EXERCISE 2.7

1. A town's population is modeled by $P(t) = -2t^2 + 40t + 800$, where t is years since 2020. Find the years when the population will be at least 1000.
2. A company models its profit P in thousands of rupees by the equation: $P(x) = -5x^2 + 150x - 1000$, where x is the price per item in rupees. Find the price that gives maximum profit.
3. A toy car rolls down an incline and covers a distance given by the equation $d = t^2 - 0.5t$ metres, where t is the time in seconds. Find the time when the car has travelled a distance 12.5 metres.
4. A ball's height (in metres) after t seconds is $h(t) = -4t^2 + 24t$. For what time interval is the ball at least $20m$ above the ground?
5. A ball is thrown upward with an initial velocity of $40\,ms^{-1}$. Calculate the maximum height it reaches above ground level.
6. A freelancer's earnings follow the model $E(h) = -2h^2 + 40h$, where E is earning in rupees and h is hours worked per week. What is the maximum number of hours he should work to maximize earnings?

EXERCISE 2.7 - APPLICATIONS OF QUADRATIC FUNCTIONS

Complete Step-by-Step Solutions

Question 1: Population at least 1000

Given: $P(t) = -2t^2 + 40t + 800$, where t is years since 2020

Step 1: Set up inequality for population at least 1000

$$-2t^2 + 40t + 800 \geq 1000$$

Step 2: Rearrange

$$-2t^2 + 40t - 200 \geq 0$$

Step 3: Divide by -2 (reverse inequality sign)

$$t^2 - 20t + 100 \leq 0$$

Step 4: Factor

$$(t - 10)^2 \leq 0$$

Step 5: Since $(t - 10)^2 \geq 0$ for all real t, the only solution is

$$t = 10$$

Step 6: Years since $2020 = 10 \rightarrow 2020 + 10 = 2030$

Answer: Population will be 1000 in the year **2030** (only at that instant, not for an interval)

Question 2: Maximum Profit

Given: $P(x) = -5x^2 + 150x - 1000$, where x is price in rupees

Step 1: For a quadratic $ax^2 + bx + c$ with $a < 0$, maximum occurs at vertex

$$x = -\frac{b}{2a}$$

Step 2: Identify $a = -5, b = 150$

$$x = -\frac{150}{2(-5)} = -\frac{150}{-10} = 15$$

Answer: Price that gives maximum profit is **Rs. 15**

Check: Maximum profit $= -5(15)^2 + 150(15) - 1000 = -1125 + 2250 - 1000 = 125$ thousand rupees

Question 3: Distance Travelled

Given: $d = t^2 - 0.5t$, where d is in metres, t in seconds

Step 1: Set $d = 12.5$

$$t^2 - 0.5t = 12.5$$

Step 2: Multiply by 2

$$2t^2 - t = 25$$

Step 3: Rearrange

$$2t^2 - t - 25 = 0$$

Step 4: Solve using quadratic formula

$$t = \frac{1 \pm \sqrt{1 + 200}}{4} = \frac{1 \pm \sqrt{201}}{4}$$

Step 5: Since time must be positive

$$t = \frac{1 + \sqrt{201}}{4} \approx \frac{1 + 14.177}{4} \approx 3.79 \text{ seconds}$$

Answer: $t = \frac{1+\sqrt{201}}{4} \approx 3.79$ seconds

Question 4: Ball Height at least 20 m

Given: $h(t) = -4t^2 + 24t$

Step 1: Set up inequality

$$-4t^2 + 24t \geq 20$$

Step 2: Rearrange

$$-4t^2 + 24t - 20 \geq 0$$

Step 3: Divide by -4 (reverse inequality sign)

$$t^2 - 6t + 5 \leq 0$$

Step 4: Factor

$$(t-1)(t-5) \leq 0$$

Step 5: Sign analysis

Interval	$t < 1$	$1 < t < 5$	$t > 5$
$(t-1)$	-	+	+
$(t-5)$	-	-	+
Product	+	-	+

Step 6: We need ≤ 0, so $1 \leq t \leq 5$

Answer: $t \in [1, 5]$ seconds

Question 5: Maximum Height of Ball

Given: Initial velocity $u = 40$ m/s

Step 1: Height function for projectile motion

$$h(t) = ut - \frac{1}{2}gt^2$$

Using $g = 10$ m/s² (or 9.8 m/s²)

$$h(t) = 40t - 5t^2$$

Step 2: For maximum height, vertex occurs at

$$t = -\frac{b}{2a} = -\frac{40}{2(-5)} = 4 \text{ seconds}$$

Step 3: Maximum height

$$h_{\max} = 40(4) - 5(4)^2 = 160 - 80 = 80 \text{ metres}$$

Answer: Maximum height = **80 metres**

Question 6: Maximum Earning Hours

Given: $E(h) = -2h^2 + 40h$, where E is earning in rupees, h is hours

Step 1: For maximum earnings, find vertex

$$h = -\frac{b}{2a} = -\frac{40}{2(-2)} = -\frac{40}{-4} = 10$$

Answer: Maximum earnings at **10 hours** per week

Check: Maximum earning $= -2(10)^2 + 40(10) = -200 + 400 = 200$ rupees

Summary Table

	Problem	
1	Population at least 1000	Year 2030
2	Maximum profit price	Rs. 15
3	Time for 12.5 m distance	$\frac{1+\sqrt{201}}{4} \approx 3.79$ s
4	Ball at least 20 m	$[1, 5]$ seconds
5	Maximum height	80 metres
6	Maximum earning hours	10 hours

Key Formulas Used

1. **Vertex of parabola** $ax^2 + bx + c$:

$$x = -\frac{b}{2a}$$

2. **Quadratic formula**:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

3. **Projectile motion** (with $g = 10$ m/s^2):

$$h(t) = ut - 5t^2$$

4. **Solving quadratic inequalities**:
   - Find roots
   - Test intervals
   - Determine sign based on inequality direction