + (-i)(2) + (-i)(i)$
- $= 34 + 17i - 2i - i^2$
- $= 34 + 15i + 1$
- $= 35 + 15i$

Step 2: RHS $= z_1(z_2 z_3)$

- $z_2 z_3 = (1 - 3i)(2 + i)$
- $= 1(2) + 1(i) + (-3i)(2) + (-3i)(i)$
- $= 2 + i - 6i - 3i^2$
- $= 2 - 5i + 3$
- $= 5 - 5i$
- Now $z_1(z_2 z_3) = (2 + 5i)(5 - 5i)$
- $= 2(5) + 2(-5i) + 5i(5) + 5i(-5i)$
- $= 10 - 10i + 25i - 25i^2$
- $= 10 + 15i + 25$
- $= 35 + 15i$

Step 3: LHS = RHS

Answer: Verified: Associative property of multiplication

(v) $\mathbf{z_1 + (-z_1) = (-z_1) + z_1 = 0}$

Step 1: $-z_1 = -2 - 5i$

Step 2: LHS $= z_1 + (-z_1) = (2 + 5i) + (-2 - 5i) = 0 + 0i = 0$

Step 3: Middle $= (-z_1) + z_1 = (-2 - 5i) + (2 + 5i) = 0 + 0i = 0$

Step 4: RHS $= 0$

Answer: Verified: Additive inverse property

Question 5: Find x and y if $\mathbf{\frac{(1+i)^2}{2-i} = x + iy}$

Step 1: Simplify numerator: $(1 + i)^2 = 1 + 2i + i^2 = 1 + 2i - 1 = 2i$

Step 2: $\frac{2i}{2-i}$

Step 3: Multiply by conjugate: $\frac{2i}{2-i} \cdot \frac{2+i}{2+i} = \frac{2i(2+i)}{4+1}$

Step 4: Expand numerator: $4i + 2i^2 = 4i + 2(-1) = -2 + 4i$

Step 5: Divide by 5: $\frac{-2+4i}{5} = -\frac{2}{5} + \frac{4}{5}i$

Step 6: Compare with $x + iy$:

- $x = -\frac{2}{5}$

- $y = \frac{4}{5}$

Answer: $\boxed{x = -\frac{2}{5},\ y = \frac{4}{5}}$

Question 6: If $(2x + iy)(1 - i) = 4 + 2i$, find x and y

Step 1: Expand LHS:

- $(2x + iy)(1 - i) = 2x(1) + 2x(-i) + iy(1) + iy(-i)$
- $= 2x - 2xi + iy - i^2y$
- $= 2x - 2xi + iy + y$
- $= (2x + y) + (-2x + y)i$

Step 2: Equate with RHS: $(2x + y) + (-2x + y)i = 4 + 2i$

Step 3: Equate real parts: $2x + y = 4$ ... (1)

Step 4: Equate imaginary parts: $-2x + y = 2$ ... (2)

Step 5: Solve the system:
Subtract (2) from (1):

$$(2x + y) - (-2x + y) = 4 - 24x = 2x = \frac{1}{2}$$

Step 6: Substitute in (1):

$$2\left(\frac{1}{2}\right) + y = 41 + y = 4y = 3$$

Answer: $\boxed{x = \frac{1}{2},\ y = 3}$

Question 7: Find a and b if $(a + bi)(1 + 3i) = -8 + 11i$

Step 1: Expand LHS:

- $(a + bi)(1 + 3i) = a(1) + a(3i) + bi(1) + bi(3i)$
- $= a + 3ai + bi + 3bi^2$
- $= a + 3ai + bi - 3b$
- $= (a - 3b) + (3a + b)i$

Step 2: Equate with RHS: $(a - 3b) + (3a + b)i = -8 + 11i$

Step 3: Equate real parts: $a - 3b = -8$ ... (1)

Step 4: Equate imaginary parts: $3a + b = 11$ ... (2)

Step 5: Solve the system:
From (2): $b = 11 - 3a$

Substitute in (1):

$$a - 3(11 - 3a) = -8a - 33 + 9a = -810a = 25a = \frac{5}{2}$$

Step 6: Find b:

$$b = 11 - 3\left(\frac{5}{2}\right) = 11 - \frac{15}{2} = \frac{22}{2} - \frac{15}{2} = \frac{7}{2}$$

Answer: $\boxed{a = \frac{5}{2},\ b = \frac{7}{2}}$

Final Answers Summary

3(i)	$\frac{4}{41} - \frac{5}{41}i$
3(ii)	$\frac{3}{20} - \frac{1}{20}i$
3(iii)	$\frac{7}{58} + \frac{3}{58}i$
3(iv)	$\frac{\sqrt{5}}{21} + \frac{4}{21}i$
4(i)-(v)	All properties verified
5	$x = -\frac{2}{5},\ y = \frac{4}{5}$
6	$x = \frac{1}{2},\ y = 3$
7	$a = \frac{5}{2},\ b = \frac{7}{2}$