# EXERCISE 3.4

1. Find $AB$ and $BA$, if possible.

(i) $A=\begin{bmatrix}1 & 2\\ -1 & 0\end{bmatrix}$, $B=\begin{bmatrix}3 & 2\\ 1 & -1\end{bmatrix}$ (ii) $A=[1 \quad -2]$, $B=\begin{bmatrix}3\\ -4\end{bmatrix}$

(iii) $A=\begin{bmatrix}4\\ 4\end{bmatrix}$, $B=[2 \quad 5]$ (iv) $A=\begin{bmatrix}1 & 2\\ -1 & 1\\ 3 & 0\end{bmatrix}$, $B=\begin{bmatrix}-1 & 4 & 1\\ 2 & 3 & 1\end{bmatrix}$

(v) $A=\begin{bmatrix}1 & 2 & 2\\ 3 & 1 & 1\end{bmatrix}$, $B=\begin{bmatrix}1 & 5\\ 2 & 4\\ 1 & 6\end{bmatrix}$

2. Verify each statement, using $A=\begin{bmatrix}5 & 1\\ -1 & 4\end{bmatrix}$, $B=\begin{bmatrix}2 & -1\\ 0 & 3\end{bmatrix}$ and $C=\begin{bmatrix}5 & 1\\ 1 & 4\end{bmatrix}$.

(i) $AB \neq BA$ (ii) $A(B-C)=AB-AC$

(iii) $A(BC)=(AB)C$ (iv) $(BC)^t=C^tB^t$

(v) $(B+C)A=BA+CA$

3. If $\begin{bmatrix}4 & a\\ b & 3\end{bmatrix}\begin{bmatrix}6\\ 6\end{bmatrix}=\begin{bmatrix}6\\ 3\end{bmatrix}$, then find the values of $a$ and $b$.

4. If $\begin{bmatrix}x & 1\\ y & 2\end{bmatrix}\begin{bmatrix}1 & 0\\ 3 & -1\end{bmatrix}=\begin{bmatrix}7 & -1\\ 4 & -2\end{bmatrix}$, then find the values of $x$ and $y$.

**EXERCISE 3.4 - MATRIX MULTIPLICATION**

**Complete Step-by-Step Solutions**

## Question 1: Find AB and BA if possible

**Condition for Multiplication:**

- $AB$ is possible if **number of columns of A = number of rows of B**
- $BA$ is possible if **number of columns of B = number of rows of A**

### (i) $\mathbf{A}=\begin{bmatrix}\mathbf{1} & \mathbf{2}\\ \mathbf{-1} & \mathbf{0}\end{bmatrix}, \mathbf{B}=\begin{bmatrix}\mathbf{3} & \mathbf{2}\\ \mathbf{1} & \mathbf{-1}\end{bmatrix}$

**Step 1:** Check orders

- A: $2\times 2$, B: $2\times 2$
- Both AB and BA are possible

**Step 2:** Find $AB$

$$AB=\begin{bmatrix}1(3)+2(1) & 1(2)+2(-1)\\ -1(3)+0(1) & -1(2)+0(-1)\end{bmatrix} AB=\begin{bmatrix}3+2 & 2-2\\ -3+0 & -2+0\end{bmatrix}=\begin{bmatrix}5 & 0\\ -3 & -2\end{bmatrix}$$

**Step 3:** Find $BA$

$$BA=\begin{bmatrix}3(1)+2(-1) & 3(2)+2(0)\\ 1(1)+(-1)(-1) & 1(2)+(-1)(0)\end{bmatrix} BA=\begin{bmatrix}3-2 & 6+0\\ 1+1 & 2+0\end{bmatrix}=\begin{bmatrix}1 & 6\\ 2 & 2\end{bmatrix}$$

**Answer:** $AB = \begin{bmatrix} 5 & 0 \\ -3 & -2 \end{bmatrix}, BA = \begin{bmatrix} 1 & 6 \\ 2 & 2 \end{bmatrix}$

**(ii)** $\mathbf{A} = [\mathbf{1} \ -\mathbf{2}], \mathbf{B} = \begin{bmatrix} \mathbf{3} \\ -\mathbf{4} \end{bmatrix}$

**Step 1:** Check orders

- A: $1 \times 2$, B: $2 \times 1$
- AB: possible ($1{\times}2 \times 2{\times}1 \to 1{\times}1$)
- BA: NOT possible ($2{\times}1 \times 1{\times}2 \to 2{\times}2$ would work actually)

Wait: BA is possible: $2{\times}1 \times 1{\times}2 \to 2{\times}2$

**Step 2:** Find $AB$

$$AB = [1(3) + (-2)(-4)] = [3 + 8] = [11]$$

**Step 3:** Find $BA$

$$BA = \begin{bmatrix} 3(1) & 3(-2) \\ -4(1) & -4(-2) \end{bmatrix} = \begin{bmatrix} 3 & -6 \\ -4 & 8 \end{bmatrix}$$

**Answer:** $AB = [11], BA = \begin{bmatrix} 3 & -6 \\ -4 & 8 \end{bmatrix}$

**(iii)** $\mathbf{A} = \begin{bmatrix} \mathbf{4} \\ \mathbf{4} \end{bmatrix}, \mathbf{B} = [\mathbf{2} \ \mathbf{5}]$

**Step 1:** Check orders

- A: $2 \times 1$, B: $1 \times 2$
- AB: possible ($2{\times}1 \times 1{\times}2 \to 2{\times}2$)
- BA: possible ($1{\times}2 \times 2{\times}1 \to 1{\times}1$)

**Step 2:** Find $AB$

$$AB = \begin{bmatrix} 4(2) & 4(5) \\ 4(2) & 4(5) \end{bmatrix} = \begin{bmatrix} 8 & 20 \\ 8 & 20 \end{bmatrix}$$

**Step 3:** Find $BA$

$$BA = [2(4) + 5(4)] = [8 + 20] = [28]$$

**Answer:** $AB = \begin{bmatrix} 8 & 20 \\ 8 & 20 \end{bmatrix}, BA = [28]$

**(iv)** $\mathbf{A} = \begin{bmatrix} \mathbf{1} & \mathbf{2} \\ -\mathbf{1} & \mathbf{1} \end{bmatrix}, \mathbf{B} = \begin{bmatrix} -\mathbf{1} & \mathbf{4} \\ \mathbf{2} & \mathbf{3} \end{bmatrix}$

**Step 1:** Both are $2 \times 2$, so both products are possible

**Step 2:** Find $AB$

$$AB = \begin{bmatrix} 1(-1)+2(2) & 1(4)+2(3) \\ -1(-1)+1(2) & -1(4)+1(3) \end{bmatrix} AB = \begin{bmatrix} -1+4 & 4+6 \\ 1+2 & -4+3 \end{bmatrix} = \begin{bmatrix} 3 & 10 \\ 3 & -1 \end{bmatrix}$$

**Step 3:** Find $BA$

$$BA = \begin{bmatrix} -1(1)+4(-1) & -1(2)+4(1) \\ 2(1)+3(-1) & 2(2)+3(1) \end{bmatrix} BA = \begin{bmatrix} -1-4 & -2+4 \\ 2-3 & 4+3 \end{bmatrix} = \begin{bmatrix} -5 & 2 \\ -1 & 7 \end{bmatrix}$$

**Answer:** $AB = \begin{bmatrix} 3 & 10 \\ 3 & -1 \end{bmatrix}, BA = \begin{bmatrix} -5 & 2 \\ -1 & 7 \end{bmatrix}$

### (v) $\mathbf{A} = \begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{2} \\ \mathbf{3} & \mathbf{1} & \mathbf{1} \end{bmatrix}, \mathbf{B} = \begin{bmatrix} \mathbf{1} & \mathbf{5} \\ \mathbf{2} & \mathbf{4} \\ \mathbf{1} & \mathbf{6} \end{bmatrix}$

**Step 1:** Check orders

- A: $2 \times 3$, B: $3 \times 2$
- AB: possible (2×3 × 3×2 → 2×2)
- BA: possible (3×2 × 2×3 → 3×3)

**Step 2:** Find $AB$

$$AB = \begin{bmatrix} 1(1)+2(2)+2(1) & 1(5)+2(4)+2(6) \\ 3(1)+1(2)+1(1) & 3(5)+1(4)+1(6) \end{bmatrix} AB = \begin{bmatrix} 1+4+2 & 5+8+12 \\ 3+2+1 & 15+4+6 \end{bmatrix} = \begin{bmatrix} 7 & 25 \\ 6 & 25 \end{bmatrix}$$

**Step 3:** Find $BA$

$$BA = \begin{bmatrix} 1(1)+5(3) & 1(2)+5(1) & 1(2)+5(1) \\ 2(1)+4(3) & 2(2)+4(1) & 2(2)+4(1) \\ 1(1)+6(3) & 1(2)+6(1) & 1(2)+6(1) \end{bmatrix} BA = \begin{bmatrix} 1+15 & 2+5 & 2+5 \\ 2+12 & 4+4 & 4+4 \\ 1+18 & 2+6 & 2+6 \end{bmatrix} = \begin{bmatrix} 16 & 7 & 7 \\ 14 & 8 & 8 \\ 19 & 8 & 8 \end{bmatrix}$$

**Answer:** $AB = \begin{bmatrix} 7 & 25 \\ 6 & 25 \end{bmatrix}, BA = \begin{bmatrix} 16 & 7 & 7 \\ 14 & 8 & 8 \\ 19 & 8 & 8 \end{bmatrix}$

## Question 2: Verify Properties

**Given:**

$$A = \begin{bmatrix} 5 & 1 \\ -1 & 4 \end{bmatrix}, B = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}, C = \begin{bmatrix} 5 & 1 \\ 1 & 4 \end{bmatrix}$$

### (i) AB≠BA

**Step 1:** Find $AB$

$$AB = \begin{bmatrix} 5(2)+1(0) & 5(-1)+1(3) \\ -1(2)+4(0) & -1(-1)+4(3) \end{bmatrix} = \begin{bmatrix} 10 & -2 \\ -2 & 13 \end{bmatrix}$$

**Step 2:** Find $BA$

$$BA = \begin{bmatrix} 2(5)+(-1)(-1) & 2(1)+(-1)(4) \\ 0(5)+3(-1) & 0(1)+3(4) \end{bmatrix} = \begin{bmatrix} 10+1 & 2-4 \\ -3 & 12 \end{bmatrix} = \begin{bmatrix} 11 & -2 \\ -3 & 12 \end{bmatrix}$$

**Step 3:** Compare

$$AB = \begin{bmatrix} 10 & -2 \\ -2 & 13 \end{bmatrix} \neq \begin{bmatrix} 11 & -2 \\ -3 & 12 \end{bmatrix} = BA$$

**Verified**

### (ii) $\mathbf{AB - C = AB - AC}$ — Wait, this should be $\mathbf{A(B - C) = AB - AC}$

Let's verify $A(B-C) = AB - AC$

**Step 1:** Find $B - C$

$$B - C = \begin{bmatrix} 2-5 & -1-1 \\ 0-1 & 3-4 \end{bmatrix} = \begin{bmatrix} -3 & -2 \\ -1 & -1 \end{bmatrix}$$

**Step 2:** Find $A(B-C)$

$$A(B-C) = \begin{bmatrix} 5 & 1 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} -3 & -2 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} -15-1 & -10-1 \\ 3-4 & 2-4 \end{bmatrix} = \begin{bmatrix} -16 & -11 \\ -1 & -2 \end{bmatrix}$$

**Step 3:** Find $AC$

$$AC = \begin{bmatrix} 5(5)+1(1) & 5(1)+1(4) \\ -1(5)+4(1) & -1(1)+4(4) \end{bmatrix} = \begin{bmatrix} 26 & 9 \\ -1 & 15 \end{bmatrix}$$

**Step 4:** Find $AB - AC$

$$AB - AC = \begin{bmatrix} 10 & -2 \\ -2 & 13 \end{bmatrix} - \begin{bmatrix} 26 & 9 \\ -1 & 15 \end{bmatrix} = \begin{bmatrix} -16 & -11 \\ -1 & -2 \end{bmatrix}$$

**Step 5:** Compare

$$A(B-C) = \begin{bmatrix} -16 & -11 \\ -1 & -2 \end{bmatrix} = AB - AC$$

**Verified**

### (iii) $\mathbf{A(BC) = (AB)C}$

**Step 1:** Find $BC$

$$BC = \begin{bmatrix} 2(5)+(-1)(1) & 2(1)+(-1)(4) \\ 0(5)+3(1) & 0(1)+3(4) \end{bmatrix} = \begin{bmatrix} 10-1 & 2-4 \\ 3 & 12 \end{bmatrix} = \begin{bmatrix} 9 & -2 \\ 3 & 12 \end{bmatrix}$$

**Step 2:** Find $A(BC)$

$$A(BC) = \begin{bmatrix} 5 & 1 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 9 & -2 \\ 3 & 12 \end{bmatrix} = \begin{bmatrix} 45+3 & -10+12 \\ -9+12 & 2+48 \end{bmatrix} = \begin{bmatrix} 48 & 2 \\ 3 & 50 \end{bmatrix}$$

**Step 3:** Find $AB$ (from above)

$$AB = \begin{bmatrix} 10 & -2 \\ -2 & 13 \end{bmatrix}$$

**Step 4:** Find $(AB)C$

$$(AB)C = \begin{bmatrix} 10 & -2 \\ -2 & 13 \end{bmatrix} \begin{bmatrix} 5 & 1 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 50-2 & 10-8 \\ -10+13 & -2+52 \end{bmatrix} = \begin{bmatrix} 48 & 2 \\ 3 & 50 \end{bmatrix}$$

**Step 5:** Compare

$$A(BC) = \begin{bmatrix} 48 & 2 \\ 3 & 50 \end{bmatrix} = (AB)C$$

**Verified**

**(iv)** $\left(\mathbf{BC}\right)^{\mathbf{t}} = \mathbf{C^t B^t}$

**Step 1:** Find $BC$ (from above)

$$BC = \begin{bmatrix} 9 & -2 \\ 3 & 12 \end{bmatrix}$$

**Step 2:** Find $(BC)^t$

$$\left(BC\right)^t = \begin{bmatrix} 9 & 3 \\ -2 & 12 \end{bmatrix}$$

**Step 3:** Find $C^t$ and $B^t$

$$C^t = \begin{bmatrix} 5 & 1 \\ 1 & 4 \end{bmatrix}, B^t = \begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix}$$

**Step 4:** Find $C^tB^t$

$$C^tB^t = \begin{bmatrix} 5 & 1 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} 10-1 & 0+3 \\ 2-4 & 0+12 \end{bmatrix} = \begin{bmatrix} 9 & 3 \\ -2 & 12 \end{bmatrix}$$

**Step 5:** Compare

$$\left(BC\right)^t = \begin{bmatrix} 9 & 3 \\ -2 & 12 \end{bmatrix} = C^tB^t$$

**Verified**

**(v)** $\mathbf{(B+C)A = BA + CA}$

**Step 1:** Find $B + C$

$$B + C = \begin{bmatrix} 2+5 & -1+1 \\ 0+1 & 3+4 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 1 & 7 \end{bmatrix}$$

**Step 2:** Find $(B+C)A$

$$(B+C)A = \begin{bmatrix} 7 & 0 \\ 1 & 7 \end{bmatrix} \begin{bmatrix} 5 & 1 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 35+0 & 7+0 \\ 5-7 & 1+28 \end{bmatrix} = \begin{bmatrix} 35 & 7 \\ -2 & 29 \end{bmatrix}$$

**Step 3:** Find $BA$ (from above)

$$BA = \begin{bmatrix} 11 & -2 \\ -3 & 12 \end{bmatrix}$$

**Step 4:** Find $CA$

$$CA = \begin{bmatrix} 5 & 1 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 5 & 1 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 25-1 & 5+4 \\ 5-4 & 1+16 \end{bmatrix} = \begin{bmatrix} 24 & 9 \\ 1 & 17 \end{bmatrix}$$

**Step 5:** Find $BA+CA$

$$BA+CA = \begin{bmatrix} 11+24 & -2+9 \\ -3+1 & 12+17 \end{bmatrix} = \begin{bmatrix} 35 & 7 \\ -2 & 29 \end{bmatrix}$$

**Step 6:** Compare

$$(B+C)A = \begin{bmatrix} 35 & 7 \\ -2 & 29 \end{bmatrix} = BA+CA$$

**Verified**

## Question 3: Find a and b

**Given:**

$$\begin{bmatrix} 4 & a \\ b & 3 \end{bmatrix} \begin{bmatrix} 6 \\ 6 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

**Step 1:** Multiply left side

$$\begin{bmatrix} 4(6)+a(6) \\ b(6)+3(6) \end{bmatrix} = \begin{bmatrix} 24+6a \\ 6b+18 \end{bmatrix}$$

**Step 2:** Equate to right side

$$\begin{bmatrix} 24+6a \\ 6b+18 \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

**Step 3:** Form equations

- $24+6a=6$
- $6b+18=3$

**Step 4:** Solve

- $6a = 6-24 = -18 \rightarrow a = -3$
- $6b = 3-18 = -15 \rightarrow b = -\frac{15}{6} = -\frac{5}{2}$

**Answer:** $a = -3, b = -\frac{5}{2}$

## Question 4: Find x and y

**Given:**

$$\begin{bmatrix} x & 1 \\ y & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} 7 & -1 \\ 4 & -2 \end{bmatrix}$$

**Step 1:** Multiply left side

$$\begin{bmatrix} x(1) + 1(3) & x(0) + 1(-1) \\ y(1) + 2(3) & y(0) + 2(-1) \end{bmatrix} = \begin{bmatrix} x+3 & -1 \\ y+6 & -2 \end{bmatrix}$$

**Step 2:** Equate to right side

$$\begin{bmatrix} x+3 & -1 \\ y+6 & -2 \end{bmatrix} = \begin{bmatrix} 7 & -1 \\ 4 & -2 \end{bmatrix}$$

**Step 3:** Form equations

- $x + 3 = 7$
- $y + 6 = 4$

**Step 4:** Solve

- $x = 4$
- $y = -2$

**Answer:** $x = 4, y = -2$

**Summary Table**

| | |
|---|---|
| 1(i) AB | $\left.\begin{matrix} 5 & 0 \\ -3 & -2 \end{matrix}\right]$ |
| 1(i) BA | $\left.\begin{matrix} 1 & 6 \\ 2 & 2 \end{matrix}\right]$ |
| 1(ii) AB | $[11]$ |
| 1(ii) BA | $\left.\begin{matrix} 3 & -6 \\ -4 & 8 \end{matrix}\right]$ |
| 1(iii) AB | $\left.\begin{matrix} 8 & 20 \\ 8 & 20 \end{matrix}\right]$ |
| 1(iii) BA | $[28]$ |
| 1(iv) AB | $\left.\begin{matrix} 3 & 10 \\ 3 & -1 \end{matrix}\right]$ |
| 1(iv) BA | $\left.\begin{matrix} -5 & 2 \\ -1 & 7 \end{matrix}\right]$ |
| 1(v) AB | $\left.\begin{matrix} 7 & 25 \\ 6 & 25 \end{matrix}\right]$ |
| 1(v) BA | $\left.\begin{matrix} 16 & 7 & 7 \\ 14 & 8 & 8 \\ 19 & 8 & 8 \end{matrix}\right]$ |
| 2(i) | Verified |
| 2(ii) | Verified |

| | |
|---|---|
| 2(iii) | Verified |
| 2(iv) | Verified |
| 2(v) | Verified |
| 3 | $a = -3, b = -\frac{5}{2}$ |
| 4 | $x = 4, y = -2$ |

**Key Concepts: Matrix Multiplication**

**1. Condition for Multiplication**

- $A_{m\times n} \times B_{n\times p} = C_{m\times p}$
- Number of columns of A = Number of rows of B

**2. Properties (when defined)**

- **Not Commutative:** $AB \neq BA$ (in general)
- **Associative:** $A(BC) = (AB)C$
- **Distributive:** $A(B + C) = AB + AC$, $(A + B)C = AC + BC$
- **Transpose:** $(AB)^t = B^t A^t$