

EXERCISE 5.2

1. Simplify:

(i) $\frac{3}{x-y} + \frac{1}{y-x}$

(ii) $\frac{4x}{x^2-3x+2} - \frac{4}{1-x} - \frac{5}{x-2}$

(iii) $\frac{x+y}{12x-6y} + \frac{x-y}{18x-9y}$

(iv) $\frac{2}{x+2y} - \frac{x-6y}{x^2-4y^2}$

(v) $\frac{1}{x+1} - \frac{2}{x+2} - \frac{2x+3}{x^2+3x+2}$

(vi) $\frac{5}{x^2+x-6} - \frac{1}{2x^2-7x+6}$

(vii) $\frac{2x}{x+y} - \frac{y}{x-y} - \frac{2y^2}{x^2-y^2}$

(viii) $\frac{7}{2x^2-x-6} - \frac{8}{3x^2-4x-4}$

(ix) $\frac{x+2}{x^2-x-12} - \frac{x}{x^2+6x+9}$

(x) $\frac{1}{x^2-4x+3} + \frac{1}{x^2-3x+2} - \frac{1}{x^2-5x+6}$

(xi) $\frac{x^3}{x-y} + \frac{y^3}{y-x}$

2. Subtract $\frac{1}{x^2+2}$ from $\frac{2x^3+x^2+3}{(x^2+2)^2}$

EXERCISE 5.2 - Simplifying Rational Expressions

Question 1(i)

$$\frac{3}{x-y} + \frac{1}{y-x}$$

Solution:

Step 1: Note that $y-x = -(x-y)$:

$$\frac{1}{y-x} = \frac{1}{-(x-y)} = -\frac{1}{x-y}$$

Step 2: Combine:

$$\frac{3}{x-y} - \frac{1}{x-y} = \frac{3-1}{x-y}$$

Answer: $\frac{2}{x-y}$, where $x \neq y$

Question 1(ii)

$$\frac{x+y}{12x-6y} + \frac{x-y}{18x-9y}$$

Solution:

Step 1: Factor denominators:

$$12x - 6y = 6(2x - y) \quad 18x - 9y = 9(2x - y)$$

Step 2: Rewrite:

$$\frac{x+y}{6(2x-y)} + \frac{x-y}{9(2x-y)}$$

Step 3: Find LCD = $18(2x - y)$:

$$\frac{3(x+y)}{18(2x-y)} + \frac{2(x-y)}{18(2x-y)}$$

Step 4: Combine numerators:

$$\frac{3x+3y+2x-2y}{18(2x-y)} = \frac{5x+y}{18(2x-y)}$$

Answer: $\frac{5x+y}{18(2x-y)}$, where $x \neq \frac{y}{2}$

Question 1(iii) and (v) [Same expression]

$$\frac{1}{x+1} - \frac{2}{x+2} - \frac{2x+3}{x^2+3x+2}$$

Solution:

Step 1: Factor denominator of third term:

$$x^2 + 3x + 2 = (x+1)(x+2)$$

Step 2: Rewrite:

$$\frac{1}{x+1} - \frac{2}{x+2} - \frac{2x+3}{(x+1)(x+2)}$$

Step 3: LCD = $(x+1)(x+2)$:

$$\frac{(x+2)}{(x+1)(x+2)} - \frac{2(x+1)}{(x+1)(x+2)} - \frac{2x+3}{(x+1)(x+2)}$$

Step 4: Combine numerators:

$$\frac{(x+2) - 2(x+1) - (2x+3)}{(x+1)(x+2)}$$

Step 5: Simplify numerator:

$$x+2-2x-2-2x-3 = x-2x-2x+2-2-3 = -3x-3 = -3(x+1)$$

Step 6: Cancel common factor $(x+1)$:

$$\frac{-3(x+1)}{(x+1)(x+2)} = \frac{-3}{x+2}$$

Answer: $-\frac{3}{x+2}$, where $x \neq -1, x \neq -2$

Question 1(iv) and (vi) [Same expression]

$$\frac{2x}{x+y} - \frac{y}{x-y} - \frac{2y^2}{x^2-y^2}$$

Solution:

Step 1: Factor denominator of third term:

$$x^2 - y^2 = (x-y)(x+y)$$

Step 2: Rewrite:

$$\frac{2x}{x+y} - \frac{y}{x-y} - \frac{2y^2}{(x-y)(x+y)}$$

Step 3: LCD = $(x-y)(x+y)$:

$$\frac{2x(x-y)}{(x-y)(x+y)} - \frac{y(x+y)}{(x-y)(x+y)} - \frac{2y^2}{(x-y)(x+y)}$$

Step 4: Combine numerators:

$$\frac{2x(x-y) - y(x+y) - 2y^2}{(x-y)(x+y)}$$

Step 5: Expand numerator:

$$2x^2 - 2xy - xy - y^2 - 2y^2 = 2x^2 - 3xy - 3y^2$$

Answer: $\frac{2x^2-3xy-3y^2}{(x-y)(x+y)}$, where $x \neq \pm y$

Question 1(vii)

$$\frac{4x}{x^2-3x+2} - \frac{4}{1-x} - \frac{5}{x-2}$$

Solution:

Step 1: Factor denominators:

$$x^2 - 3x + 2 = (x-1)(x-2) \quad 1-x = -(x-1)$$

Step 2: Rewrite with LCD = $(x-1)(x-2)$:

$$\frac{4x}{(x-1)(x-2)} - \frac{4}{-(x-1)} - \frac{5}{x-2}$$

Note: $-\frac{4}{1-x} = \frac{4}{x-1}$ since $-(1-x) = x-1$, so $-\frac{4}{1-x} = \frac{4}{x-1}$

$$\frac{4x}{(x-1)(x-2)} + \frac{4}{x-1} - \frac{5}{x-2}$$

Step 3: LCD = $(x-1)(x-2)$:

$$\frac{4x}{(x-1)(x-2)} + \frac{4(x-2)}{(x-1)(x-2)} - \frac{5(x-1)}{(x-1)(x-2)}$$

Step 4: Combine numerators:

$$\frac{4x + 4(x-2) - 5(x-1)}{(x-1)(x-2)}$$

Step 5: Simplify numerator:

$$4x + 4x - 8 - 5x + 5 = 3x - 3 = 3(x-1)$$

Step 6: Cancel common factor $(x-1)$:

$$\frac{3(x-1)}{(x-1)(x-2)} = \frac{3}{x-2}$$

Answer: $\frac{3}{x-2}$, where $x \neq 1, x \neq 2$

Question 1(viii) [First one]

$$\frac{2}{x+2y} - \frac{x-6y}{x^2-4y^2}$$

Solution:

Step 1: Factor denominator of second term:

$$x^2 - 4y^2 = (x-2y)(x+2y)$$

Step 2: Rewrite:

$$\frac{2}{x+2y} - \frac{x-6y}{(x-2y)(x+2y)}$$

Step 3: LCD = $(x-2y)(x+2y)$:

$$\frac{2(x-2y)}{(x-2y)(x+2y)} - \frac{x-6y}{(x-2y)(x+2y)}$$

Step 4: Combine numerators:

$$\frac{2(x-2y) - (x-6y)}{(x-2y)(x+2y)}$$

Step 5: Simplify numerator:

$$2x - 4y - x + 6y = x + 2y$$

Step 6: Cancel common factor $(x + 2y)$:

$$\frac{x + 2y}{(x - 2y)(x + 2y)} = \frac{1}{x - 2y}$$

Answer: $\frac{1}{x-2y}$, where $x \neq \pm 2y$

Question 1(viii) [Second one - labeled as (ix)]

$$\frac{5}{x^2 + x - 6} - \frac{1}{2x^2 - 7x + 6}$$

Solution:

Step 1: Factor denominators:

$$x^2 + x - 6 = (x + 3)(x - 2) \quad 2x^2 - 7x + 6 = (2x - 3)(x - 2)$$

Step 2: Rewrite:

$$\frac{5}{(x + 3)(x - 2)} - \frac{1}{(2x - 3)(x - 2)}$$

Step 3: LCD = $(x - 2)(x + 3)(2x - 3)$:

$$\frac{5(2x - 3)}{(x - 2)(x + 3)(2x - 3)} - \frac{1(x + 3)}{(x - 2)(x + 3)(2x - 3)}$$

Step 4: Combine numerators:

$$\frac{5(2x - 3) - (x + 3)}{(x - 2)(x + 3)(2x - 3)}$$

Step 5: Simplify numerator:

$$10x - 15 - x - 3 = 9x - 18 = 9(x - 2)$$

Step 6: Cancel common factor $(x - 2)$:

$$\frac{9(x - 2)}{(x - 2)(x + 3)(2x - 3)} = \frac{9}{(x + 3)(2x - 3)}$$

Answer: $\frac{9}{(x+3)(2x-3)}$, where $x \neq -3, x \neq 2, x \neq \frac{3}{2}$

Question 1(viii) [Third one - labeled as (x)]

$$\frac{7}{2x^2 - x - 6} - \frac{8}{3x^2 - 4x - 4}$$

Solution:

Step 1: Factor denominators:

$$2x^2 - x - 6 = (2x + 3)(x - 2) \quad 3x^2 - 4x - 4 = (3x + 2)(x - 2)$$

Step 2: Rewrite:

$$\frac{7}{(2x + 3)(x - 2)} - \frac{8}{(3x + 2)(x - 2)}$$

Step 3: LCD = $(x - 2)(2x + 3)(3x + 2)$:

$$\frac{7(3x + 2)}{(x - 2)(2x + 3)(3x + 2)} - \frac{8(2x + 3)}{(x - 2)(2x + 3)(3x + 2)}$$

Step 4: Combine numerators:

$$\frac{7(3x + 2) - 8(2x + 3)}{(x - 2)(2x + 3)(3x + 2)}$$

Step 5: Simplify numerator:

$$21x + 14 - 16x - 24 = 5x - 10 = 5(x - 2)$$

Step 6: Cancel common factor $(x - 2)$:

$$\frac{5(x - 2)}{(x - 2)(2x + 3)(3x + 2)} = \frac{5}{(2x + 3)(3x + 2)}$$

Answer: $\frac{5}{(2x+3)(3x+2)}$, where $x \neq 2, x \neq -\frac{3}{2}, x \neq -\frac{2}{3}$

Summary Table

	Original Expression	Simplified Form	Restrictions
(i)	$\frac{3}{x-y} + \frac{1}{y-x}$	$\frac{2}{x-y}$	$x \neq y$
(ii)	$\frac{x+y}{12x-6y} + \frac{x-y}{18x-9y}$	$\frac{5x+y}{18(2x-y)}$	$x \neq \frac{y}{2}$
(iii)/(v)	$\frac{1}{x+1} - \frac{2}{x+2} - \frac{2x+3}{x^2+3x+2}$	$-\frac{3}{x+2}$	$x \neq -1, -2$
(iv)/(vi)	$\frac{2x}{x+y} - \frac{y}{x-y} - \frac{2y^2}{x^2-y^2}$	$\frac{2x^2-3xy-3y^2}{(x-y)(x+y)}$	$x \neq \pm y$
(vii)	$\frac{4x}{x^2-3x+2} - \frac{4}{1-x} - \frac{5}{x-2}$	$\frac{3}{x-2}$	$x \neq 1, 2$
(viii)	$\frac{2}{x+2y} - \frac{x-6y}{x^2-4y^2}$	$\frac{1}{x-2y}$	$x \neq \pm 2y$
(ix)	$\frac{5}{x^2+x-6} - \frac{1}{2x^2-7x+6}$	$\frac{9}{(x+3)(2x-3)}$	$x \neq -3, 2, \frac{3}{2}$
(x)	$\frac{7}{2x^2-x-6} - \frac{8}{3x^2-4x-4}$	$\frac{5}{(2x+3)(3x+2)}$	$x \neq 2, -\frac{3}{2}, -\frac{2}{3}$

Key Techniques Used:

1. **Finding Common Denominators:** Always factor denominators first
2. **LCD Method:** Use LCM of all denominators
3. **Combining Fractions:** Add/subtract numerators after converting to LCD
4. **Simplifying:** Combine like terms and cancel common factors

5. **Domain Restrictions:** Exclude values that make any denominator zero

EXERCISE 5.2 (Continued) - Simplifying Rational Expressions

Question 1(ix)

$$\frac{x+2}{x^2-x-12} - \frac{x}{x^2+6x+9}$$

Solution:

Step 1: Factor denominators:

$$x^2 - x - 12 = (x - 4)(x + 3) \quad x^2 + 6x + 9 = (x + 3)^2$$

Step 2: Rewrite:

$$\frac{x+2}{(x-4)(x+3)} - \frac{x}{(x+3)^2}$$

Step 3: LCD = $()^2$:

$$\frac{(x+2)(x+3)}{()^2} - \frac{x(x-4)}{()^2}$$

Step 4: Combine numerators:

$$\frac{(x+2)(x+3) - x(x-4)}{()^2}$$

Step 5: Expand numerator:

$$(x^2 + 5x + 6) - (x^2 - 4x) = x^2 + 5x + 6 - x^2 + 4x = 9x + 6 = 3(3x + 2)$$

Answer: $\frac{9x+6}{()^2} = \frac{3(3x+2)}{()^2}$, where $x \neq 4, x \neq -3$

Question 1(x)

$$\frac{1}{x^2-4x+3} + \frac{1}{x^2-3x+2} - \frac{1}{x^2-5x+6}$$

Solution:

Step 1: Factor denominators:

$$x^2 - 4x + 3 = (x - 1)(x - 3) \quad x^2 - 3x + 2 = (x - 1)(x - 2) \quad x^2 - 5x + 6 = (x - 2)(x - 3)$$

Step 2: Rewrite:

$$\frac{1}{(x-1)(x-3)} + \frac{1}{(x-1)(x-2)} - \frac{1}{(x-2)(x-3)}$$

Step 3: LCD = $(x - 1)(x - 2)(x - 3)$:

$$\frac{(x-2)}{(x-1)(x-2)(x-3)} + \frac{(x-3)}{(x-1)(x-2)(x-3)} - \frac{(x-1)}{(x-1)(x-2)(x-3)}$$

Step 4: Combine numerators:

$$\frac{(x-2) + (x-3) - (x-1)}{(x-1)(x-2)(x-3)}$$

Step 5: Simplify numerator:

$$x-2+x-3-x+1=x-4$$

Answer: $\frac{x-4}{(x-1)(x-2)(x-3)}$, where $x \neq 1, 2, 3$

Question 1(xi)

$$\frac{x^3}{x-y} + \frac{y^3}{y-x}$$

Solution:

Step 1: Note that $y-x=-(x-y)$:

$$\frac{y^3}{y-x} = \frac{y^3}{-(x-y)} = -\frac{y^3}{x-y}$$

Step 2: Rewrite:

$$\frac{x^3}{x-y} - \frac{y^3}{x-y}$$

Step 3: Combine numerators:

$$\frac{x^3 - y^3}{x-y}$$

Step 4: Factor numerator using difference of cubes:

$$x^3 - y^3 = (x-y)(x^2 + xy + y^2)$$

Step 5: Cancel common factor $(x-y)$:

$$\frac{(x-y)(x^2 + xy + y^2)}{x-y} = x^2 + xy + y^2$$

Answer: $x^2 + xy + y^2$, where $x \neq y$

Question 2

Subtract $\frac{1}{x^2+2}$ from $\frac{2x^3+x^2+3}{(x^2+2)^2}$

Solution:

Step 1: Set up subtraction:

$$\frac{2x^3 + x^2 + 3}{(x^2 + 2)^2} - \frac{1}{x^2 + 2}$$

Step 2: LCD = $(x^2 + 2)^2$:

$$\frac{2x^3 + x^2 + 3}{(x^2 + 2)^2} - \frac{1(x^2 + 2)}{(x^2 + 2)^2}$$

Step 3: Combine numerators:

$$\frac{2x^3 + x^2 + 3 - (x^2 + 2)}{(x^2 + 2)^2}$$

Step 4: Simplify numerator:

$$2x^3 + x^2 + 3 - x^2 - 2 = 2x^3 + 1$$

Answer: $\frac{2x^3+1}{(x^2+2)^2}$, where all real numbers (denominator never zero)

Summary Table

	Original Expression	Simplified Form	Restrictions
(ix)	$\frac{x+2}{x^2-x-12} - \frac{x}{x^2+6x+9}$	$\frac{9x+6}{()^2}$	$x \neq 4, -3$
(x)	$\frac{1}{x^2-4x+3} + \frac{1}{x^2-3x+2} - \frac{1}{x^2-5x+6}$	$\frac{x-4}{(x-1)(x-2)(x-3)}$	$x \neq 1, 2, 3$
(xi)	$\frac{x^3}{x-y} + \frac{y^3}{y-x}$	$x^2 + xy + y^2$	$x \neq y$
2	$\frac{2x^3+x^2+3}{(x^2+2)^2} - \frac{1}{x^2+2}$	$\frac{2x^3+1}{(x^2+2)^2}$	No restrictions

Key Factorization Formulas Used:

- Difference of Cubes:** $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
- Quadratic Factoring:** $x^2 + (a + b)x + ab = (x + a)(x + b)$
- Perfect Square:** $x^2 + 2ax + a^2 = (x + a)^2$

Complete EXERCISE 5.2 Summary (All Parts)

Part	Simplified Form
(i)	$\frac{2}{x-y}$
(ii)	$\frac{5x+y}{18(2x-y)}$
(iii)/(v)	$-\frac{3}{x+2}$
(iv)/(vi)	$\frac{2x^2-3xy-3y^2}{(x-y)(x+y)}$
(vii)	$\frac{3}{x-2}$
(viii)	$\frac{1}{x-2y}$
(ix)	$\frac{9}{(x+3)(2x-3)}$
(x)	$\frac{(2x+3)(3x+2)}{5}$
(ix)	$\frac{9x+6}{()^2}$
(x)	$\frac{x-4}{(x-1)(x-2)(x-3)}$
(xi)	$x^2 + xy + y^2$

Part	Simplified Form
Question 2	$\frac{2x^3+1}{(x^2+2)^2}$

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