

EXERCISE 3.5

1. Find the values of each of the determinant.

(i) $\begin{vmatrix} 10 & 5 \\ 4 & 6 \end{vmatrix}$

(ii) $\begin{vmatrix} -5 & 8 \\ -3 & -7 \end{vmatrix}$

(iii) $\begin{vmatrix} 3 & 8 \\ 0 & 2 \end{vmatrix}$
2. Find whether the following matrices are singular or non-singular.
 $A = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 7 & 21 \\ 2 & 6 \end{bmatrix}$, $C = \begin{bmatrix} 13 & 5 \\ 7 & 3 \end{bmatrix}$, $D = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$
3. Find the value of x when $A = \begin{bmatrix} x & 6 \\ 5 & 15 \end{bmatrix}$ is a singular matrix.
4. Find the adjoint of the following matrices:
 $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $B = \begin{bmatrix} 5 & -2 \\ 3 & 7 \end{bmatrix}$, $C = \begin{bmatrix} -3 & 5 \\ -3 & 2 \end{bmatrix}$
5. Find multiplicative inverse of the following matrices:

(i) $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

(ii) $\begin{bmatrix} -4 & 8 \\ 7 & 2 \end{bmatrix}$

(iii) $\begin{bmatrix} 40 & 8 \\ 5 & 2 \end{bmatrix}$

(iv) $\begin{bmatrix} 3 & 5 \\ 5 & -3 \end{bmatrix}$

(v) $\begin{bmatrix} 10 & 8 \\ 3 & 3 \end{bmatrix}$

(vi) $\begin{bmatrix} -2 & -3 \\ 4 & 5 \end{bmatrix}$
6. If $A = \begin{bmatrix} 5 & -3 \\ 2 & -1 \end{bmatrix}$, then find A^{-1} and prove that $AA^{-1} = A^{-1}A = I$.
7. Show that the following matrices are multiplicative inverse of each other.

(i) $\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$, $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$

(ii) $\begin{bmatrix} 4 & 15 \\ 2 & 8 \end{bmatrix}$, $\begin{bmatrix} 4 & -\frac{15}{2} \\ -1 & 2 \end{bmatrix}$
8. Prove that $(AB)^{-1} = B^{-1}A^{-1}$, if

(i) $A = \begin{bmatrix} -3 & -2 \\ 5 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$

(ii) $A = \begin{bmatrix} 1 & 2 \\ 8 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -1 \\ 5 & 2 \end{bmatrix}$

EXERCISE 3.5 - DETERMINANTS AND INVERSE OF MATRICES

Complete Step-by-Step Solutions

Question 1: Determinants

Formula:

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

(i) $\begin{vmatrix} 10 & 5 \\ 4 & 6 \end{vmatrix}$

Step 1: Apply formula

$$= 10(6) - 5(4) = 60 - 20 = 40$$

Answer: 40

(ii) $\begin{vmatrix} -5 & 8 \\ -3 & -7 \end{vmatrix}$

Step 1: Apply formula

$$= (-5)(-7) - 8(-3) = 35 + 24 = 59$$

Answer: 59

(iii) $\begin{vmatrix} 3 & 8 \\ 0 & 2 \end{vmatrix}$

Step 1: Apply formula

$$= 3(2) - 8(0) = 6 - 0 = 6$$

Answer: 6

Question 2: Singular or Non-Singular Matrices

Definition:

- **Singular Matrix:** Determinant = 0
- **Non-Singular Matrix:** Determinant $\neq 0$

Matrix A:

$$A = \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$$

Step 1: Find determinant

$$|A| = 5(2) - 3(3) = 10 - 9 = 1$$

Step 2: Since $|A| \neq 0$

Answer: Non-Singular

Matrix B:

$$B = \begin{bmatrix} 7 & 21 \\ 2 & 6 \end{bmatrix}$$

Step 1: Find determinant

$$|B| = 7(6) - 21(2) = 42 - 42 = 0$$

Step 2: Since $|B| = 0$

Answer: Singular

Matrix C:

$$C = \begin{bmatrix} 13 & 5 \\ 7 & 3 \end{bmatrix}$$

Step 1: Find determinant

$$|C| = 13(3) - 5(7) = 39 - 35 = 4$$

Step 2: Since $|C| \neq 0$

Answer: Non-Singular

Matrix D:

$$D = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

Step 1: Find determinant

$$|D| = 2(0) - 0(0) = 0$$

Step 2: Since $|D| = 0$

Answer: Singular

Question 3: Singular Matrix

Given:

$$A = \begin{bmatrix} x & 6 \\ 5 & 15 \end{bmatrix}$$

Step 1: For singular matrix, $|A| = 0$

$$|A| = x(15) - 6(5) = 15x - 30$$

Step 2: Set equal to 0

$$15x - 30 = 0 \quad 15x = 30 \quad x = 2$$

Answer: $x = 2$

Question 4: Adjoint of Matrices

Formula:

$$\text{For } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \text{adjoint} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Matrix A:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$\text{Answer: } \text{adj}(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Matrix B:

$$B = \begin{bmatrix} 5 & -2 \\ 3 & 7 \end{bmatrix}$$

Step 1: Identify $a = 5, b = -2, c = 3, d = 7$

Step 2: Apply formula

$$\text{adj}(B) = \begin{bmatrix} 7 & 2 \\ -3 & 5 \end{bmatrix}$$

Answer: $\begin{bmatrix} 7 & 2 \\ -3 & 5 \end{bmatrix}$

Matrix C:

$$C = \begin{bmatrix} -3 & 5 \\ -3 & 2 \end{bmatrix}$$

Step 1: Identify $a = -3, b = 5, c = -3, d = 2$

Step 2: Apply formula

$$\text{adj}(C) = \begin{bmatrix} 2 & -5 \\ 3 & -3 \end{bmatrix}$$

Answer: $\begin{bmatrix} 2 & -5 \\ 3 & -3 \end{bmatrix}$

Question 5: Multiplicative Inverse

Formula:

$$A^{-1} = \frac{1}{|A|} \text{adj}(A) = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

(i) $\begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}$

Step 1: Find determinant

$$|A| = 5(5) - 0(0) = 25$$

Step 2: Find inverse

$$A^{-1} = \frac{1}{25} \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} \frac{1}{5} & 0 \\ 0 & \frac{1}{5} \end{bmatrix}$$

Answer: $\begin{bmatrix} \frac{1}{5} & 0 \\ 0 & \frac{1}{5} \end{bmatrix}$

(ii) $\begin{bmatrix} -4 & 8 \\ 7 & 2 \end{bmatrix}$

Step 1: Find determinant

$$|A| = (-4)(2) - 8(7) = -8 - 56 = -64$$

Step 2: Find inverse

$$A^{-1} = \frac{1}{-64} \begin{bmatrix} 2 & -8 \\ -7 & -4 \end{bmatrix} = \begin{bmatrix} -\frac{1}{32} & \frac{1}{8} \\ \frac{7}{64} & \frac{1}{16} \end{bmatrix}$$

Answer: $\begin{bmatrix} -\frac{1}{32} & \frac{1}{8} \\ \frac{7}{64} & \frac{1}{16} \end{bmatrix}$

(iii) $\begin{bmatrix} 40 & 8 \\ 5 & 2 \end{bmatrix}$

Step 1: Find determinant

$$|A| = 40(2) - 8(5) = 80 - 40 = 40$$

Step 2: Find inverse

$$A^{-1} = \frac{1}{40} \begin{bmatrix} 2 & -8 \\ -5 & 40 \end{bmatrix} = \begin{bmatrix} \frac{1}{20} & -\frac{1}{5} \\ -\frac{1}{8} & 1 \end{bmatrix}$$

Answer: $\begin{bmatrix} \frac{1}{20} & -\frac{1}{5} \\ -\frac{1}{8} & 1 \end{bmatrix}$

(iv) $\begin{bmatrix} 3 & 5 \\ 5 & -3 \end{bmatrix}$

Step 1: Find determinant

$$|A| = 3(-3) - 5(5) = -9 - 25 = -34$$

Step 2: Find inverse

$$A^{-1} = \frac{1}{-34} \begin{bmatrix} -3 & -5 \\ -5 & 3 \end{bmatrix} = \begin{bmatrix} \frac{3}{34} & \frac{5}{34} \\ \frac{5}{34} & -\frac{3}{34} \end{bmatrix}$$

Answer: $\begin{bmatrix} \frac{3}{34} & \frac{5}{34} \\ \frac{5}{34} & -\frac{3}{34} \end{bmatrix}$

(v) $\begin{bmatrix} 10 & 8 \\ 3 & 3 \end{bmatrix}$

Step 1: Find determinant

$$|A| = 10(3) - 8(3) = 30 - 24 = 6$$

Step 2: Find inverse

$$A^{-1} = \frac{1}{6} \begin{bmatrix} 3 & -8 \\ -3 & 10 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{4}{3} \\ -\frac{1}{2} & \frac{5}{3} \end{bmatrix}$$

Answer: $\begin{bmatrix} \frac{1}{2} & -\frac{4}{3} \\ -\frac{1}{2} & \frac{5}{3} \end{bmatrix}$

(vi) $\begin{bmatrix} -2 & -3 \\ 4 & 5 \end{bmatrix}$

Step 1: Find determinant

$$|A| = (-2)(5) - (-3)(4) = -10 + 12 = 2$$

Step 2: Find inverse

$$A^{-1} = \frac{1}{2} \begin{bmatrix} 5 & 3 \\ -4 & -2 \end{bmatrix} = \begin{bmatrix} \frac{5}{2} & \frac{3}{2} \\ -2 & -1 \end{bmatrix}$$

Answer: $\begin{bmatrix} \frac{5}{2} & \frac{3}{2} \\ -2 & -1 \end{bmatrix}$

Question 6: Verify $AA^{-1} = A^{-1}A = I$

Given:

$$A = \begin{bmatrix} 5 & -3 \\ 2 & -1 \end{bmatrix}$$

Step 1: Find determinant

$$|A| = 5(-1) - (-3)(2) = -5 + 6 = 1$$

Step 2: Find inverse

$$A^{-1} = \frac{1}{1} \begin{bmatrix} -1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ -2 & 5 \end{bmatrix}$$

Step 3: Verify AA^{-1}

$$AA^{-1} = \begin{bmatrix} 5 & -3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} -1 & 3 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 5(-1) + (-3)(-2) & 5(3) + (-3)(5) \\ 2(-1) + (-1)(-2) & 2(3) + (-1)(5) \end{bmatrix} = \begin{bmatrix} -5 + 6 & 15 - 15 \\ -2 + 2 & 6 - 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Step 4: Verify $A^{-1}A$

$$A^{-1}A = \begin{bmatrix} -1 & 3 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} 5 & -3 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} -1(5) + 3(2) & -1(-3) + 3(-1) \\ -2(5) + 5(2) & -2(-3) + 5(-1) \end{bmatrix} = \begin{bmatrix} -5 + 6 & 3 - 3 \\ -10 + 10 & 6 - 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Verified

Question 7: Multiplicative Inverses

(i) $A = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$

Step 1: Find AB

$$AB = \begin{bmatrix} 2(2) + (-1)(3) & 2(1) + (-1)(2) \\ -3(2) + 2(3) & -3(1) + 2(2) \end{bmatrix} = \begin{bmatrix} 4-3 & 2-2 \\ -6+6 & -3+4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Step 2: Find BA

$$BA = \begin{bmatrix} 2(2) + 1(-3) & 2(-1) + 1(2) \\ 3(2) + 2(-3) & 3(-1) + 2(2) \end{bmatrix} = \begin{bmatrix} 4-3 & -2+2 \\ 6-6 & -3+4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Therefore, B is inverse of A

(ii) $A = \begin{bmatrix} 4 & 15 \\ 2 & 8 \end{bmatrix}, B = \begin{bmatrix} 4 & -\frac{15}{2} \\ -1 & 2 \end{bmatrix}$

Step 1: Find AB

$$AB = \begin{bmatrix} 4(4) + 15(-1) & 4(-\frac{15}{2}) + 15(2) \\ 2(4) + 8(-1) & 2(-\frac{15}{2}) + 8(2) \end{bmatrix} = \begin{bmatrix} 16-15 & -30+30 \\ 8-8 & -15+16 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

Therefore, B is inverse of A

Question 8: Prove $(AB)^{-1} = B^{-1}A^{-1}$

(i) $A = \begin{bmatrix} -3 & -2 \\ 5 & 6 \end{bmatrix}, B = \begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$

Step 1: Find AB

$$AB = \begin{bmatrix} -3(2) + (-2)(-3) & -3(-1) + (-2)(2) \\ 5(2) + 6(-3) & 5(-1) + 6(2) \end{bmatrix} = \begin{bmatrix} -6+6 & 3-4 \\ 10-18 & -5+12 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -8 & 7 \end{bmatrix}$$

Step 2: Find $|AB|$

$$|AB| = 0(7) - (-1)(-8) = -8$$

Step 3: Find $(AB)^{-1}$

$$(AB)^{-1} = \frac{1}{-8} \begin{bmatrix} 7 & 1 \\ 8 & 0 \end{bmatrix} = \begin{bmatrix} -\frac{7}{8} & -\frac{1}{8} \\ -1 & 0 \end{bmatrix}$$

Step 4: Find A^{-1}

$$|A| = (-3)(6) - (-2)(5) = -18 + 10 = -8 A^{-1} = \frac{1}{-8} \begin{bmatrix} 6 & 2 \\ -5 & -3 \end{bmatrix} = \begin{bmatrix} -\frac{3}{4} & -\frac{1}{4} \\ \frac{5}{8} & \frac{3}{8} \end{bmatrix}$$

Step 5: Find B^{-1}

$$|B| = 2(2) - (-1)(-3) = 4 - 3 = 1 B^{-1} = \frac{1}{1} \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$$

Step 6: Find $B^{-1}A^{-1}$

$$B^{-1}A^{-1} = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} -\frac{3}{8} & -\frac{1}{4} \\ \frac{5}{8} & \frac{3}{8} \end{bmatrix} = \begin{bmatrix} 2\left(-\frac{3}{8}\right) + 1\left(\frac{5}{8}\right) & 2\left(-\frac{1}{4}\right) + 1\left(\frac{3}{8}\right) \\ 3\left(-\frac{3}{8}\right) + 2\left(\frac{5}{8}\right) & 3\left(-\frac{1}{4}\right) + 2\left(\frac{3}{8}\right) \end{bmatrix} = \begin{bmatrix} -\frac{3}{4} + \frac{5}{8} & -\frac{1}{2} + \frac{3}{8} \\ -\frac{9}{8} + \frac{5}{4} & -\frac{3}{4} + \frac{3}{4} \end{bmatrix} = \begin{bmatrix} -\frac{12}{8} + \frac{5}{8} & -\frac{4}{8} + \frac{3}{8} \\ -\frac{9}{8} + \frac{10}{8} & 0 \end{bmatrix} = \begin{bmatrix} -\frac{7}{8} & -\frac{1}{8} \\ \frac{1}{8} & 0 \end{bmatrix}$$

Step 7: Compare

$$(AB)^{-1} = \begin{bmatrix} -\frac{7}{8} & -\frac{1}{8} \\ \frac{1}{8} & 0 \end{bmatrix} = B^{-1}A^{-1}$$

Verified

(ii) $A = \begin{bmatrix} 1 & 2 \\ 8 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & -1 \\ 5 & 2 \end{bmatrix}$

Step 1: Find AB

$$AB = \begin{bmatrix} 1(0) + 2(5) & 1(-1) + 2(2) \\ 8(0) + 0(5) & 8(-1) + 0(2) \end{bmatrix} = \begin{bmatrix} 10 & 3 \\ 0 & -8 \end{bmatrix}$$

Step 2: Find $|AB|$

$$|AB| = 10(-8) - 3(0) = -80$$

Step 3: Find $(AB)^{-1}$

$$(AB)^{-1} = \frac{1}{-80} \begin{bmatrix} -8 & -3 \\ 0 & 10 \end{bmatrix} = \begin{bmatrix} \frac{1}{10} & \frac{3}{80} \\ 0 & -\frac{1}{8} \end{bmatrix}$$

Step 4: Find A^{-1}

$$|A| = 1(0) - 2(8) = -16 \quad A^{-1} = \frac{1}{-16} \begin{bmatrix} 0 & -2 \\ -8 & 1 \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{8} \\ \frac{1}{2} & -\frac{1}{16} \end{bmatrix}$$

Step 5: Find B^{-1}

$$|B| = 0(2) - (-1)(5) = 5 \quad B^{-1} = \frac{1}{5} \begin{bmatrix} 2 & 1 \\ -5 & 0 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & \frac{1}{5} \\ -1 & 0 \end{bmatrix}$$

Step 6: Find $B^{-1}A^{-1}$

$$B^{-1}A^{-1} = \begin{bmatrix} \frac{2}{5} & \frac{1}{5} \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 0 & \frac{1}{8} \\ \frac{1}{2} & -\frac{1}{16} \end{bmatrix} = \begin{bmatrix} \frac{2}{5}(0) + \frac{1}{5}\left(\frac{1}{2}\right) & \frac{2}{5}\left(\frac{1}{8}\right) + \frac{1}{5}\left(-\frac{1}{16}\right) \\ -1(0) + 0\left(\frac{1}{2}\right) & -1\left(\frac{1}{8}\right) + 0\left(-\frac{1}{16}\right) \end{bmatrix} = \begin{bmatrix} 0 + \frac{1}{10} & \frac{1}{20} - \frac{1}{80} \\ 0 & -\frac{1}{8} \end{bmatrix} = \begin{bmatrix} \frac{1}{10} & \frac{4}{80} - \frac{1}{80} \\ 0 & -\frac{1}{8} \end{bmatrix} = \begin{bmatrix} \frac{1}{10} & \frac{3}{80} \\ 0 & -\frac{1}{8} \end{bmatrix}$$

Step 7: Compare

$$(AB)^{-1} = \begin{bmatrix} \frac{1}{10} & \frac{3}{80} \\ 0 & -\frac{1}{8} \end{bmatrix} = B^{-1}A^{-1}$$

Verified

Summary Table

1(i)	40
1(ii)	59
1(iii)	6
2(A)	Non-Singular
2(B)	Singular
2(C)	Non-Singular
2(D)	Singular
3	$x = 2$
4(A)	$\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$
4(B)	$\begin{bmatrix} 7 & 2 \\ -3 & 5 \end{bmatrix}$
4(C)	$\begin{bmatrix} 2 & -5 \\ 3 & -3 \end{bmatrix}$
5(i)	$\begin{bmatrix} \frac{1}{5} & 0 \\ 0 & \frac{1}{5} \end{bmatrix}$
5(ii)	$\begin{bmatrix} -\frac{1}{32} & \frac{1}{8} \\ \frac{1}{64} & \frac{1}{16} \end{bmatrix}$
5(iii)	$\begin{bmatrix} \frac{20}{1} & -\frac{1}{5} \\ -\frac{1}{8} & 1 \end{bmatrix}$
5(iv)	$\begin{bmatrix} \frac{3}{34} & \frac{5}{34} \\ \frac{34}{1} & -\frac{34}{3} \end{bmatrix}$
5(v)	$\begin{bmatrix} \frac{1}{2} & \frac{1}{3} \\ -\frac{1}{2} & \frac{1}{3} \end{bmatrix}$
5(vi)	$\begin{bmatrix} \frac{5}{2} & \frac{3}{2} \\ -2 & -1 \end{bmatrix}$
6	Verified
7(i)	Verified
7(ii)	Verified
8(i)	Verified
8(ii)	Verified

Key Formulas

1. Determinant (2×2)

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

2. Adjoint (2×2)

$$\text{adj} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

3. Inverse (2×2)

$$A^{-1} = \frac{1}{|A|} \text{adj}(A)$$

4. Inverse of Product

$$(AB)^{-1} = B^{-1}A^{-1}$$

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