EXERCISE 3.3

1. Which of the following matrices are conformable for addition and subtraction?

$A=\begin{bmatrix}3 & 4\\ -2 & 1\end{bmatrix}$, $B=\begin{bmatrix}3\\ 4\end{bmatrix}$, $C=\begin{bmatrix}5 & 2\end{bmatrix}$, $D=\begin{bmatrix}1 & 7\\ 2 & 5\end{bmatrix}$ $E=\begin{bmatrix}2\end{bmatrix}$,

$F=\begin{bmatrix}7 & 11\end{bmatrix}$, $G=\begin{bmatrix}a\\ b\end{bmatrix}$, $H=\begin{bmatrix}3\end{bmatrix}$, $M=\begin{bmatrix}l\\ m\end{bmatrix}$

2. If $X=\begin{bmatrix}1 & -1\\ -2 & 2\end{bmatrix}$, $Y=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$ and $Z=\begin{bmatrix}3 & 0\\ 0 & -2\end{bmatrix}$, then find the following:

(i) $X+Y$ (ii) $Y+7Z$ (iii) $4X-Z$

(iv) $X+2Y+3Z$ (v) $X-4Y+Z$ (vi) $Z-Z$

3. Find the additive inverse of the following matrices:

(i) $P=\begin{bmatrix}5\\ -7\end{bmatrix}$ (ii) $Q=\begin{bmatrix}9 & -3\end{bmatrix}$ (iii) $R=\begin{bmatrix}2 & -1\\ -2 & 3\end{bmatrix}$ (iv) $S=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$

4. If $A=\begin{bmatrix}2 & 3\\ -3 & 2\end{bmatrix}$, $B=\begin{bmatrix}3 & 4\\ 5 & 6\end{bmatrix}$ and $C=\begin{bmatrix}1 & -2\\ 0 & 5\end{bmatrix}$, then verify the following:

(i) $A+B=B+A$ (ii) $(A+B)+C=A+(B+C)$

(iii) $(2A+B)+C=2A+(B+C)$ (iv) $3(A+B)=3A+3B$

5. If $A=\begin{bmatrix}5 & 6\\ 7 & -2\end{bmatrix}$ and $B=\begin{bmatrix}-5 & -6\\ -7 & 2\end{bmatrix}$, then show that B is additive inverse of A and A is additive inverse of B.

6. If $A=\begin{bmatrix}6 & -2\\ 0 & 3\end{bmatrix}$ and $B=\begin{bmatrix}8 & -1\\ 3 & 0\end{bmatrix}$, then verify that

(i) $(A+B)^t=A^t+B^t$ (ii) $(A-B)^t=A^t-B^t$

EXERCISE 3.3 - MATRIX ADDITION AND SUBTRACTION

Complete Step-by-Step Solutions

Question 1: Conformable Matrices for Addition and Subtraction

Definition:

Two matrices are **conformable for addition and subtraction** if and only if they have the **same order** (same number of rows and columns).

Given Matrices:

Matrix	Order
A = [3 4; -2 1]	2×2

Matrix	Order
B = [3; 4]	2×1
C = [5 2]	1×2
D = [1 7; 2 5]	2×2
E = [2]	1×1
F = [7 11]	1×2
G = [a; b]	2×1
H = [3]	1×1
M = [1; m]	2×1

Step 1: Group by Order

2×2 **Matrices:** A, D

- A and D are conformable for addition/subtraction

2×1 **Matrices:** B, G, M

- B, G, M are conformable for addition/subtraction

1×2 **Matrices:** C, F

- C and F are conformable for addition/subtraction

1×1 **Matrices:** E, H

- E and H are conformable for addition/subtraction

Answer:

Group	Matrices	Conformable?
2×2	A, D	Yes
2×1	B, G, M	Yes
1×2	C, F	Yes
1×1	E, H	Yes

Different orders cannot be added or subtracted.

Question 2: Matrix Operations

Given:

$$X = \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix}, Y = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, Z = \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix}$$

(i) $\mathbf{X + Y}$

Step 1: Add corresponding entries

$$X + Y = \begin{bmatrix} 1+1 & -1+2 \\ -2+3 & 2+4 \end{bmatrix}$$

Step 2: Simplify

$$X + Y = \begin{bmatrix} 2 & 1 \\ 1 & 6 \end{bmatrix}$$

Answer: $\begin{bmatrix} 2 & 1 \\ 1 & 6 \end{bmatrix}$

(ii) $\mathbf{Y + 7Z}$

Step 1: First find $7Z$

$$7Z = 7\begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 21 & 0 \\ 0 & -14 \end{bmatrix}$$

Step 2: Add $Y + 7Z$

$$Y + 7Z = \begin{bmatrix} 1+21 & 2+0 \\ 3+0 & 4+(-14) \end{bmatrix}$$

Step 3: Simplify

$$Y + 7Z = \begin{bmatrix} 22 & 2 \\ 3 & -10 \end{bmatrix}$$

Answer: $\begin{bmatrix} 22 & 2 \\ 3 & -10 \end{bmatrix}$

(iii) $\mathbf{4X - Z}$

Step 1: First find $4X$

$$4X = 4\begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ -8 & 8 \end{bmatrix}$$

Step 2: Subtract Z

$$4X - Z = \begin{bmatrix} 4-3 & -4-0 \\ -8-0 & 8-(-2) \end{bmatrix}$$

Step 3: Simplify

$$4X - Z = \begin{bmatrix} 1 & -4 \\ -8 & 10 \end{bmatrix}$$

Answer: $\begin{bmatrix} 1 & -4 \\ -8 & 10 \end{bmatrix}$

Summary Table

	Expression	
1	Conformable matrices	Same order matrices are conformable
2(i)	$X + Y$	$\begin{bmatrix} 2 & 1 \\ 1 & 6 \end{bmatrix}$
2(ii)	$Y + 7Z$	$\begin{bmatrix} 22 & 2 \\ 3 & -10 \end{bmatrix}$
2(iii)	$4X - Z$	$\begin{bmatrix} 1 & -4 \\ -8 & 10 \end{bmatrix}$

Key Concepts

1. Conformability for Addition/Subtraction

- Matrices must have the **same order**
- Add or subtract **corresponding entries**

2. Scalar Multiplication

- Multiply **every entry** by the scalar
- $kA = [k \cdot a_{ij}]$

3. Rules for Addition and Subtraction

- **Commutative:** $A + B = B + A$
- **Associative:** $(A + B) + C = A + (B + C)$
- **Distributive:** $k(A + B) = kA + kB$

EXERCISE 3.3 - MATRIX ADDITION AND SUBTRACTION (Continued)

Complete Step-by-Step Solutions

Question 3: Additive Inverse of Matrices

Definition:

The additive inverse of a matrix A is $-A$, where each entry is multiplied by -1.

(i) $\mathbf{P} = \begin{bmatrix} \mathbf{5} \\ \mathbf{-7} \end{bmatrix}$

Step 1: Multiply each entry by -1

$$-P = \begin{bmatrix} -5 \\ 7 \end{bmatrix}$$

Answer: $-P = \begin{bmatrix} -5 \\ 7 \end{bmatrix}$

(ii) $\mathbf{Q} = \begin{bmatrix} \mathbf{9} & \mathbf{-3} \end{bmatrix}$

Step 1: Multiply each entry by -1

$$-Q = \begin{bmatrix} -9 & 3 \end{bmatrix}$$

Answer: $-Q = \begin{bmatrix} -9 & 3 \end{bmatrix}$

(iii) $\mathbf{R} = \begin{bmatrix} \mathbf{2} & \mathbf{-1} \\ \mathbf{-2} & \mathbf{3} \end{bmatrix}$

Step 1: Multiply each entry by -1

$$-R = \begin{bmatrix} -2 & 1 \\ 2 & -3 \end{bmatrix}$$

Answer: $-R = \begin{bmatrix} -2 & 1 \\ 2 & -3 \end{bmatrix}$

(iv) $\mathbf{S} = \begin{bmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix}$

Step 1: Multiply each entry by -1

$$-S = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

Answer: $-S = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

Question 4: Verify Properties of Matrix Addition

Given:

$$A = \begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix}, B = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}, C = \begin{bmatrix} 1 & -2 \\ 0 & 5 \end{bmatrix}$$

(i) $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$ (Commutative Property)

Step 1: Find $A + B$

$$A + B = \begin{bmatrix} 2+3 & 3+4 \\ -3+5 & 2+6 \end{bmatrix} = \begin{bmatrix} 5 & 7 \\ 2 & 8 \end{bmatrix}$$

Step 2: Find $B + A$

$$B + A = \begin{bmatrix} 3+2 & 4+3 \\ 5+(-3) & 6+2 \end{bmatrix} = \begin{bmatrix} 5 & 7 \\ 2 & 8 \end{bmatrix}$$

Step 3: Compare

$$A + B = \begin{bmatrix} 5 & 7 \\ 2 & 8 \end{bmatrix} = B + A$$

Verified

(ii) $(\mathbf{A} + \mathbf{B}) + \mathbf{C} = \mathbf{A} + (\mathbf{B} + \mathbf{C})$ (Associative Property)

Step 1: Find $A + B$ (from above)

$$A + B = \begin{bmatrix} 5 & 7 \\ 2 & 8 \end{bmatrix}$$

Step 2: Find $(A + B) + C$

$$(A + B) + C = \begin{bmatrix} 5+1 & 7+(-2) \\ 2+0 & 8+5 \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 2 & 13 \end{bmatrix}$$

Step 3: Find $B + C$

$$B + C = \begin{bmatrix} 3+1 & 4+(-2) \\ 5+0 & 6+5 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 5 & 11 \end{bmatrix}$$

Step 4: Find $A + (B + C)$

$$A + (B + C) = \begin{bmatrix} 2+4 & 3+2 \\ -3+5 & 2+11 \end{bmatrix} = \begin{bmatrix} 6 & 5 \\ 2 & 13 \end{bmatrix}$$

Step 5: Compare

$$\Big(A + B) + C = \begin{bmatrix} 6 & 5 \\ 2 & 13 \end{bmatrix} = A + (B + C\Big)$$

Verified

(iii) $(\mathbf{2A} + \mathbf{B}) + \mathbf{C} = \mathbf{2A} + (\mathbf{B} + \mathbf{C})$

Step 1: Find $2A$

$$2A = \begin{bmatrix} 4 & 6 \\ -6 & 4 \end{bmatrix}$$

Step 2: Find $2A + B$

$$2A + B = \begin{bmatrix} 4+3 & 6+4 \\ -6+5 & 4+6 \end{bmatrix} = \begin{bmatrix} 7 & 10 \\ -1 & 10 \end{bmatrix}$$

Step 3: Find $(2A + B) + C$

$$(2A + B) + C = \begin{bmatrix} 7+1 & 10+(-2) \\ -1+0 & 10+5 \end{bmatrix} = \begin{bmatrix} 8 & 8 \\ -1 & 15 \end{bmatrix}$$

Step 4: Find $B + C$ (from above)

$$B + C = \begin{bmatrix} 4 & 2 \\ 5 & 11 \end{bmatrix}$$

Step 5: Find $2A + (B + C)$

$$2A + (B + C) = \begin{bmatrix} 4+4 & 6+2 \\ -6+5 & 4+11 \end{bmatrix} = \begin{bmatrix} 8 & 8 \\ -1 & 15 \end{bmatrix}$$

Step 6: Compare

$$\Big(2A + B) + C = \begin{bmatrix} 8 & 8 \\ -1 & 15 \end{bmatrix} = 2A + (B + C\Big)$$

Verified

(iv) $\mathbf{3}(\mathbf{A} + \mathbf{B}) = \mathbf{3A} + \mathbf{3B}$ (Distributive Property)

Step 1: Find $A + B$

$$A + B = \begin{bmatrix} 5 & 7 \\ 2 & 8 \end{bmatrix}$$

Step 2: Find $3(A+B)$

$$3(A+B) = \begin{bmatrix} 15 & 21 \\ 6 & 24 \end{bmatrix}$$

Step 3: Find $3A$

$$3A = \begin{bmatrix} 6 & 9 \\ -9 & 6 \end{bmatrix}$$

Step 4: Find $3B$

$$3B = \begin{bmatrix} 9 & 12 \\ 15 & 18 \end{bmatrix}$$

Step 5: Find $3A+3B$

$$3A+3B = \begin{bmatrix} 6+9 & 9+12 \\ -9+15 & 6+18 \end{bmatrix} = \begin{bmatrix} 15 & 21 \\ 6 & 24 \end{bmatrix}$$

Step 6: Compare

$$3(A+B) = \begin{bmatrix} 15 & 21 \\ 6 & 24 \end{bmatrix} = 3A+3B$$

Verified

Question 5: Additive Inverse

Given:

$$A = \begin{bmatrix} 5 & 6 \\ 7 & -2 \end{bmatrix}, B = \begin{bmatrix} -5 & -6 \\ -7 & 2 \end{bmatrix}$$

Step 1: Check if $B = -A$

$$-A = \begin{bmatrix} -5 & -6 \\ -7 & 2 \end{bmatrix} = B$$

Step 2: Check if $A = -B$

$$-B = \begin{bmatrix} 5 & 6 \\ 7 & -2 \end{bmatrix} = A$$

Step 3: Check $A+B=0$

$$A+B = \begin{bmatrix} 5+(-5) & 6+(-6) \\ 7+(-7) & -2+2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Therefore, B is the additive inverse of A and A is the additive inverse of B

Question 6: Verify Transpose Properties

Given:

$$A = \begin{bmatrix} 6 & -2 \\ 0 & 3 \end{bmatrix}, B = \begin{bmatrix} 8 & -1 \\ 3 & 0 \end{bmatrix}$$

(i) $\left(\mathbf{A+B}\right)^{\mathbf{t}} = \mathbf{A^t} + \mathbf{B^t}$

Step 1: Find $A + B$

$$A + B = \begin{bmatrix} 6+8 & -2+(-1) \\ 0+3 & 3+0 \end{bmatrix} = \begin{bmatrix} 14 & -3 \\ 3 & 3 \end{bmatrix}$$

Step 2: Find $(A + B)^t$

$$\left(A + B\right)^t = \begin{bmatrix} 14 & 3 \\ -3 & 3 \end{bmatrix}$$

Step 3: Find A^t and B^t

$$A^t = \begin{bmatrix} 6 & 0 \\ -2 & 3 \end{bmatrix}, B^t = \begin{bmatrix} 8 & 3 \\ -1 & 0 \end{bmatrix}$$

Step 4: Find $A^t + B^t$

$$A^t + B^t = \begin{bmatrix} 6+8 & 0+3 \\ -2+(-1) & 3+0 \end{bmatrix} = \begin{bmatrix} 14 & 3 \\ -3 & 3 \end{bmatrix}$$

Step 5: Compare

$$\left(A + B\right)^t = \begin{bmatrix} 14 & 3 \\ -3 & 3 \end{bmatrix} = A^t + B^t$$

Verified

(ii) $\left(\mathbf{A-B}\right)^{\mathbf{t}} = \mathbf{A^t} - \mathbf{B^t}$

Step 1: Find $A - B$

$$A - B = \begin{bmatrix} 6-8 & -2-(-1) \\ 0-3 & 3-0 \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ -3 & 3 \end{bmatrix}$$

Step 2: Find $(A - B)^t$

$$\left(A - B\right)^t = \begin{bmatrix} -2 & -3 \\ -1 & 3 \end{bmatrix}$$

Step 3: Find $A^t - B^t$

$$A^t - B^t = \begin{bmatrix} 6-8 & 0-3 \\ -2-(-1) & 3-0 \end{bmatrix} = \begin{bmatrix} -2 & -3 \\ -1 & 3 \end{bmatrix}$$

Step 4: Compare

$$(A-B)^t = \begin{bmatrix} -2 & -3 \\ -1 & 3 \end{bmatrix} = A^t - B^t$$

Verified

Summary Table

3(i)	$\begin{bmatrix} -5 \\ 7 \end{bmatrix}$
3(ii)	$\begin{bmatrix} -9 & 3 \end{bmatrix}$
3(iii)	$\begin{bmatrix} -2 & 1 \\ 2 & -3 \end{bmatrix}$
3(iv)	$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$
4(i)	Verified
4(ii)	Verified
4(iii)	Verified
4(iv)	Verified
5	Verified
6(i)	Verified
6(ii)	Verified

Key Properties Summary

Matrix Addition Properties

1. **Commutative:** $A + B = B + A$
2. **Associative:** $(A + B) + C = A + (B + C)$
3. **Additive Identity:** $A + 0 = A$
4. **Additive Inverse:** $A + (-A) = 0$
5. **Distributive:** $k(A + B) = kA + kB$

Transpose Properties

1. $(A^t)^t = A$
2. $(A + B)^t = A^t + B^t$
3. $(A - B)^t = A^t - B^t$
4. $(kA)^t = kA^t$