

EXERCISE 5.5

1. A train travels a distance of 240 km at a uniform rate, if it had finished 4 km an hour slower, it would have taken 2 hours more over the journey. Find its rate of travelling.
2. Arshia and Ibraheem complete a job together in 4 hours. If Arshia takes 6 hours, then find how much time will Ibraheem take?
3. One pipe fills water in 5 hours, another in 8 hours. Second pipe closed after 2 hours. Find the total time.
4. Huria can complete a project in 12 hours by working alone. If Abdul Hadi join her and they finish it together in 5 hours, how long would it take Abdul Hadi to do the project alone?
5. Two cars start from opposite towns and head towards each other. The distance between them is 240 km. One travels at x km/h and the other at $(x + 10)$ km/h. They meet after 2 hours. Find the speed of each car.
6. Rashid can paint a house in 6 days, but if he gets a helper he can do it in 4 days. How long would it take the helper to paint the house alone?
7. Ishmal runs 10 km in $\frac{2x}{x+4}$ hours. If her average speed is 8 km/h, find the value of x .
8. Fahad runs 600 m at a certain pace, and then doubling his pace, does another 600 m. If he took $2\frac{1}{2}$ to cover the distance 1200 m, find the pace he started at, in metres per seconds.
9. A cyclist travels 30 km at a certain speed. If the speed had been 5 km/h faster, the journey would have taken 1 hour less. Find the speed.

EXERCISE 5.5 - Word Problems (Rational Equations)

Question 1

A train travels a distance of 240 km at a uniform rate. If it had finished 4 km/h slower, it would have taken 2 hours more over the journey. Find its rate of travelling.

Solution:

Step 1: Let the original speed be x km/h.

Step 2: Time at original speed:

$$t_1 = \frac{240}{x}$$

Step 3: Time at slower speed $(x - 4 \text{ km/h})$:

$$t_2 = \frac{240}{x - 4}$$

Step 4: Given: $t_2 = t_1 + 2$:

$$\frac{240}{x-4} = \frac{240}{x} + 2$$

Step 5: Solve:

$$\frac{240}{x-4} - \frac{240}{x} = 2 \frac{240x - 240(x-4)}{x(x-4)} = 2 \frac{240x - 240x + 960}{x(x-4)} = 2 \frac{960}{x(x-4)} = 2960 = 2x(x-4)480 = x^2 - 4x^2 - 4x - 480 = 0$$

Step 6: Solve quadratic:

$$x^2 - 4x - 480 = 0(x-24)(x+20) = 0x = 24 \text{ or } x = -20 \text{ (reject, speed cannot be negative)}$$

Answer: Speed = **24 km/h**

Question 2

Arshia and Ibraheem complete a job together in 4 hours. If Arshia takes 6 hours, then find how much time will Ibraheem take?

Solution:

Step 1: Let Ibraheem take x hours alone.

Step 2: Rate of Arshia = $\frac{1}{6}$ job/hour

Rate of Ibraheem = $\frac{1}{x}$ job/hour

Combined rate = $\frac{1}{4}$ job/hour

Step 3: Equation:

$$\frac{1}{6} + \frac{1}{x} = \frac{1}{4}$$

Step 4: Solve:

$$\frac{1}{x} = \frac{1}{4} - \frac{1}{6} = \frac{3-2}{12} = \frac{1}{12}x = 12$$

Answer: Ibraheem takes **12 hours**

Question 3

One pipe fills water in 5 hours, another in 8 hours. Second pipe closed after 2 hours. Find the total time.

Solution:

Step 1: Let the total time be x hours.

Pipe 1 rate = $\frac{1}{5}$ tank/hour

Pipe 2 rate = $\frac{1}{8}$ tank/hour

Step 2: Pipe 1 works for x hours. Pipe 2 works for 2 hours only.

Step 3: Equation (total work = 1):

$$\frac{x}{5} + \frac{2}{8} = 1 \frac{x}{5} + \frac{1}{4} = 1$$

Step 4: Solve:

$$\frac{x}{5} = 1 - \frac{1}{4} = \frac{3}{4} \Rightarrow x = \frac{3}{4} \times 5 = \frac{15}{4} = 3.75$$

Answer: Total time = **3.75 hours** (3 hours 45 minutes)

Question 4

Huria can complete a project in 12 hours by working alone. If Abdul Hadi joins her and they finish it together in 5 hours, how long would it take Abdul Hadi to do the project alone?

Solution:

Step 1: Let Abdul Hadi take x hours alone.

Step 2: Rate of Huria = $\frac{1}{12}$ project/hour

Rate of Abdul Hadi = $\frac{1}{x}$ project/hour

Combined rate = $\frac{1}{5}$ project/hour

Step 3: Equation:

$$\frac{1}{12} + \frac{1}{x} = \frac{1}{5}$$

Step 4: Solve:

$$\frac{1}{x} = \frac{1}{5} - \frac{1}{12} = \frac{12-5}{60} = \frac{7}{60} \Rightarrow x = \frac{60}{7} = 8\frac{4}{7}$$

Answer: Abdul Hadi takes $\frac{60}{7}$ hours **8.57 hours**

Question 5

Two cars start from opposite towns and head towards each other. The distance between them is 240 km. One travels at x km/h and the other at $(x + 10)$ km/h. They meet after 2 hours. Find the speed of each car.

Solution:

Step 1: Distance covered by Car 1 = $2x$ km

Distance covered by Car 2 = $2(x + 10)$ km

Step 2: Total distance = 240 km:

$$2x + 2(x + 10) = 240$$

Step 3: Solve:

$$2x + 2x + 20 = 240 \Rightarrow 4x = 220 \Rightarrow x = 55$$

Step 4: Speeds:

- Car 1: $x = 55$ km/h
- Car 2: $x + 10 = 65$ km/h

Answer: Speeds are 55 km/h and 65 km/h

Question 6

Rashid can paint a house in 6 days, but if he gets a helper he can do it in 4 days. How long would it take the helper to paint the house alone?

Solution:

Step 1: Let the helper take x days alone.

Step 2: Rate of Rashid = $\frac{1}{6}$ house/day

Rate of helper = $\frac{1}{x}$ house/day

Combined rate = $\frac{1}{4}$ house/day

Step 3: Equation:

$$\frac{1}{6} + \frac{1}{x} = \frac{1}{4}$$

Step 4: Solve:

$$\frac{1}{x} = \frac{1}{4} - \frac{1}{6} = \frac{3-2}{12} = \frac{1}{12} \Rightarrow x = 12$$

Answer: Helper takes 12 days

Summary Table

	Problem Type	
1	Speed-Distance-Time	24 km/h
2	Work (2 people)	12 hours
3	Work (2 pipes)	3.75 hours
4	Work (2 people)	$\frac{60}{7}$ hours
5	Relative Speed	55 km/h and 65 km/h
6	Work (2 people)	12 days

Key Formulas Used:

1. **Speed-Distance-Time:** Time = $\frac{\text{Distance}}{\text{Speed}}$

2. **Work Problems:** Rate = $\frac{1}{\text{Time}}$

- Combined rate = Sum of individual rates

- Time to complete together = $\frac{1}{\text{Combined Rate}}$

3. **Relative Speed (towards each other):** Time to meet = $\frac{\text{Distance}}{\text{Sum of speeds}}$

EXERCISE 5.5 (Continued) - Word Problems

Question 7

Ishmal runs 10 km in $\frac{2x}{x+4}$ hours. If her average speed is 8 km/h, find the value of x .

Solution:

Step 1: Use formula: Speed = $\frac{\text{Distance}}{\text{Time}}$

$$8 = \frac{10}{\frac{2x}{x+4}}$$

Step 2: Simplify:

$$8 = 10 \times \frac{x+4}{2x} \Rightarrow 8 = \frac{10(x+4)}{2x} \Rightarrow 8 = \frac{5(x+4)}{x}$$

Step 3: Solve:

$$8x = 5(x+4) \Rightarrow 8x = 5x + 20 \Rightarrow 3x = 20 \Rightarrow x = \frac{20}{3}$$

Answer: $x = \frac{20}{3}$

Question 8

Fahad runs 600 m at a certain pace, and then doubling his pace, does another 600 m. If he took $2\frac{1}{2}$ minutes to cover the distance 1200 m, find the pace he started at, in metres per second.

Solution:

Step 1: Let the initial pace (speed) be v m/s.

Step 2: Time for first 600 m:

$$t_1 = \frac{600}{v}$$

Step 3: Time for next 600 m (at double speed = $2v$):

$$t_2 = \frac{600}{2v} = \frac{300}{v}$$

Step 4: Total time = $2\frac{1}{2}$ minutes = $2.5 \times 60 = 150$ seconds

Step 5: Equation:

$$\frac{600}{v} + \frac{300}{v} = 150 \Rightarrow \frac{900}{v} = 150$$

Step 6: Solve:

$$v = \frac{900}{150} = 6$$

Answer: Initial pace = 6 m/s

Question 9

A cyclist travels 30 km at a certain speed. If the speed had been 5 km/h faster, the journey would have taken 1 hour less. Find the speed.

Solution:

Step 1: Let the original speed be x km/h.

Step 2: Time at original speed:

$$t_1 = \frac{30}{x}$$

Step 3: Time at faster speed ($x + 5$ km/h):

$$t_2 = \frac{30}{x + 5}$$

Step 4: Given: $t_2 = t_1 - 1$:

$$\frac{30}{x + 5} = \frac{30}{x} - 1$$

Step 5: Solve:

$$\frac{30}{x + 5} = \frac{30 - x}{x} \Rightarrow 30x = (30 - x)(x + 5) \Rightarrow 30x = 30x + 150 - x^2 - 5x \Rightarrow 30x = 25x + 150 - x^2 \Rightarrow 0 = -5x + 150 - x^2 \Rightarrow x^2 - 5x + 150 = 0$$

Step 6: Solve quadratic:

$$x^2 + 5x - 150 = 0 \Rightarrow (x + 15)(x - 10) = 0 \Rightarrow x = 10 \text{ or } x = -15 \text{ (reject, speed cannot be negative)}$$

Answer: Speed = **10 km/h**

Summary Table

	Problem Type	
7	Speed-Distance-Time (with variable)	$x = \frac{20}{3}$
8	Speed-Distance-Time (with variable)	6 m/s
9	Speed-Distance-Time (faster speed)	10 km/h

Complete EXERCISE 5.5 Summary (All Questions)

1	24 km/h
2	12 hours
3	3.75 hours
4	$\frac{60}{7}$ hours
5	55 km/h and 65 km/h
6	12 days
7	$x = \frac{20}{3}$
8	6 m/s
9	10 km/h

Key Formulas Used:

1. **Speed-Distance-Time:**

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}, \text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

2. **Unit Conversion:** 1 minute = 60 seconds

3. **Quadratic Formula:** For solving $ax^2 + bx + c = 0$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

4. **Checking Answers:** Always reject negative values for speed or time

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