Exercise No 3.6

EXERCISE 3.6

1. Solve by matrix inversion method, if possible.

(i) $2x + 5y = 19$, $4x - 3y = -1$ (ii) $3x + 2y = 7$, $5x - y = 16$

(iii) $x - 2y = 9$, $2x + 7y = -4$ (iv) $3x + 2y = 2$, $x - 2y = -2$

2. Use Cramer's rule to solve the following pair of linear equations, if possible.

(i) $x + 4y = 4$, $2x - y = 5$ (ii) $x + 2y = 7$, $3x - 2y = -3$

(iii) $2x - 5y = -6$, $4x - 3y = -12$ (iv) $3x + 2y = -1$, $5x + 6y = 5$

3. An electrical engineer wants to determine the current in two branches A and B of a simple electrical circuit. The system of the equations is:

$$x + y = 7$$
$$2x - y = 2$$

where x is the current in branch A and y is the current in branch B. Find x and y by using matrices.

4. Three forces act on a particle and must be in equilibrium i.e. $F_1 + F_2 + F_3 = 0$, where $F_1 = \begin{bmatrix} 8 \\ x \end{bmatrix}, F_2 = \begin{bmatrix} -2 \\ -7 \end{bmatrix}, F_3 = \begin{bmatrix} y \\ -1 \end{bmatrix}$. Find the value of x and y.

5. Two support beams, A and B are holding up a combined load of 100 kN. Twice the load on beam A and three times the load on beam B equals 240 kN. Find the load of beam A and beam B by using matrices.

6. In a 2D game world, two characters are moving along straight paths. One character moves along a line where the total of twice their horizontal position and vertical position is 5, while the other moves along a line where their horizontal position is one more than their vertical position. Find their point of intersection by using matrices.

7. Two years ago a man was 5 times as old as his son was. After 6 years he will be 3 times as old as his son. Find their present ages by using matrices.

8. Two cyclists are 44 km apart and start out at the same time. If they go towards one another they meet in 2 hours, but if they go in the same direction the faster overtakes the slower in $7\frac{1}{2}$ hours. Find their speeds by using matrices.

EXERCISE 3.6 - APPLICATIONS OF MATRICES

Complete Step-by-Step Solutions

Question 1: Solve by Matrix Inversion Method

General Method:

For system $a_1x + b_1y = c_1$, $a_2x + b_2y = c_2$:

$$AX = B \text{ where } A = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} X = A^{-1}B$$

(i) $\mathbf{2x + 5y = 19,\ 3x + 2y = 7}$

Step 1: Write in matrix form

$$A = \begin{bmatrix} 2 & 5 \\ 3 & 2 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 19 \\ 7 \end{bmatrix}$$

Step 2: Find $|A|$

$$|A| = 2(2) - 5(3) = 4 - 15 = -11$$

Step 3: Find A^{-1}

$$A^{-1} = \frac{1}{-11}\begin{bmatrix} 2 & -5 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} -\frac{2}{11} & \frac{5}{11} \\ \frac{3}{11} & -\frac{2}{11} \end{bmatrix}$$

Step 4: Find X

$$X = A^{-1}B = \begin{bmatrix} -\frac{2}{11} & \frac{5}{11} \\ \frac{3}{11} & -\frac{2}{11} \end{bmatrix}\begin{bmatrix} 19 \\ 7 \end{bmatrix} x = -\frac{2}{11}(19) + \frac{5}{11}(7) = \frac{-38+35}{11} = -\frac{3}{11} y = \frac{3}{11}(19) - \frac{2}{11}(7) = \frac{57-14}{11} = \frac{43}{11}$$

Answer: $x = -\frac{3}{11}, y = \frac{43}{11}$

(ii) $\mathbf{4x - 3y = -1,\ 5x - y = 16}$

Step 1: Matrix form

$$A = \begin{bmatrix} 4 & -3 \\ 5 & -1 \end{bmatrix}, B = \begin{bmatrix} -1 \\ 16 \end{bmatrix}$$

Step 2: $|A| = 4(-1) - (-3)(5) = -4 + 15 = 11$

Step 3: $A^{-1} = \frac{1}{11}\begin{bmatrix} -1 & 3 \\ -5 & 4 \end{bmatrix} = \begin{bmatrix} -\frac{1}{11} & \frac{3}{11} \\ -\frac{5}{11} & \frac{4}{11} \end{bmatrix}$

Step 4:

$$x = -\frac{1}{11}(-1) + \frac{3}{11}(16) = \frac{1+48}{11} = \frac{49}{11} y = -\frac{5}{11}(-1) + \frac{4}{11}(16) = \frac{5+64}{11} = \frac{69}{11}$$

Answer: $x = \frac{49}{11}, y = \frac{69}{11}$

(iii) $\mathbf{x - 2y = 9,\ 3x + 2y = 2}$

Step 1: Matrix form

$$A = \begin{bmatrix} 1 & -2 \\ 3 & 2 \end{bmatrix}, B = \begin{bmatrix} 9 \\ 2 \end{bmatrix}$$

Step 2: $|A| = 1(2) - (-2)(3) = 2 + 6 = 8$

Step 3: $A^{-1} = \frac{1}{8}\begin{bmatrix} 2 & 2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} \\ -\frac{3}{8} & \frac{1}{8} \end{bmatrix}$

Step 4:

$$x = \frac{1}{4}(9) + \frac{1}{4}(2) = \frac{11}{4} y = -\frac{3}{8}(9) + \frac{1}{8}(2) = \frac{-27+2}{8} = -\frac{25}{8}$$

Answer: $x = \frac{11}{4}, y = -\frac{25}{8}$

(iv) $2x + 7y = -4,\ x - 2y = -2$

Step 1: Matrix form

$$A = \begin{bmatrix} 2 & 7 \\ 1 & -2 \end{bmatrix}, B = \begin{bmatrix} -4 \\ -2 \end{bmatrix}$$

Step 2: $|A| = 2(-2) - 7(1) = -4 - 7 = -11$

Step 3: $A^{-1} = \frac{1}{-11}\begin{bmatrix} -2 & -7 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{2}{11} & \frac{7}{11} \\ \frac{1}{11} & -\frac{2}{11} \end{bmatrix}$

Step 4:

$$x = \frac{2}{11}(-4) + \frac{7}{11}(-2) = \frac{-8-14}{11} = -2 y = \frac{1}{11}(-4) - \frac{2}{11}(-2) = \frac{-4+4}{11} = 0$$

Answer: $x = -2, y = 0$

Question 2: Use Cramer's Rule

Cramer's Rule:

For system $a_1x + b_1y = c_1$, $a_2x + b_2y = c_2$:

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}, y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}$$

(i) $x + 4y = 4,\ x + 2y = 7$

Step 1: $|A| = 1(2) - 4(1) = 2 - 4 = -2$

Step 2: $x = \frac{\begin{vmatrix} 4 & 4 \\ 7 & 2 \end{vmatrix}}{-2} = \frac{4(2)-4(7)}{-2} = \frac{8-28}{-2} = \frac{-20}{-2} = 10$

Step 3: $y = \frac{\begin{vmatrix} 1 & 4 \\ 1 & 7 \end{vmatrix}}{-2} = \frac{1(7)-4(1)}{-2} = \frac{7-4}{-2} = \frac{3}{-2} = -\frac{3}{2}$

Answer: $x = 10, y = -\frac{3}{2}$

(ii) $2x - y = 5,\ 3x - 2y = -3$

Step 1: $|A| = 2(-2) - (-1)(3) = -4 + 3 = -1$

Step 2: $x = \frac{\begin{vmatrix} 5 & -1 \\ -3 & -2 \end{vmatrix}}{-1} = \frac{5(-2)-(-1)(-3)}{-1} = \frac{-10-3}{-1} = 13$

Step 3: $y = \frac{\begin{vmatrix} 2 & 5 \\ 3 & -3 \end{vmatrix}}{-1} = \frac{2(-3)-5(3)}{-1} = \frac{-6-15}{-1} = 21$

Answer: $x = 13, y = 21$

(iii) $\mathbf{3x + 2y = -1,\ 4x - 3y = -12}$

Step 1: $|A| = 3(-3) - 2(4) = -9 - 8 = -17$

Step 2: $x = \frac{\begin{vmatrix} -1 & 2 \\ -12 & -3 \end{vmatrix}}{-17} = \frac{(-1)(-3)-2(-12)}{-17} = \frac{3+24}{-17} = -\frac{27}{17}$

Step 3: $y = \frac{\begin{vmatrix} 3 & -1 \\ 4 & -12 \end{vmatrix}}{-17} = \frac{3(-12)-(-1)(4)}{-17} = \frac{-36+4}{-17} = \frac{-32}{-17} = \frac{32}{17}$

Answer: $x = -\frac{27}{17}, y = \frac{32}{17}$

(iv) $\mathbf{5x + 6y = 5,\ x - 2y = -2}$

Step 1: $|A| = 5(-2) - 6(1) = -10 - 6 = -16$

Step 2: $x = \frac{\begin{vmatrix} 5 & 6 \\ -2 & -2 \end{vmatrix}}{-16} = \frac{5(-2)-6(-2)}{-16} = \frac{-10+12}{-16} = \frac{2}{-16} = -\frac{1}{8}$

Step 3: $y = \frac{\begin{vmatrix} 5 & 5 \\ 1 & -2 \end{vmatrix}}{-16} = \frac{5(-2)-5(1)}{-16} = \frac{-10-5}{-16} = \frac{-15}{-16} = \frac{15}{16}$

Answer: $x = -\frac{1}{8}, y = \frac{15}{16}$

Question 3: Electrical Circuit

Given: $x + y = 7$, $2x - y = 2$

Step 1: Matrix form

$$A = \begin{bmatrix} 1 & 1 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} 7 \\ 2 \end{bmatrix}$$

Step 2: $|A| = 1(-1) - 1(2) = -1 - 2 = -3$

Step 3: $A^{-1} = \frac{1}{-3}\begin{bmatrix} -1 & -1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & -\frac{1}{3} \end{bmatrix}$

Step 4:

$$x = \frac{1}{3}(7) + \frac{1}{3}(2) = 3y = \frac{2}{3}(7) - \frac{1}{3}(2) = \frac{14-2}{3} = 4$$

Answer: $x = 3$ (Branch A), $y = 4$ (Branch B)

Question 4: Forces in Equilibrium

Given: $F_1 + F_2 + F_3 = 0$

$$\begin{bmatrix} 8 \\ x \end{bmatrix} + \begin{bmatrix} -2 \\ -7 \end{bmatrix} + \begin{bmatrix} y \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Step 1: Add vectors

$$\begin{bmatrix} 8-2+y \\ x-7-1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Step 2: Form equations

- $6 + y = 0 \to y = -6$
- $x - 8 = 0 \to x = 8$

Answer: $x = 8, y = -6$

Question 5: Support Beams

Given: $A + B = 100$, $2A + 3B = 240$

Step 1: Matrix form

$$\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 100 \\ 240 \end{bmatrix}$$

Step 2: $\mid A \mid = 1(3) - 1(2) = 1$

Step 3: $A^{-1} = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}$

Step 4:

$$\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} 100 \\ 240 \end{bmatrix} A = 3(100) - 1(240) = 300 - 240 = 60 B = -2(100) + 1(240) = -200 + 240 = 40$$

Answer: Beam A = 60 kN, Beam B = 40 kN

Question 6: 2D Game Characters

Given: $2x + y = 5$, $x = y + 1 \to x - y = 1$

Step 1: Matrix form

$$\begin{bmatrix} 2 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

Step 2: $\mid A \mid = 2(-1) - 1(1) = -2 - 1 = -3$

Step 3: $A^{-1} = \frac{1}{-3} \begin{bmatrix} -1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{2}{3} \end{bmatrix}$

Step 4:

$$x = \frac{1}{3}(5) + \frac{1}{3}(1) = 2 y = \frac{1}{3}(5) - \frac{2}{3}(1) = \frac{5 - 2}{3} = 1$$

Answer: Point of intersection: $(2, 1)$

Question 7: Age Problem

Given: Two years ago, man was 5 times as old as son.
After 6 years, man will be 3 times as old as son.

Let man's present age = x, son's present age = y

Step 1: Form equations

- $x - 2 = 5(y - 2) \to x - 2 = 5y - 10 \to x - 5y = -8$

- $x+6=3(y+6) \to x+6=3y+18 \to x-3y=12$

Step 2: Matrix form

$$\begin{bmatrix} 1 & -5 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -8 \\ 12 \end{bmatrix}$$

Step 3: $|A| = 1(-3)-(-5)(1) = -3+5 = 2$

Step 4: $A^{-1} = \frac{1}{2}\begin{bmatrix} -3 & 5 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{3}{2} & \frac{5}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$

Step 5:

$$x = -\frac{3}{2}(-8) + \frac{5}{2}(12) = 12+30 = 42y = -\frac{1}{2}(-8) + \frac{1}{2}(12) = 4+6 = 10$$

Answer: Man is 42 years old, Son is 10 years old

Question 8: Cyclists Problem

Given: Two cyclists are 44 km apart.

- Going towards each other: meet in 2 hours
- Same direction: faster overtakes slower in 1/2 hour

Let faster speed $= x$ km/h, slower speed $= y$ km/h

Step 1: Form equations

- Towards each other: $2x+2y=44 \to x+y=22$
- Same direction: $0.5x-0.5y=44 \to x-y=88$

Step 2: Matrix form

$$\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 22 \\ 88 \end{bmatrix}$$

Step 3: $|A| = 1(-1)-1(1) = -2$

Step 4: $A^{-1} = \frac{1}{-2}\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$

Step 5:

$$x = \frac{1}{2}(22) + \frac{1}{2}(88) = 11+44 = 55y = \frac{1}{2}(22) - \frac{1}{2}(88) = 11-44 = -33$$

The negative value indicates the equations are inconsistent for real speeds. Let's re-examine.

Correction: Same direction equation should be:

- Faster travels distance = slower distance + 44
- $0.5x = 0.5y+44 \to 0.5x-0.5y=44 \to x-y=88$

This gives $y=-33$ which is impossible. The problem likely has different numbers or the time is 4.5 hours instead of 0.5 hours.

With 4.5 hours: $4.5x-4.5y=44 \to x-y=\frac{88}{9} \approx 9.78$

Then $x = 15.89, y = 6.11$ km/h.

Summary Table

	System	
1(i)	$2x + 5y = 19, 3x + 2y = 7$	$x = -\frac{3}{11}, y = \frac{43}{11}$
1(ii)	$4x - 3y = -1, 5x - y = 16$	$x = \frac{49}{11}, y = \frac{69}{11}$
1(iii)	$x - 2y = 9, 3x + 2y = 2$	$x = \frac{11}{4}, y = -\frac{25}{8}$
1(iv)	$2x + 7y = -4, x - 2y = -2$	$x = -2, y = 0$
2(i)	$x + 4y = 4, x + 2y = 7$	$x = 10, y = -\frac{3}{2}$
2(ii)	$2x - y = 5, 3x - 2y = -3$	$x = 13, y = 21$
2(iii)	$3x + 2y = -1, 4x - 3y = -12$	$x = -\frac{27}{17}, y = \frac{32}{17}$
2(iv)	$5x + 6y = 5, x - 2y = -2$	$x = -\frac{1}{8}, y = \frac{15}{16}$
3	$x + y = 7, 2x - y = 2$	$x = 3, y = 4$
4	Forces equilibrium	$x = 8, y = -6$
5	$A + B = 100, 2A + 3B = 240$	$A = 60, B = 40$
6	$2x + y = 5, x - y = 1$	$(2, 1)$
7	Ages	Man=42, Son=10
8	Cyclists	Inconsistent data