EXERCISE 1.3

1. Find the modulus of the following complex numbers:
   (i) $4+3i$ (ii) $-5-4i$ (iii) $\frac{3}{5}-\frac{4}{5}i$ (iv) $-\sqrt{2}-\sqrt{3}i$
2. If $z_1 = 2+7i$ and $z_2 = 4-3i$, then verify that
   (i) $\overline{z_1+z_2} = \overline{z_1}+\overline{z_2}$ (ii) $\overline{z_1 z_2} = \overline{z_1}\,\overline{z_2}$ (iii) $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$
3. If $z = 5-2i$, then verify that
   (i) $\overline{\overline{z}} = z$ (ii) $|z| = |\overline{z}|$ (iii) $|z| = |-z|$
   (iv) $z\overline{z} = |z|^2$ (v) $|z| = |-\overline{z}|$
4. If $z = 4-3i$, then verify that $|z| = |-z| = |\overline{\overline{z}}| = |-\overline{z}|$.
5. If $z_1 = 2+3i$, $z_2 = -1+i$, then evaluate:
   (i) $Re(z_1 z_2)$ (ii) $Im(z_1 z_2)$

Complex Numbers - Exercise 1.3 Solutions

Question 1: Find the modulus of the following complex numbers

Recall: The modulus of $z = a+bi$ is $|z| = \sqrt{a^2+b^2}$

(i) $4+3i$

Step 1: Here $a=4$, $b=3$

Step 2: $|4+3i| = \sqrt{4^2+3^2} = \sqrt{16+9} = \sqrt{25}$

Answer: $\boxed{5}$

(ii) $-5-4i$

Step 1: Here $a=-5$, $b=-4$

Step 2: $|-5-4i| = \sqrt{(-5)^2+(-4)^2} = \sqrt{25+16} = \sqrt{41}$

Answer: $\boxed{\sqrt{41}}$

(iii) $\frac{3}{5}-\frac{4}{5}i$

Step 1: Here $a=\frac{3}{5}$, $b=-\frac{4}{5}$

Step 2: $\left|\frac{3}{5}-\frac{4}{5}i\right| = \sqrt{\left(\frac{3}{5}\right)^2+\left(-\frac{4}{5}\right)^2} = \sqrt{\frac{9}{25}+\frac{16}{25}} = \sqrt{\frac{25}{25}} = \sqrt{1}$

Answer: $\boxed{1}$

(iv) $-\sqrt{2}-\sqrt{3}i$

Step 1: Here $a=-\sqrt{2}$, $b=-\sqrt{3}$

Step 2: $|-\sqrt{2}-\sqrt{3}i| = \sqrt{(-\sqrt{2})^2+(-\sqrt{3})^2} = \sqrt{2+3} = \sqrt{5}$

Answer: $\boxed{\sqrt{5}}$

Question 2: Verify properties using $\mathbf{z_1 = 2 + 7i}$ and $\mathbf{z_2 = 4 - 3i}$

(i) $\mathbf{z_1 \bar{+} z_2 = \bar{z_1} + \bar{z_2}}$

Step 1: LHS $= z_1 \bar{+} z_2$

- $z_1 + z_2 = (2 + 7i) + (4 - 3i) = 6 + 4i$
- $z_1 \bar{+} z_2 = 6 \bar{+} 4i = 6 - 4i$

Step 2: RHS $= \bar{z_1} + \bar{z_2}$

- $\bar{z_1} = 2 \bar{+} 7i = 2 - 7i$
- $\bar{z_2} = 4 \bar{-} 3i = 4 + 3i$
- $\bar{z_1} + \bar{z_2} = (2 - 7i) + (4 + 3i) = 6 - 4i$

Step 3: LHS = RHS

Answer: $\boxed{\text{Verified}}$

(ii) $\mathbf{z_1 \bar{z_2} = \bar{z_1} z_2}$

Step 1: LHS $= z_1 \bar{z_2}$

- $\bar{z_2} = 4 \bar{-} 3i = 4 + 3i$
- $z_1 \bar{z_2} = (2 + 7i)(4 + 3i)$
- $= 2(4) + 2(3i) + 7i(4) + 7i(3i)$
- $= 8 + 6i + 28i + 21i^2$
- $= 8 + 34i - 21$
- $= -13 + 34i$

Step 2: RHS $= \bar{z_1} z_2$

- $\bar{z_1} = 2 \bar{+} 7i = 2 - 7i$
- $\bar{z_1} z_2 = (2 - 7i)(4 - 3i)$
- $= 2(4) + 2(-3i) + (-7i)(4) + (-7i)(-3i)$
- $= 8 - 6i - 28i + 21i^2$
- $= 8 - 34i - 21$
- $= -13 - 34i$

Step 3: LHS   RHS

Note: The correct property is $z_1 \bar{} z_2 = \bar{z_1} \cdot \bar{z_2}$, not $z_1 \bar{z_2} = \bar{z_1} z_2$.

The given statement appears to have a typo. If it meant $z_1 \bar{} z_2 = \bar{z_1} \cdot \bar{z_2}$, then verified.

Answer: $\boxed{\text{The given property is not true as stated}}$

(iii) $\left(\frac{\bar{\mathbf{z_1}}}{\mathbf{z_2}}\right) = \frac{\bar{\mathbf{z_1}}}{\bar{\mathbf{z_2}}}$

Step 1: LHS $= \left(\frac{\bar{z_1}}{z_2}\right)$

- First find $\frac{z_1}{z_2} = \frac{2+7i}{4-3i}$
- Multiply by conjugate: $\frac{2+7i}{4-3i} \cdot \frac{4+3i}{4+3i}$

- Numerator: $(2+7i)(4+3i) = 8+6i+28i+21i^2 = 8+34i-21 = -13+34i$
- Denominator: $(4-3i)(4+3i) = 16+9 = 25$
- $\frac{z_1}{z_2} = \frac{-13+34i}{25} = -\frac{13}{25} + \frac{34}{25}i$
- $\overline{\left(\frac{z_1}{z_2}\right)} = -\frac{13}{25} - \frac{34}{25}i$

Step 2: RHS $= \frac{\bar{z_1}}{\bar{z_2}}$

- $\bar{z_1} = 2-7i$
- $\bar{z_2} = 4+3i$
- $\frac{\bar{z_1}}{\bar{z_2}} = \frac{2-7i}{4+3i}$
- Multiply by conjugate: $\frac{2-7i}{4+3i} \cdot \frac{4-3i}{4-3i}$
- Numerator: $(2-7i)(4-3i) = 8-6i-28i+21i^2 = 8-34i-21 = -13-34i$
- Denominator: $(4+3i)(4-3i) = 16+9 = 25$
- $\frac{\bar{z_1}}{\bar{z_2}} = \frac{-13-34i}{25} = -\frac{13}{25} - \frac{34}{25}i$

Step 3: LHS = RHS

Answer: $\boxed{\text{Verified}}$

Question 3: If $\mathbf{z = 5 - 2i}$, verify the following properties

(i) $\bar{\bar{\mathbf{z}}} = \mathbf{z}$

Step 1: $z = 5 - 2i$

Step 2: $\bar{z} = \overline{5 - 2i} = 5 + 2i$

Step 3: $\bar{\bar{z}} = \overline{5 + 2i} = 5 - 2i = z$

Answer: $\boxed{\text{Verified}}$

(ii) $|\mathbf{z}| = |\bar{\mathbf{z}}|$

Step 1: $|z| = |5-2i| = \sqrt{25+4} = \sqrt{29}$

Step 2: $|\bar{z}| = |5+2i| = \sqrt{25+4} = \sqrt{29}$

Step 3: $|z| = |\bar{z}|$

Answer: $\boxed{\text{Verified}}$

(iii) $|\mathbf{z}| = |-\mathbf{z}|$

Step 1: $|z| = \sqrt{29}$

Step 2: $-z = -(5-2i) = -5+2i$

Step 3: $|-z| = |-5+2i| = \sqrt{25+4} = \sqrt{29}$

Answer: $\boxed{\text{Verified}}$

(iv) $\mathbf{z\bar{z}} = |\mathbf{z}|^2$

Step 1: $z\bar{z} = (5-2i)(5+2i) = 25-4i^2 = 25+4 = 29$

Step 2: $|z|^2 = (\sqrt{29})^2 = 29$

Step 3: $z\bar{z} = |z|^2$

Answer: $\boxed{\text{Verified}}$

(v) $|\mathbf{z}| = |-\mathbf{z}|$ **(already verified in part iii)**

Answer: $\boxed{\text{Verified}}$

Question 4: If $\mathbf{z = 4 - 3i}$, verify that $|\mathbf{z}| = |-\mathbf{z}| = |\bar{\mathbf{z}}|$

Step 1: $|z| = |4 - 3i| = \sqrt{16 + 9} = \sqrt{25} = 5$

Step 2: $-z = -4 + 3i$

- $|-z| = |-4 + 3i| = \sqrt{16 + 9} = \sqrt{25} = 5$

Step 3: $\bar{z} = 4 + 3i$

- $|\bar{z}| = |4 + 3i| = \sqrt{16 + 9} = \sqrt{25} = 5$

Step 4: Therefore, $|z| = |-z| = |\bar{z}| = 5$

Answer: $\boxed{\text{Verified}}$

Question 5: If $\mathbf{z_1 = 2 + 3i}$, $\mathbf{z_2 = -1 + i}$, evaluate:

(i) $\text{Re}(\mathbf{z_1 z_2})$

Step 1: $z_1 z_2 = (2 + 3i)(-1 + i)$

Step 2: Expand:

- $2(-1) + 2(i) + 3i(-1) + 3i(i)$
- $= -2 + 2i - 3i + 3i^2$
- $= -2 - i - 3$
- $= -5 - i$

Step 3: Real part $= -5$

Answer: $\boxed{-5}$

(ii) $\text{Im}(\mathbf{z_1 z_2})$

Step 1: From above, $z_1 z_2 = -5 - i$

Step 2: Imaginary part $= -1$

Answer: $\boxed{-1}$

Final Answers Summary

1(i)	5
1(ii)	$\sqrt{41}$
1(iii)	1
1(iv)	$\sqrt{5}$
2(i)	Verified
2(ii)	Not true as stated (property may have typo)
2(iii)	Verified
3(i)-(v)	All verified

4	Verified
5(i)	-5
5(ii)	-1