EXERCISE 5.1

1. Reduce the following rational expressions to lowest forms:

(i) $\frac{3a^2-6ab}{2a^2b-4ab^2}$ (ii) $\frac{abx+bx^2}{acx+cx^2}$ (iii) $\frac{ac}{a^2x^2-ax}$

(iv) $\frac{15a^2b^2c}{100(a^2-a^2b)}$ (v) $\frac{4x^2-9y^2}{4x^2+6xy}$ (vi) $\frac{20(x^3-y^3)}{5x^2+5xy+5y^2}$

(vii) $\frac{x(2a^2-3ax)}{a(4a^2x-9x^3)}$ (viii) $\frac{x^2-5x}{x^2-4x-5}$ (ix) $\frac{3x^2+6x}{x^2+4x+4}$

(x) $\frac{x^2+xy-2y^2}{x^3-y^3}$ (xi) $\frac{2x^2+17x+21}{3x^2+26x+35}$

EXERCISE 5.1 - Reducing Rational Expressions to Lowest Forms

Question 1(i)

$$\frac{3a^2-6ab}{2a^2b-4ab^2}$$

Solution:

Step 1: Factor numerator:

$$3a^2-6ab=3a(a-2b)$$

Step 2: Factor denominator:

$$2a^2b-4ab^2=2ab(a-2b)$$

Step 3: Cancel common factor $(a-2b)$:

$$\frac{3a(a-2b)}{2ab(a-2b)}=\frac{3a}{2ab}$$

Step 4: Cancel common factor a:

$$\frac{3a}{2ab}=\frac{3}{2b}$$

Answer: $\frac{3}{2b}$, where $a\neq 0, b\neq 0, a\neq 2b$

Question 1(ii)

$$\frac{abx+bx^2}{acx+cx^2}$$

Solution:

Step 1: Factor numerator:

$$abx + bx^2 = bx(a + x)$$

Step 2: Factor denominator:

$$acx + cx^2 = cx(a + x)$$

Step 3: Cancel common factor $(a + x)$:

$$\frac{bx(a + x)}{cx(a + x)} = \frac{bx}{cx}$$

Step 4: Cancel common factor x:

$$\frac{bx}{cx} = \frac{b}{c}$$

Answer: $\frac{b}{c}$, where $c \neq 0, x \neq 0, a \neq -x$

Question 1(iii)

$$\frac{ac}{a^2x^2 - ax}$$

Solution:

Step 1: Factor denominator:

$$a^2x^2 - ax = ax(ax - 1)$$

Step 2: Cancel common factor a:

$$\frac{ac}{ax(ax - 1)} = \frac{c}{x(ax - 1)}$$

Answer: $\frac{c}{x(ax-1)}$, where $a \neq 0, x \neq 0, ax \neq 1$

Question 1(iv)

$$\frac{15a^2b^2c}{100(a^2 - a^2b)}$$

Solution:

Step 1: Factor numerator:

$$15a^2b^2c = 15a^2b^2c$$

Step 2: Factor denominator:

$$100(a^2 - a^2b) = 100a^2(1 - b)$$

Step 3: Cancel common factor a^2:

$$\frac{15a^2b^2c}{100a^2(1-b)} = \frac{15b^2c}{100(1-b)}$$

Step 4: Simplify coefficients:

$$\frac{15b^2c}{100(1-b)} = \frac{3b^2c}{20(1-b)}$$

Answer: $\frac{3b^2c}{20(1-b)}$, where $a \neq 0, b \neq 1$

Question 1(v)

$$\frac{4x^2-9y^2}{4x^2+6xy}$$

Solution:

Step 1: Factor numerator using difference of squares:

$$4x^2-9y^2 = (2x)^2-(3y)^2 = (2x-3y)(2x+3y)$$

Step 2: Factor denominator:

$$4x^2+6xy = 2x(2x+3y)$$

Step 3: Cancel common factor $(2x+3y)$:

$$\frac{(2x-3y)(2x+3y)}{2x(2x+3y)} = \frac{2x-3y}{2x}$$

Answer: $\frac{2x-3y}{2x}$, where $x \neq 0, 2x \neq -3y$

Question 1(vi)

$$\frac{20(x^3-y^3)}{5x^2+5xy+5y^2}$$

Solution:

Step 1: Factor numerator using difference of cubes:

$$x^3-y^3 = (x-y)(x^2+xy+y^2)20(x^3-y^3) = 20(x-y)(x^2+xy+y^2)$$

Step 2: Factor denominator:

$$5x^2+5xy+5y^2 = 5(x^2+xy+y^2)$$

Step 3: Cancel common factor (x^2+xy+y^2):

$$\frac{20(x-y)(x^2+xy+y^2)}{5(x^2+xy+y^2)} = \frac{20(x-y)}{5}$$

Step 4: Simplify:

$$\frac{20(x-y)}{5} = 4(x-y)$$

Answer: $4(x-y)$, where $x^2+xy+y^2 \neq 0$

Question 1(vii)

$$\frac{x(2a^2-3ax)}{a(4a^2x-9x^3)}$$

Solution:

Step 1: Factor numerator:

$$x(2a^2-3ax) = x \cdot a(2a-3x) = ax(2a-3x)$$

Step 2: Factor denominator:

$$a(4a^2x-9x^3) = a \cdot x(4a^2-9x^2) = ax(4a^2-9x^2)$$

Step 3: Factor $4a^2-9x^2$ using difference of squares:

$$4a^2-9x^2 = (2a)^2-(3x)^2 = (2a-3x)(2a+3x)$$

Step 4: Cancel common factor $ax(2a-3x)$:

$$\frac{ax(2a-3x)}{ax(2a-3x)(2a+3x)} = \frac{1}{2a+3x}$$

Answer: $\frac{1}{2a+3x}$, where $a \neq 0, x \neq 0, 2a \neq 3x, 2a \neq -3x$

Question 1(viii)

$$\frac{x^2-5x}{x^2-4x-5}$$

Solution:

Step 1: Factor numerator:

$$x^2-5x = x(x-5)$$

Step 2: Factor denominator:

$$x^2-4x-5 = (x-5)(x+1)$$

Step 3: Cancel common factor $(x-5)$:

$$\frac{x(x-5)}{(x-5)(x+1)} = \frac{x}{x+1}$$

Answer: $\frac{x}{x+1}$, where $x \neq 5, x \neq -1$

Question 1(ix)

$$\frac{3x^2 + 6x}{x^2 + 4x + 4}$$

Solution:

Step 1: Factor numerator:

$$3x^2 + 6x = 3x(x + 2)$$

Step 2: Factor denominator:

$$x^2 + 4x + 4 = (x + 2)^2 = (x + 2)(x + 2)$$

Step 3: Cancel common factor $(x + 2)$:

$$\frac{3x(x + 2)}{(x + 2)(x + 2)} = \frac{3x}{x + 2}$$

Answer: $\frac{3x}{x+2}$, where $x \neq -2$

Summary Table

	Original Expression	Simplified Form	Restrictions
(i)	$\frac{3a^2-6ab}{2a^2b-4ab^2}$	$\frac{3}{2b}$	$a \neq 0, b \neq 0, a \neq 2b$
(ii)	$\frac{abx+bx^2}{acx+cx^2}$	$\frac{b}{c}$	$c \neq 0, x \neq 0, a \neq -x$
(iii)	$\frac{ac}{a^2x^2-ax}$	$\frac{c}{x(ax-1)}$	$a \neq 0, x \neq 0, ax \neq 1$
(iv)	$\frac{15a^2b^2c}{100(a^2-a^2b)}$	$\frac{3b^2c}{20(1-b)}$	$a \neq 0, b \neq 1$
(v)	$\frac{4x^2-9y^2}{4x^2+6xy}$	$\frac{2x-3y}{2x}$	$x \neq 0, 2x \neq -3y$
(vi)	$\frac{20(x^3-y^3)}{5x^2+5xy+5y^2}$	$4(x - y)$	$x^2 + xy + y^2 \neq 0$
(vii)	$\frac{x(2a^2-3ax)}{a(4a^2x-9x^3)}$	$\frac{1}{2a+3x}$	$a \neq 0, x \neq 0, 2a \neq \pm 3x$
(viii)	$\frac{x^2-5x}{x^2-4x-5}$	$\frac{x}{x+1}$	$x \neq 5, x \neq -1$
(ix)	$\frac{3x^2+6x}{x^2+4x+4}$	$\frac{3x}{x+2}$	$x \neq -2$

Key Factorization Formulas Used:

1. **Difference of Squares:** $a^2 - b^2 = (a - b)(a + b)$
2. **Difference of Cubes:** $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
3. **Quadratic Factoring:** $x^2 + (a + b)x + ab = (x + a)(x + b)$
4. **Perfect Square:** $x^2 + 2ax + a^2 = (x + a)^2$

EXERCISE 5.1 (Continued) - Reducing Rational Expressions to Lowest Forms

Question 1(x)

$$\frac{x^2 + xy - 2y^2}{x^3 - y^3}$$

Solution:

Step 1: Factor numerator:

$$x^2 + xy - 2y^2$$

Find two numbers that multiply to $-2y^2$ and add to y:

$$x^2 + xy - 2y^2 = (x + 2y)(x - y)$$

Check: $(x + 2y)(x - y) = x^2 - xy + 2xy - 2y^2 = x^2 + xy - 2y^2$

Step 2: Factor denominator using difference of cubes:

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

Step 3: Cancel common factor $(x - y)$:

$$\frac{(x + 2y)(x - y)}{(x - y)(x^2 + xy + y^2)} = \frac{x + 2y}{x^2 + xy + y^2}$$

Answer: $\frac{x+2y}{x^2+xy+y^2}$, where $x \neq y$

Question 1(xi)

$$\frac{2x^2 + 17x + 21}{3x^2 + 26x + 35}$$

Solution:

Step 1: Factor numerator:

$$2x^2 + 17x + 21$$

Find two numbers whose product is $2 \times 21 = 42$ and sum is 17:

- Numbers: 3 and 14
- $3 \times 14 = 42$, $3 + 14 = 17$

Rewrite:

$$2x^2 + 17x + 21 = 2x^2 + 3x + 14x + 21$$

Group and factor:

$$= x(2x + 3) + 7(2x + 3) = (2x + 3)(x + 7)$$

Check: $(2x + 3)(x + 7) = 2x^2 + 14x + 3x + 21 = 2x^2 + 17x + 21$

Step 2: Factor denominator:

$$3x^2 + 26x + 35$$

Find two numbers whose product is $3 \times 35 = 105$ and sum is 26:

- Numbers: 5 and 21
- $5 \times 21 = 105$, $5 + 21 = 26$

Rewrite:

$$3x^2 + 26x + 35 = 3x^2 + 5x + 21x + 35$$

Group and factor:

$$= x(3x + 5) + 7(3x + 5) = (3x + 5)(x + 7)$$

Check: $(3x + 5)(x + 7) = 3x^2 + 21x + 5x + 35 = 3x^2 + 26x + 35$

Step 3: Cancel common factor $(x + 7)$:

$$\frac{(2x + 3)(x + 7)}{(3x + 5)(x + 7)} = \frac{2x + 3}{3x + 5}$$

Answer: $\frac{2x+3}{3x+5}$, where $x \neq -7, x \neq -\frac{5}{3}$

Updated Summary Table

	Original Expression	Simplified Form	Restrictions
(i)	$\frac{3a^2-6ab}{2a^2b-4ab^2}$	$\frac{3}{2b}$	$a \neq 0, b \neq 0, a \neq 2b$
(ii)	$\frac{abx+bx^2}{acx+cx^2}$	$\frac{b}{c}$	$c \neq 0, x \neq 0, a \neq -x$
(iii)	$\frac{ac}{a^2x^2-ax}$	$\frac{c}{x(ax-1)}$	$a \neq 0, x \neq 0, ax \neq 1$
(iv)	$\frac{15a^2b^2c}{100(a^2-a^2b)}$	$\frac{3b^2c}{20(1-b)}$	$a \neq 0, b \neq 1$
(v)	$\frac{4x^2-9y^2}{4x^2+6xy}$	$\frac{2x-3y}{2x}$	$x \neq 0, 2x \neq -3y$
(vi)	$\frac{20(x^3-y^3)}{5x^2+5xy+5y^2}$	$4(x - y)$	$x^2 + xy + y^2 \neq 0$
(vii)	$\frac{x(2a^2-3ax)}{a(4a^2x-9x^3)}$	$\frac{1}{2a+3x}$	$a \neq 0, x \neq 0, 2a \neq \pm 3x$
(viii)	$\frac{x^2-5x}{x^2-4x-5}$	$\frac{x}{x+1}$	$x \neq 5, x \neq -1$
(ix)	$\frac{3x^2+6x}{x^2+4x+4}$	$\frac{3x}{x+2}$	$x \neq -2$
(x)	$\frac{x^2+xy-2y^2}{x^3-y^3}$	$\frac{x+2y}{x^2+xy+y^2}$	$x \neq y$
(xi)	$\frac{2x^2+17x+21}{3x^2+26x+35}$	$\frac{2x+3}{3x+5}$	$x \neq -7, x \neq -\frac{5}{3}$

Key Factorization Formulas Used:

1. **Difference of Squares:** $a^2 - b^2 = (a - b)(a + b)$
2. **Difference of Cubes:** $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
3. **Quadratic Factoring (AC Method):** $ax^2 + bx + c$ where product $= ac$, sum $= b$
4. **Perfect Square:** $x^2 + 2ax + a^2 = (x + a)^2$
5. **Factoring by Grouping:** Used for quadratics with leading coefficient $\neq 1$