

EXERCISE 5.3

1. Simplify:

- (i) $\frac{24lm}{5l+10m} \times \frac{5l^2-20m^2}{16mn}$ (ii) $\frac{x^2-3x+2}{x^2+3x-4} \times \frac{2x^2+8x}{3x+6}$
- (iii) $\frac{a^2-4b^2}{a^2+2ba} \times \frac{2a^2+10ab}{a^2+3ab-10b^2}$ (iv) $\frac{(a+b)^2-c^2}{(a+c)^2-b^2} \times \frac{a^2-(b-c)^2}{(a+c)^2-c^2}$
- (v) $\frac{x^3-8}{x^2-4} \div \frac{x^2+2x+4}{x^2+4x+4}$ (vi) $\frac{(a^2+ab)^2}{(a^2-ab)^2} \div \left(\frac{a+b}{a-b}\right)^2$
- (vii) $\frac{x^2-x-6}{x^2-x-20} \div \left\{ \frac{x^3-3x^2}{x^2+4x} \times \frac{x^3-5x^2-14x}{x^2-12x+35} \right\}$
- (viii) $\frac{3x^2-3xy}{10x^2+10xy-20y^2} \times \frac{5xy+10y^2}{6x^2+6xy+6y^2} \div \frac{2x^2-2y^2}{4x^3-4y^3}$
- (ix) $\frac{2x^2-98}{x^3-125} \times \frac{x^2-3x-10}{3x-21} \div \frac{x^2+5x-14}{x^2+5x+25}$
- (x) $\frac{x^2-2x}{2x+6} \times \frac{x^2+x-6}{x^2-5x} \times \frac{6x-30}{x^2-2x}$

EXERCISE 5.3 - Simplifying Rational Expressions (Multiplication & Division)

Question 1(i)

$$\frac{24lm}{5l+10m} \times \frac{5l^2-20m^2}{16mn}$$

Solution:

Step 1: Factor all expressions:

$$5l+10m = 5(l+2m) \quad 5l^2-20m^2 = 5(l^2-4m^2) = 5(l-2m)(l+2m)$$

Step 2: Rewrite:

$$\frac{24lm}{5(l+2m)} \times \frac{5(l-2m)(l+2m)}{16mn}$$

Step 3: Cancel common factors:

- Cancel 5: $24lm$ and $5(l-2m)(l+2m) / 5(l+2m) \times 16mn$
- Cancel $(l+2m)$
- Cancel m

$$\frac{24l}{1} \times \frac{(l-2m)}{16n} = \frac{24l(l-2m)}{16n}$$

Step 4: Simplify coefficient:

$$\frac{24}{16} = \frac{3}{2}$$

Answer: $\frac{3l(l-2m)}{2n}$, where $l \neq -2m, m \neq 0, n \neq 0$

Question 1(ii)

$$\frac{x^2 - 3x + 2}{x^2 + 3x - 4} \times \frac{2x^2 + 8x}{3x + 6}$$

Solution:

Step 1: Factor all expressions:

$$x^2 - 3x + 2 = (x - 1)(x - 2), x^2 + 3x - 4 = (x + 4)(x - 1), 2x^2 + 8x = 2x(x + 4), 3x + 6 = 3(x + 2)$$

Step 2: Rewrite:

$$\frac{(x - 1)(x - 2)}{(x + 4)(x - 1)} \times \frac{2x(x + 4)}{3(x + 2)}$$

Step 3: Cancel common factors:

- Cancel $(x - 1)$
- Cancel $(x + 4)$

$$\frac{(x - 2)}{1} \times \frac{2x}{3(x + 2)} = \frac{2x(x - 2)}{3(x + 2)}$$

Answer: $\frac{2x(x-2)}{3(x+2)}$, where $x \neq -4, 1, -2$

Question 1(iii)

$$\frac{a^2 - 4b^2}{a^2 + 2ba} \times \frac{2a^2 + 10ab}{a^2 + 3ab - 10b^2}$$

Solution:

Step 1: Factor all expressions:

$$a^2 - 4b^2 = (a - 2b)(a + 2b), a^2 + 2ba = a(a + 2b), 2a^2 + 10ab = 2a(a + 5b), a^2 + 3ab - 10b^2 = (a + 5b)(a - 2b)$$

Step 2: Rewrite:

$$\frac{(a - 2b)(a + 2b)}{a(a + 2b)} \times \frac{2a(a + 5b)}{(a + 5b)(a - 2b)}$$

Step 3: Cancel common factors:

- Cancel $(a - 2b)$
- Cancel $(a + 2b)$

- Cancel $(a + 5b)$
- Cancel a

$$\frac{2}{1}$$

Answer: 2, where $a \neq 0, a \neq -2b, a \neq -5b, a \neq 2b$

Question 1(iv)

$$\frac{(a+b)^2 - c^2}{(a+c)^2 - b^2} \times \frac{a^2 - (b-c)^2}{(a+c)^2 - c^2}$$

Solution:

Step 1: Factor using difference of squares:

$$(a+b)^2 - c^2 = (a+b-c)(a+b+c)(a+c)^2 - b^2 = (a+c-b)(a+c+b) = (a-b+c)(a+b+c)a^2 - (b-c)^2 = (a-(b-$$

Step 2: Rewrite:

$$\frac{(a+b-c)(a+b+c)}{(a-b+c)(a+b+c)} \times \frac{(a-b+c)(a+b-c)}{a(a+2c)}$$

Step 3: Cancel common factors:

- Cancel $(a + b + c)$
- Cancel $(a - b + c)$
- Cancel $(a + b - c)$

$$\frac{1}{a(a+2c)}$$

Answer: $\frac{1}{a(a+2c)}$, where $a \neq 0, a \neq -2c, a+b+c \neq 0, a-b+c \neq 0, a+b-c \neq 0$

Question 1(v)

$$\frac{x^3 - 8}{x^2 - 4} \div \frac{x^2 + 2x + 4}{x^2 + 4x + 4}$$

Solution:

Step 1: Change division to multiplication by reciprocal:

$$\frac{x^3 - 8}{x^2 - 4} \times \frac{x^2 + 4x + 4}{x^2 + 2x + 4}$$

Step 2: Factor all expressions:

$$x^3 - 8 = x^3 - 2^3 = (x - 2)(x^2 + 2x + 4)x^2 - 4 = (x - 2)(x + 2)x^2 + 4x + 4 = (x + 2)^2$$

Step 3: Rewrite:

$$\frac{(x-2)(x^2+2x+4)}{(x-2)(x+2)} \times \frac{(x+2)^2}{x^2+2x+4}$$

Step 4: Cancel common factors:

- Cancel $(x-2)$
- Cancel (x^2+2x+4)
- Cancel one $(x+2)$ from numerator and denominator

$$\frac{x+2}{1}$$

Answer: $x+2$, where $x \neq \pm 2$

Question 1(vi)

$$\frac{(a^2+ab)^2}{(a^2-ab)^2} \div \frac{(a+b)^2}{(a-b)^2}$$

Solution:

Step 1: Change division to multiplication by reciprocal:

$$\frac{(a^2+ab)^2}{(a^2-ab)^2} \times \frac{(a-b)^2}{(a+b)^2}$$

Step 2: Factor:

$$a^2+ab = a(a+b)a^2-ab = a(a-b)$$

Step 3: Rewrite:

$$\frac{[a(a+b)]^2}{[a(a-b)]^2} \times \frac{(a-b)^2}{(a+b)^2} = \frac{a^2(a+b)^2}{a^2(a-b)^2} \times \frac{(a-b)^2}{(a+b)^2}$$

Step 4: Cancel common factors:

- Cancel a^2
- Cancel $(a+b)^2$
- Cancel $(a-b)^2$

Answer: 1, where $a \neq 0, a \neq \pm b$

Question 1(vii)

$$\frac{x^2-x-6}{x^2-x-20} \div \frac{x^3-3x^2}{x^2+4x} \times \frac{x^3-5x^2-14x}{x^2-12x+35}$$

Solution:

Step 1: Change division to multiplication by reciprocal:

$$\frac{x^2 - x - 6}{x^2 - x - 20} \times \frac{x^2 + 4x}{x^3 - 3x^2} \times \frac{x^3 - 5x^2 - 14x}{x^2 - 12x + 35}$$

Step 2: Factor all expressions:

$$x^2 - x - 6 = (x - 3)(x + 2) \quad x^2 - x - 20 = (x - 5)(x + 4) \quad x^3 - 3x^2 = x^2(x - 3) \quad x^3 - 5x^2 - 14x = x(x^2 - 5x - 14)$$

Step 3: Rewrite:

$$\frac{(x - 3)(x + 2)}{(x - 5)(x + 4)} \times \frac{x(x + 4)}{x^2(x - 3)} \times \frac{x(x - 7)(x + 2)}{(x - 5)(x - 7)}$$

Step 4: Cancel common factors:

- Cancel $(x - 3)$
- Cancel $(x + 4)$
- Cancel one x from numerator and denominator
- Cancel $(x + 2)$
- Cancel $(x - 7)$

$$\frac{1}{(x - 5)} \times \frac{1}{x} \times \frac{1}{(x - 5)} = \frac{1}{x(x - 5)^2}$$

Answer: $\frac{1}{x(x-5)^2}$, where $x \neq 0, 5, -4, 3, 7$

Question 1(viii)

$$\frac{3x^2 - 3xy}{10x^2 + 10xy - 20y^2} \times \frac{5xy + 10y^2}{6x^2 + 6xy + 6y^2} \div \frac{2x^2 - 2y^2}{4x^3 - 4y^3}$$

Solution:

Step 1: Change division to multiplication by reciprocal:

$$\frac{3x^2 - 3xy}{10x^2 + 10xy - 20y^2} \times \frac{5xy + 10y^2}{6x^2 + 6xy + 6y^2} \times \frac{4x^3 - 4y^3}{2x^2 - 2y^2}$$

Step 2: Factor all expressions:

$$3x^2 - 3xy = 3x(x - y) \quad 10x^2 + 10xy - 20y^2 = 10(x^2 + xy - 2y^2) = 10(x + 2y)(x - y) \quad 5xy + 10y^2 = 5y(x + 2y) \quad 6x^2 + 6xy + 6y^2 = 6(x + y)^2 \quad 4x^3 - 4y^3 = 4(x - y)(x^2 + xy + y^2) \quad 2x^2 - 2y^2 = 2(x - y)(x + y)$$

Step 3: Rewrite:

$$\frac{3x(x - y)}{10(x + 2y)(x - y)} \times \frac{5y(x + 2y)}{6(x + y)^2} \times \frac{4(x - y)(x^2 + xy + y^2)}{2(x - y)(x + y)}$$

Step 4: Cancel common factors:

- Cancel $(x - y)$

- Cancel $(x + 2y)$
- Cancel $(x^2 + xy + y^2)$
- Cancel another $(x - y)$

$$\frac{3x}{10} \times \frac{5y}{6} \times \frac{4}{2(x+y)}$$

Step 5: Simplify:

$$\frac{3x \cdot 5y \cdot 4}{10 \cdot 6 \cdot 2(x+y)} = \frac{60xy}{120(x+y)} = \frac{xy}{2(x+y)}$$

Answer: $\frac{xy}{2(x+y)}$, where $x \neq -y, x \neq y, x \neq -2y$

Question 1(ix)

$$\frac{2x^2 - 98}{x^3 - 125} \times \frac{x^2 - 3x - 10}{3x - 21} \div \frac{x^2 + 5x - 14}{x^2 + 5x + 25}$$

Solution:

Step 1: Change division to multiplication by reciprocal:

$$\frac{2x^2 - 98}{x^3 - 125} \times \frac{x^2 - 3x - 10}{3x - 21} \times \frac{x^2 + 5x + 25}{x^2 + 5x - 14}$$

Step 2: Factor all expressions:

$$2x^2 - 98 = 2(x^2 - 49) = 2(x - 7)(x + 7) \quad x^3 - 125 = x^3 - 5^3 = (x - 5)(x^2 + 5x + 25) \quad x^2 - 3x - 10 = (x - 5)(x + 2) \quad 3x - 21 = 3(x - 7)$$

Step 3: Rewrite:

$$\frac{2(x-7)(x+7)}{(x-5)(x^2+5x+25)} \times \frac{(x-5)(x+2)}{3(x-7)} \times \frac{x^2+5x+25}{(x+7)(x-2)}$$

Step 4: Cancel common factors:

- Cancel $(x - 7)$
- Cancel $(x + 7)$
- Cancel $(x - 5)$
- Cancel $(x^2 + 5x + 25)$

$$\frac{2}{1} \times \frac{(x+2)}{3} \times \frac{1}{(x-2)} = \frac{2(x+2)}{3(x-2)}$$

Answer: $\frac{2(x+2)}{3(x-2)}$, where $x \neq 5, 7, -7, 2$

Question 1(x)

$$\frac{x^2 - 2x}{2x + 6} \times \frac{x^2 + x - 6}{x^2 - 5x} \times \frac{6x - 30}{x^2 - 2x}$$

Solution:

Step 1: Factor all expressions:

$$x^2 - 2x = x(x - 2)2x + 6 = 2(x + 3)x^2 + x - 6 = (x + 3)(x - 2)x^2 - 5x = x(x - 5)6x - 30 = 6(x - 5)$$

Step 2: Rewrite:

$$\frac{x(x - 2)}{2(x + 3)} \times \frac{(x + 3)(x - 2)}{x(x - 5)} \times \frac{6(x - 5)}{x(x - 2)}$$

Step 3: Cancel common factors:

- Cancel x from first numerator and second denominator
- Cancel $(x - 2)$ from first numerator and second numerator and third denominator
- Cancel $(x + 3)$
- Cancel $(x - 5)$

$$\frac{6}{2x} = \frac{3}{x}$$

Answer: $\frac{3}{x}$, where $x \neq 0, -3, 5, 2$

Summary Table

	Simplified Form	Restrictions
(i)	$\frac{3l(l-2m)}{2n}$	$l \neq -2m, m \neq 0, n \neq 0$
(ii)	$\frac{2x(x-2)}{3(x+2)}$	$x \neq -4, 1, -2$
(iii)	2	$a \neq 0, \pm 2b, -5b$
(iv)	$\frac{1}{a(a+2c)}$	$a \neq 0, -2c$
(v)	$x + 2$	$x \neq \pm 2$
(vi)	1	$a \neq 0, \pm b$
(vii)	$\frac{1}{x(x-5)^2}$	$x \neq 0, 5, -4, 3, 7$
(viii)	$\frac{xy}{2(x+y)}$	$x \neq -y, y, -2y$
(ix)	$\frac{2(x+2)}{3(x-2)}$	$x \neq 5, 7, -7, 2$
(x)	$\frac{3}{x}$	$x \neq 0, -3, 5, 2$

Key Techniques Used:

1. **Factor Everything:** Factor numerators and denominators completely
2. **Change Division to Multiplication:** $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$
3. **Cancel Common Factors:** Cancel identical factors from numerator and denominator
4. **Simplify Coefficients:** Reduce numerical fractions
5. **State Restrictions:** Exclude values that make any denominator zero