EXERCISE 2.5

1. Solve the following inequalities:

(i)	$x^2+3x-4>0$	(ii)	$2x^2-8x+6>0$
(iii)	$x^2+x-6<0$	(iv)	$x^2-6x+9<0$
(v)	$4x^2-16x+15\leq 0$	(vi)	$-x^2+3x-2\geq 0$

EXERCISE 2.5 - QUADRATIC INEQUALITIES

Complete Step-by-Step Solutions

Concept: Solving Quadratic Inequalities

For quadratic inequality $ax^2+bx+c>0$ (or $<0, \geq 0, \leq 0$):

1. Find the roots of the quadratic equation $ax^2+bx+c=0$
2. Plot the roots on a number line
3. Determine the sign of the quadratic expression in each interval
4. Select intervals satisfying the inequality

Question 1: Solve the following inequalities

(i) $\mathbf{x^2+3x-4>0}$

Step 1: Factor the quadratic

$$x^2+3x-4=(x+4)(x-1)$$

Step 2: Find roots

$$(x+4)(x-1)=0 x=-4 \text{or} x=1$$

Step 3: Sign analysis

Interval	$x<-4$	$-4<x<1$	$x>1$
$(x+4)$	-	+	+
$(x-1)$	-	-	+
Product	+	-	+

Step 4: We need >0, so select intervals where product is positive

Answer: $x\in(-\infty,-4)\cup(1,\infty)$

(ii) $\mathbf{2x^2-8x+6>0}$

Step 1: Factor the quadratic

$$2x^2-8x+6=2(x^2-4x+3)=2(x-1)(x-3)$$

Step 2: Find roots

$$2(x-1)(x-3)=0x=1\text{or}x=3$$

Step 3: Sign analysis

Interval	$x<1$	$1<x<3$	$x>3$
$(x-1)$	-	+	+
$(x-3)$	-	-	+
Product	+	-	+

Step 4: We need >0, so select intervals where product is positive

Answer: $x\in(-\infty,1)\cup(3,\infty)$

(iii) $\mathbf{x^2+x-6<0}$

Step 1: Factor the quadratic

$$x^2+x-6=(x+3)(x-2)$$

Step 2: Find roots

$$(x+3)(x-2)=0x=-3\text{or}x=2$$

Step 3: Sign analysis

Interval	$x<-3$	$-3<x<2$	$x>2$
$(x+3)$	-	+	+
$(x-2)$	-	-	+
Product	+	-	+

Step 4: We need <0 , so select interval where product is negative

Answer: $x\in(-3,2)$

(iv) $\mathbf{x^2-6x+9<0}$

Step 1: Factor the quadratic

$$x^2-6x+9=(x-3)^2$$

Step 2: Find roots

$$(x-3)^2=0\ \ x=3(\text{double root})$$

Step 3: Since $(x-3)^2\geq 0$ for all real x, and equals 0 only at $x=3$

Step 4: We need <0 , which is impossible since a square is never negative

Answer: No solution (empty set, $\emptyset$)

(v) $\mathbf{4x^2 - 16x + 15 \leq 0}$

Step 1: Factor the quadratic

$$4x^2 - 16x + 15 = (2x - 3)(2x - 5)$$

Step 2: Find roots

$$(2x - 3)(2x - 5) = 0 2x - 3 = 0 \Rightarrow x = \frac{3}{2} 2x - 5 = 0 \Rightarrow x = \frac{5}{2}$$

Step 3: Sign analysis

Interval	$x < \frac{3}{2}$	$\frac{3}{2} < x < \frac{5}{2}$	$x > \frac{5}{2}$
$(2x - 3)$	-	+	+
$(2x - 5)$	-	-	+
Product	+	-	+

Step 4: We need ≤ 0, so select interval where product is negative (including roots)

Answer: $x \in [\frac{3}{2}, \frac{5}{2}]$

(vi) $\mathbf{-x^2 + 3x - 2 \geq 0}$

Step 1: Multiply by -1 (reverse inequality sign)

$$x^2 - 3x + 2 \leq 0$$

Step 2: Factor the quadratic

$$x^2 - 3x + 2 = (x - 1)(x - 2)$$

Step 3: Find roots

$$(x - 1)(x - 2) = 0 x = 1 \text{or} x = 2$$

Step 4: Sign analysis

Interval	$x < 1$	$1 < x < 2$	$x > 2$
$(x - 1)$	-	+	+
$(x - 2)$	-	-	+
Product	+	-	+

Step 5: We need ≤ 0, so select interval where product is negative (including roots)

Answer: $x \in [1, 2]$

Summary Table

	Inequality	
(i)	$x^2 + 3x - 4 > 0$	$(-\infty, -4) \cup (1, \infty)$
(ii)	$2x^2 - 8x + 6 > 0$	$(-\infty, 1) \cup (3, \infty)$
(iii)	$x^2 + x - 6 < 0$	$(-3, 2)$
(iv)	$x^2 - 6x + 9 < 0$	$\emptyset$ (No solution)
(v)	$4x^2 - 16x + 15 \leq 0$	$[\frac{3}{2}, \frac{5}{2}]$
(vi)	$-x^2 + 3x - 2 \geq 0$	$[1, 2]$

Quick Tips for Solving Quadratic Inequalities

1. **Always factor** the quadratic expression
2. **Find the roots** (where expression = 0)
3. **Test a point** in each interval
4. **Remember:** For $ax^2 + bx + c > 0$:
   - If $a > 0$: expression is positive outside the roots
   - If $a < 0$: expression is positive between the roots
5. **Pay attention** to whether endpoints are included ($\leq$ or $\geq$) or excluded ($<$ or $>$)