

EXERCISE 2.4

1. Examine the nature of roots of the following quadratic equations:

(i) $3x^2 - 9x - 2 = 0$

(ii) $x^2 + 6x + 9 = 0$

(iii) $2x^2 + 4x + 5 = 0$

(iv) $7x^2 - 6x - 1 = 0$

(v) $5x^2 - 2x + 10 = 0$

(vi) $x^2 - 8x + 16 = 0$

2. For what values of t , the roots of $3x^2 + x + 9t = 0$ are real and unequal?
3. If the quadratic equation $16x^2 + 7px + 49 = 0$ has equal roots, then find the values of p .
4. If the quadratic equation $4u^2 + 8u + q = 0$ has unequal and real roots, find the possible values for q .
5. Find the value for m , if the quadratic equation $mx^2 - 8x + 1 = 0$ has real and equal roots.

EXERCISE 2.4 - NATURE OF ROOTS

Complete Step-by-Step Solutions

Concept: Nature of Roots

For quadratic equation $ax^2 + bx + c = 0$:

- **Discriminant:** $D = b^2 - 4ac$

Condition	Nature of Roots
$D > 0$	Real, distinct (unequal)
$D = 0$	Real, equal (repeated)
$D < 0$	No real roots (complex conjugates)
$D > 0$ and perfect square	Real, rational, distinct
$D > 0$ and not perfect square	Real, irrational, distinct

Question 1: Examine the nature of roots

(i) $3x^2 - 9x - 2 = 0$

Step 1: Identify a, b, c

- $a = 3, b = -9, c = -2$

Step 2: Calculate discriminant

$$D = b^2 - 4ac = (-9)^2 - 4(3)(-2) = 81 + 24 = 105$$

Step 3: Analyze

- $D = 105 > 0$
- 105 is not a perfect square

Answer: Roots are **real, irrational, and unequal**

(ii) $x^2 + 6x + 9 = 0$

Step 1: Identify a, b, c

- $a = 1, b = 6, c = 9$

Step 2: Calculate discriminant

$$D = (6)^2 - 4(1)(9) D = 36 - 36 = 0$$

Step 3: Analyze

- $D = 0$

Answer: Roots are **real and equal** (repeated)

Note: $x^2 + 6x + 9 = (x + 3)^2$, root: $x = -3$ (double)

(iii) $2x^2 + 4x + 5 = 0$

Step 1: Identify a, b, c

- $a = 2, b = 4, c = 5$

Step 2: Calculate discriminant

$$D = (4)^2 - 4(2)(5) D = 16 - 40 = -24$$

Step 3: Analyze

- $D = -24 < 0$

Answer: Roots are **complex conjugates** (no real roots)

(iv) $7x^2 - 6x - 1 = 0$

Step 1: Identify a, b, c

- $a = 7, b = -6, c = -1$

Step 2: Calculate discriminant

$$D = (-6)^2 - 4(7)(-1) D = 36 + 28 = 64$$

Step 3: Analyze

- $D = 64 > 0$
- 64 is a perfect square (8^2)

Answer: Roots are **real, rational, and unequal**

(v) $5x^2 - 2x + 10 = 0$

Step 1: Identify a, b, c

- $a = 5, b = -2, c = 10$

Step 2: Calculate discriminant

$$D = (-2)^2 - 4(5)(10) = 4 - 200 = -196$$

Step 3: Analyze

- $D = -196 < 0$

Answer: Roots are **complex conjugates** (no real roots)

(vi) $x^2 - 8x + 16 = 0$

Step 1: Identify a, b, c

- $a = 1, b = -8, c = 16$

Step 2: Calculate discriminant

$$D = (-8)^2 - 4(1)(16) = 64 - 64 = 0$$

Step 3: Analyze

- $D = 0$

Answer: Roots are **real and equal** (repeated)

Note: $x^2 - 8x + 16 = (x - 4)^2$, root: $x = 4$ (double)

Summary Table

	Equation	Discriminant	Nature of Roots
(i)	$3x^2 - 9x - 2 = 0$	105	Real, irrational, unequal
(ii)	$x^2 + 6x + 9 = 0$	0	Real, equal
(iii)	$2x^2 + 4x + 5 = 0$	-24	Complex conjugates
(iv)	$7x^2 - 6x - 1 = 0$	64	Real, rational, unequal
(v)	$5x^2 - 2x + 10 = 0$	-196	Complex conjugates
(vi)	$x^2 - 8x + 16 = 0$	0	Real, equal

Quick Tip

To quickly determine nature of roots:

1. **Calculate** $D = b^2 - 4ac$
2. **If** $D > 0$: Real and unequal
 - Perfect square $\rightarrow$ Rational
 - Not perfect square $\rightarrow$ Irrational
3. **If** $D = 0$: Real and equal
4. **If** $D < 0$: Complex conjugates (no real roots)

EXERCISE 2.4 - NATURE OF ROOTS (Continued)

Complete Step-by-Step Solutions

Question 2: For what values of t , the roots of $3x^2 + x + 9t = 0$ are real and unequal?

Step 1: Identify a, b, c

- $a = 3, b = 1, c = 9t$

Step 2: For roots to be real and unequal, $D > 0$

$$D = b^2 - 4ac > 0 \Rightarrow D = (1)^2 - 4(3)(9t) > 0 \Rightarrow D = 1 - 108t > 0$$

Step 3: Solve the inequality

$$1 - 108t > 0 \Rightarrow -108t > -1 \Rightarrow 108t < 1 \Rightarrow t < \frac{1}{108}$$

Answer: $t < \frac{1}{108}$

Question 3: If $16x^2 + 7px + 49 = 0$ has equal roots, find p

Step 1: Identify a, b, c

- $a = 16, b = 7p, c = 49$

Step 2: For equal roots, $D = 0$

$$D = b^2 - 4ac = 0 \Rightarrow (7p)^2 - 4(16)(49) = 0$$

Step 3: Simplify

$$49p^2 - 4(16)(49) = 0 \Rightarrow 49p^2 - 3136 = 0 \Rightarrow 49p^2 = 3136 \Rightarrow p^2 = \frac{3136}{49} = 64$$

Step 4: Solve

$$p = \pm 8$$

Answer: $p = \pm 8$

Question 4: If $4u^2 + 8u + q = 0$ has unequal and real roots, find possible values for q

Step 1: Identify a, b, c

- $a = 4, b = 8, c = q$

Step 2: For real and unequal roots, $D > 0$

$$D = b^2 - 4ac > 0 \Rightarrow D = (8)^2 - 4(4)(q) > 0 \Rightarrow D = 64 - 16q > 0$$

Step 3: Solve the inequality

$$64 - 16q > 0 \Rightarrow -16q > -64 \Rightarrow 16q < 64 \Rightarrow q < 4$$

Answer: $q < 4$

Question 5: Find m , if $mx^2 - 8x + 1 = 0$ has real and equal roots

Step 1: Identify a, b, c

- $a = m, b = -8, c = 1$

Step 2: For equal roots, $D = 0$

$$D = b^2 - 4ac = 0(-8)^2 - 4(m)(1) = 064 - 4m = 0$$

Step 3: Solve

$$4m = 64 \quad m = 16$$

Answer: $m = 16$

Summary Table

	Given Condition	
2	Roots real and unequal	$t < \frac{1}{108}$
3	Equal roots	$p = \pm 8$
4	Unequal and real roots	$q < 4$
5	Real and equal roots	$m = 16$

Key Formulas Reminder

For quadratic equation $ax^2 + bx + c = 0$:

Condition	Discriminant
Real and unequal	$D > 0$
Real and equal	$D = 0$
Complex conjugates	$D < 0$
Real (equal or unequal)	$D \geq 0$