EXERCISE 3.2

1. From the following matrices identify unit matrices, row matrices, column matrices and null matrices.

$$A=\begin{bmatrix}5 & 7 & 8\end{bmatrix}, \quad B=\begin{bmatrix}0\end{bmatrix}, \quad C=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix},$$

$$D=\begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}, \quad E=\begin{bmatrix}6\\0\\8\end{bmatrix}, \quad F=\begin{bmatrix}7\\1\\9\end{bmatrix}$$

2. Identify type of the given matrices as row, column, square and rectangular matrices.

$$A=\begin{bmatrix}5\\3\\4\end{bmatrix}, \quad B=\begin{bmatrix}1 & 6\\4 & 1\end{bmatrix}, \quad C=\begin{bmatrix}3 & 5 & 8\\0 & 4 & -2\end{bmatrix}, \quad D=\begin{bmatrix}5 & -5\\2 & 7\end{bmatrix},$$

$$E=\begin{bmatrix}3 & 2\\4 & 1\\5 & 0\end{bmatrix}, \quad F=\begin{bmatrix}5 & -3 & 7\end{bmatrix}, \quad G=\begin{bmatrix}3 & 0 & 1\\1 & 3 & 4\\5 & 2 & -3\end{bmatrix}, \quad H=\begin{bmatrix}3 & 5\\4 & 4\\5 & 2\end{bmatrix}$$

3. Identify diagonal, scalar and unit matrices.

$$A=\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}, \quad B=\begin{bmatrix}5 & 0\\0 & 5\end{bmatrix}, \quad C=\begin{bmatrix}3 & 0\\0 & 0\end{bmatrix},$$

$$D=\begin{bmatrix}7 & 0\\0 & 6\end{bmatrix}, \quad E=\begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}$$

4. Find transpose of each of the following matrices:

$$A=\begin{bmatrix}2 & 3\\4 & 5\end{bmatrix}, \quad B=\begin{bmatrix}3\\7\\6\end{bmatrix}, \quad C=\begin{bmatrix}5 & -2 & 4\end{bmatrix}, \quad D=\begin{bmatrix}2 & 5\\3 & 6\\4 & 7\end{bmatrix}$$

5. Find negative of the following matrices:

$$A=\begin{bmatrix}-3 & 0\\5 & 6\end{bmatrix}, \quad B=\begin{bmatrix}-3 & 3\\-2 & 2\end{bmatrix}, \quad C=\begin{bmatrix}-9 & 1\\1 & -7\end{bmatrix}$$

6. If $A=\begin{bmatrix}2 & 3\\4 & 5\end{bmatrix}$ and $B=\begin{bmatrix}7 & 6\\5 & 8\end{bmatrix}$, then verify that

(i) $(A^t)^t = A$ (ii) $(B^t)^t = B$

7. Show that $L=\begin{bmatrix}2 & 3 & 4\\3 & -2 & 5\\4 & 5 & 0\end{bmatrix}$ is a symmetric matrix.

EXERCISE 3.2 - TYPES OF MATRICES

Complete Step-by-Step Solutions

Question 1: Identify Unit, Row, Column and Null Matrices

Definitions:

Type	Definition
Row Matrix	A matrix with exactly one row
Column Matrix	A matrix with exactly one column
Unit (Identity) Matrix	Square matrix with 1s on main diagonal and 0s elsewhere
Null (Zero) Matrix	All entries are zero

Matrix A:

$$A = \begin{bmatrix} 5 & 7 & 8 \end{bmatrix}$$

- 1 row, 3 columns
- **Answer:** Row Matrix

Matrix B:

$$B = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

- 1 row, 2 columns
- All entries are 0
- **Answer:** Row Matrix and Null Matrix

Matrix C:

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- 2 rows, 2 columns
- Main diagonal: 1, 1
- Other entries: 0
- **Answer:** Unit (Identity) Matrix

Matrix D:

$$D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

- 2 rows, 2 columns
- All entries are 0
- **Answer:** Null Matrix

Matrix E:

$$E = \begin{bmatrix} 6 \\ 0 \\ 8 \end{bmatrix}$$

- 3 rows, 1 column
- **Answer:** Column Matrix

Matrix F:

$$F = \begin{bmatrix} 7 \\ 1 \\ 9 \end{bmatrix}$$

- 3 rows, 1 column
- **Answer:** Column Matrix

Question 2: Identify Types of Matrices

Definitions:

Type	Definition
Row Matrix	Exactly one row
Column Matrix	Exactly one column
Square Matrix	Same number of rows and columns
Rectangular Matrix	Different number of rows and columns

Matrix A:

$$A = \begin{bmatrix} 5 \\ 3 \\ 4 \end{bmatrix}$$

- 3 rows, 1 column
- **Answer:** Column Matrix

Matrix B:

$$B = \begin{bmatrix} 1 & 6 \\ 4 & 1 \end{bmatrix}$$

- 2 rows, 2 columns
- **Answer:** Square Matrix

Matrix C:

$$C = \begin{bmatrix} 3 & 5 & 8 \\ 0 & 4 & -2 \end{bmatrix}$$

- 2 rows, 3 columns
- **Answer:** Rectangular Matrix

Matrix D:

$$D = \begin{bmatrix} 5 & -5 \\ 2 & 7 \end{bmatrix}$$

- 2 rows, 2 columns
- **Answer:** Square Matrix

Matrix E:

$$E = \begin{bmatrix} 3 & 2 \\ 4 & 1 \\ 5 & 0 \end{bmatrix}$$

- 3 rows, 2 columns
- **Answer:** Rectangular Matrix

Matrix F:

$$F = \begin{bmatrix} 5 & -3 & 7 \end{bmatrix}$$

- 1 row, 3 columns
- **Answer:** Row Matrix

Matrix G:

$$G = \begin{bmatrix} 3 & 0 & 1 \\ 1 & 3 & 4 \\ 5 & 2 & -3 \end{bmatrix}$$

- 3 rows, 3 columns
- **Answer:** Square Matrix

Matrix H:

$$H = \begin{bmatrix} 3 & 5 \\ 4 & 4 \\ 5 & 2 \end{bmatrix}$$

- 3 rows, 2 columns
- **Answer:** Rectangular Matrix

Question 3: Identify Diagonal, Scalar and Unit Matrices

Definitions:

Type	Definition
Diagonal Matrix	Square matrix with non-zero entries only on the main diagonal
Scalar Matrix	Diagonal matrix where all diagonal entries are equal
Unit (Identity) Matrix	Scalar matrix with all diagonal entries = 1

Matrix A:

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- Diagonal entries: 1, 1 (all equal to 1)
- **Answer:** Diagonal Matrix, Scalar Matrix, Unit Matrix

Matrix B:

$$B = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$$

- Diagonal entries: 5, 5 (all equal)
- **Answer:** Diagonal Matrix, Scalar Matrix

Matrix C:

$$C = \begin{bmatrix} 3 & 0 \\ 0 & 0 \end{bmatrix}$$

- Diagonal entries: 3, 0 (not all equal)
- **Answer:** Diagonal Matrix only

Summary Tables

Question 1 Summary:

Matrix	Type(s)
A = [5 7 8]	Row Matrix
B = [0 0]	Row Matrix, Null Matrix
C = [1 0; 0 1]	Unit Matrix
D = [0 0; 0 0]	Null Matrix
E = [6; 0; 8]	Column Matrix
F = [7; 1; 9]	Column Matrix

Question 2 Summary:

Matrix	Type(s)
A = [5; 3; 4]	Column Matrix
B = [1 6; 4 1]	Square Matrix
C = [3 5 8; 0 4 -2]	Rectangular Matrix
D = [5 -5; 2 7]	Square Matrix
E = [3 2; 4 1; 5 0]	Rectangular Matrix
F = [5 -3 7]	Row Matrix
G = [3 0 1; 1 3 4; 5 2 -3]	Square Matrix
H = [3 5; 4 4; 5 2]	Rectangular Matrix

Question 3 Summary:

Matrix	Type(s)
A = [1 0; 0 1]	Diagonal, Scalar, Unit
B = [5 0; 0 5]	Diagonal, Scalar
C = [3 0; 0 0]	Diagonal

Quick Reference: Matrix Types

Type	Description	Example
Row	One row	$[1\ 2\ 3]$
Column	One column	$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$
Square	Equal rows and columns	$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
Rectangular	Unequal rows and columns	$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$
Diagonal	Non-zero only on main diagonal	$\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$
Scalar	Diagonal with all equal entries	$\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$
Unit/Identity	Scalar with all diagonal entries = 1	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
Null/Zero	All entries are zero	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

EXERCISE 3.2 - TYPES OF MATRICES (Continued)

Complete Step-by-Step Solutions

Question 4: Find Transpose of Matrices

Definition:

For a matrix $A = [a_{ij}]$, its transpose A^t is obtained by interchanging rows and columns:

$$(A^t)_{ij} = a_{ji}$$

Matrix A:

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$

Step 1: Interchange rows and columns

- Row 1 becomes Column 1: [2, 4]
- Row 2 becomes Column 2: [3, 5]

Answer: $A^t = \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}$

Matrix B:

$$B = \begin{bmatrix} 3 \\ 7 \\ 6 \end{bmatrix}$$

Step 1: Interchange rows and columns

- 3 rows, 1 column → 1 row, 3 columns

Answer: $B^t = \begin{bmatrix} 3 & 7 & 6 \end{bmatrix}$

Matrix C:

$$C = \begin{bmatrix} 5 & -2 & 4 \end{bmatrix}$$

Step 1: Interchange rows and columns

- 1 row, 3 columns → 3 rows, 1 column

Answer: $C^t = \begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix}$

Matrix D:

$$D = \begin{bmatrix} 2 & 5 \\ 3 & 6 \\ 4 & 7 \end{bmatrix}$$

Step 1: Interchange rows and columns

- 3 rows, 2 columns → 2 rows, 3 columns
- Column 1: [2, 3, 4] → Row 1
- Column 2: [5, 6, 7] → Row 2

Answer: $D^t = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix}$

Question 5: Find Negative of Matrices

Definition:

The negative of a matrix A is $-A$, where each entry is multiplied by -1.

Matrix A:

$$A = \begin{bmatrix} -3 & 0 \\ 5 & 6 \end{bmatrix}$$

Step 1: Multiply each entry by -1

- $(-3) \times (-1) = 3$
- $0 \times (-1) = 0$
- $5 \times (-1) = -5$
- $6 \times (-1) = -6$

Answer: $-A = \begin{bmatrix} 3 & 0 \\ -5 & -6 \end{bmatrix}$

Matrix B:

$$B = \begin{bmatrix} -3 & 3 \\ -2 & 2 \end{bmatrix}$$

Step 1: Multiply each entry by -1

- $-3 \times (-1) = 3$
- $3 \times (-1) = -3$
- $-2 \times (-1) = 2$
- $2 \times (-1) = -2$

Answer: $-B = \begin{bmatrix} 3 & -3 \\ 2 & -2 \end{bmatrix}$

Matrix C:

$$C = \begin{bmatrix} -9 & 1 \\ 1 & -7 \end{bmatrix}$$

Step 1: Multiply each entry by -1

- $-9 \times (-1) = 9$
- $1 \times (-1) = -1$
- $1 \times (-1) = -1$
- $-7 \times (-1) = 7$

Answer: $-C = \begin{bmatrix} 9 & -1 \\ -1 & 7 \end{bmatrix}$

Question 6: Verify $\left(\mathbf{A^t}\right)^{\mathbf{t}} = \mathbf{A}$

Given:

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}, B = \begin{bmatrix} 7 & 6 \\ 5 & 8 \end{bmatrix}$$

(i) $\left(\mathbf{A^t}\right)^{\mathbf{t}} = \mathbf{A}$

Step 1: Find A^t

$$A^t = \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}$$

Step 2: Find $\left(A^t\right)^t$

$$\left(A^t\right)^t = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} = A$$

Verified

(ii) $\left(\mathbf{B^t}\right)^{\mathbf{t}} = \mathbf{B}$

Step 1: Find B^t

$$B^t = \begin{bmatrix} 7 & 5 \\ 6 & 8 \end{bmatrix}$$

Step 2: Find $\left(B^t\right)^t$

$$\left(B^t\right)^t = \begin{bmatrix} 7 & 6 \\ 5 & 8 \end{bmatrix} = B$$

Verified

Question 7: Show L is a Symmetric Matrix

Given:

$$L = \begin{bmatrix} 2 & 3 & 4 \\ 3 & -2 & 5 \\ 4 & 5 & 0 \end{bmatrix}$$

Definition:

A matrix is **symmetric** if $L = L^t$

Step 1: Find L^t

Interchange rows and columns:

- Row 1 [2, 3, 4] $\rightarrow$ Column 1
- Row 2 [3, -2, 5] $\rightarrow$ Column 2
- Row 3 [4, 5, 0] $\rightarrow$ Column 3

$$L^t = \begin{bmatrix} 2 & 3 & 4 \\ 3 & -2 & 5 \\ 4 & 5 & 0 \end{bmatrix}$$

Step 2: Compare L and L^t

$$L = \begin{bmatrix} 2 & 3 & 4 \\ 3 & -2 & 5 \\ 4 & 5 & 0 \end{bmatrix} = L^t$$

Therefore, L is a symmetric matrix

Summary Table

4(A)	$\begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}$
4(B)	$\begin{bmatrix} 3 & 7 & 6 \end{bmatrix}$
4(C)	$\begin{bmatrix} 5 \\ -2 \\ 4 \end{bmatrix}$
4(D)	$\begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix}$
5(A)	$\begin{bmatrix} 3 & 0 \\ -5 & -6 \end{bmatrix}$
5(B)	$\begin{bmatrix} 3 & -3 \\ 2 & -2 \end{bmatrix}$
5(C)	$\begin{bmatrix} 9 & -1 \\ -1 & 7 \end{bmatrix}$

6(i)	Verified
6(ii)	Verified
7	L is symmetric

Key Concepts

1. Transpose of a Matrix

- Interchange rows and columns
- $(A^t)^t = A$

2. Negative of a Matrix

- Multiply every entry by -1
- $A + (-A) = 0$ (null matrix)

3. Symmetric Matrix

- Square matrix where $A = A^t$
- Entries are symmetric about the main diagonal: $a_{ij} = a_{ji}$