

EXERCISE 6.2

- If $\underline{a} = 7\underline{i} - 3\underline{j}$ and $\underline{b} = \underline{i} + 5\underline{j}$, then find the following vectors:
 - $\underline{a} + \underline{b}$
 - $\underline{a} + 3\underline{b}$
 - $3\underline{a} + \frac{1}{2}\underline{b}$
 - $\underline{b} - \underline{a}$
 - $4\underline{b} - 5\underline{a}$
 - $\frac{3}{2}\underline{a} - \underline{b}$
- If $\underline{a} = 6\underline{i} - \underline{j}$, $\underline{b} = \underline{i} + 5\underline{j}$ and $\underline{c} = 3\underline{i} + 5\underline{j}$, then find the magnitudes of the following vectors:
 - $\underline{b} - \underline{c}$
 - $\underline{a} - 2\underline{b} + \underline{c}$
 - $\underline{c} - \underline{b} - \underline{a}$
- Find the values of x and y in each of the following equations:
 - $(x\underline{i} + y\underline{j}) + (2\underline{i} + 3\underline{j}) = 7\underline{i} + 6\underline{j}$
 - $(x\underline{i} - 5\underline{j}) + (3\underline{i} + 5\underline{j}) = -8\underline{i} + y\underline{j}$
 - $(y\underline{i} + 3\underline{j}) + (-5\underline{i} + 2x\underline{j}) = 9\underline{i} + 7\underline{j}$
- If $\underline{a} = \underline{i} + 3\underline{j}$, $\underline{c} = 2\underline{i} + \underline{j}$ and $\underline{a} + 2\underline{b} = \underline{c}$, then find $|\underline{b}|$.
- If $\underline{a} = -2\underline{i} + 7\underline{j}$ and $\underline{b} = 3\underline{i} - 5\underline{j}$, then find the vector components of $\underline{a} + 5\underline{b}$.
- If $5\underline{i} - 3\underline{j} = m(\underline{i} - 10\underline{j}) + n(4\underline{i} - 3\underline{j})$, then find the values of m and n .

EXERCISE 6.2 - Vector Operations

Question 1: Vector Addition and Scalar Multiplication

Given: $\underline{a} = 7\underline{i} - 3\underline{j}$, $\underline{b} = \underline{i} + 5\underline{j}$

1(i) $\underline{a} + \underline{b}$

Solution:

$$\underline{a} + \underline{b} = (7\underline{i} - 3\underline{j}) + (\underline{i} + 5\underline{j}) = (7 + 1)\underline{i} + (-3 + 5)\underline{j} = 8\underline{i} + 2\underline{j}$$

Answer: $8\underline{i} + 2\underline{j}$

1(ii) $\underline{a} + 3\underline{b}$

Solution:

$$3\underline{b} = 3(\underline{i} + 5\underline{j}) = 3\underline{i} + 15\underline{j} \quad \underline{a} + 3\underline{b} = (7\underline{i} - 3\underline{j}) + (3\underline{i} + 15\underline{j}) = (7 + 3)\underline{i} + (-3 + 15)\underline{j} = 10\underline{i} + 12\underline{j}$$

Answer: $10\underline{i} + 12\underline{j}$

1(iii) $3\underline{a} + \frac{1}{2}\underline{b}$

Solution:

$$3\underline{a} = 3(7\underline{i} - 3\underline{j}) = 21\underline{i} - 9\underline{j} \quad \frac{1}{2}\underline{b} = \frac{1}{2}(\underline{i} + 5\underline{j}) = \frac{1}{2}\underline{i} + \frac{5}{2}\underline{j} \quad 3\underline{a} + \frac{1}{2}\underline{b} = (21\underline{i} - 9\underline{j}) + \left(\frac{1}{2}\underline{i} + \frac{5}{2}\underline{j}\right) = \left(21 + \frac{1}{2}\right)\underline{i} + \left(-9 + \frac{5}{2}\right)\underline{j} = \frac{43}{2}\underline{i} - \frac{13}{2}\underline{j}$$

Answer: $\frac{43}{2}\underline{i} - \frac{13}{2}\underline{j}$

1(iv) $\underline{b} - \underline{a}$

Solution:

$$\underline{b} - \underline{a} = (\underline{i} + 5\underline{j}) - (7\underline{i} - 3\underline{j}) = (1 - 7)\underline{i} + (5 - (-3))\underline{j} = -6\underline{i} + 8\underline{j}$$

Answer: $-6i + 8j$

1(v) $4b - 5a$

Solution:

$$4b = 4(i + 5j) = 4i + 20j$$

$$5a = 5(7i - 3j) = 35i - 15j$$

$$4b - 5a = (4i + 20j) - (35i - 15j) = (4 - 35)i + (20 - (-15))j = -31i + 35j$$

Answer: $-31i + 35j$

1(vi) $\frac{3}{2}a - b$

Solution:

$$\frac{3}{2}a = \frac{3}{2}(7i - 3j) = \frac{21}{2}i - \frac{9}{2}j$$

$$\frac{3}{2}a - b = \left(\frac{21}{2}i - \frac{9}{2}j\right) - (i + 5j) = \left(\frac{21}{2} - 1\right)i + \left(-\frac{9}{2} - 5\right)j = \left(\frac{21}{2} - \frac{2}{2}\right)i + \left(-\frac{9}{2} - \frac{10}{2}\right)j = \frac{19}{2}i - \frac{19}{2}j$$

Answer: $\frac{19}{2}i - \frac{19}{2}j$

Question 2: Finding Magnitudes

Given: $a = 6i - j$, $b = i + 5j$, $c = 3i + 5j$

Formula: For vector $v = xi + yj$, $|v| = \sqrt{x^2 + y^2}$

2(i) $b - c$

Solution:

$$b - c = (i + 5j) - (3i + 5j) = (1 - 3)i + (5 - 5)j = -2i + 0j = -2i$$

$$|b - c| = \sqrt{(-2)^2 + 0^2} = \sqrt{4} = 2$$

Answer: 2

2(ii) $a - 2b + c$

Solution:

$$a - 2b + c = (6i - j) - 2(i + 5j) + (3i + 5j) = (6i - j) + (-2i - 10j) + (3i + 5j) = (6 - 2 + 3)i + (-1 - 10 + 5)j = 7i - 6j$$

Answer: $\sqrt{85}$

2(iii) $c - b - a$

Solution:

$$c - b - a = (3i + 5j) - (i + 5j) - (6i - j) = 3i + 5j - i - 5j - 6i + j = (3 - 1 - 6)i + (5 - 5 + 1)j = -4i + j$$

$$|c - b - a| = \sqrt{(-4)^2 + 1^2} = \sqrt{17}$$

Answer: $\sqrt{17}$

Question 3: Finding x and y

3(i) $(xi + yj) + (2i + 3j) = 7i + 6j$

Solution:

$$(xi + yj) + (2i + 3j) = (x + 2)i + (y + 3)j = 7i + 6j$$

Equating components:

$$x + 2 = 7 \Rightarrow x = 5, y + 3 = 6 \Rightarrow y = 3$$

Answer: $x = 5, y = 3$

3(ii) $(xi - 5j) + (3i + 5j) = -8i + yj$

Solution:

$$(xi - 5j) + (3i + 5j) = (x + 3)i + (-5 + 5)j = (x + 3)i + 0j = -8i + yj$$

Equating components:

$$x + 3 = -8 \Rightarrow x = -11, y = 0$$

Answer: $x = -11, y = 0$

3(iii) $(yi + 3j) + (-5i + 2xj) = 9i + 7j$

Solution:

$$(yi + 3j) + (-5i + 2xj) = (y - 5)i + (3 + 2x)j = 9i + 7j$$

Equating components:

$$y - 5 = 9 \Rightarrow y = 14, 3 + 2x = 7 \Rightarrow 2x = 4 \Rightarrow x = 2$$

Answer: $x = 2, y = 14$

Question 4: Find $|b|$

Given: $a = i + 3j$, $c = 2i + j$, and $a + 2b = c$

Solution:

Step 1: Solve for b :

$$a + 2b = c \Rightarrow 2b = c - a \Rightarrow 2b = (2i + j) - (i + 3j) = (2 - 1)i + (1 - 3)j = i - 2j \Rightarrow b = \frac{1}{2}i - j$$

Step 2: Find magnitude:

$$|b| = \sqrt{\left(\frac{1}{2}\right)^2 + (-1)^2} = \sqrt{\frac{1}{4} + 1} = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$$

Answer: $\frac{\sqrt{5}}{2}$

Question 5: Find vector components of $a + 5b$

Given: $a = -2i + 7j$, $b = 3i - 5j$

Solution:

$$a + 5b = (-2i + 7j) + 5(3i - 5j) = (-2i + 7j) + (15i - 25j) = (-2 + 15)i + (7 - 25)j = 13i - 18j$$

Answer: $13i - 18j$

Question 6: Find m and n

Given: $5i - 3j = m(i - 10j) + n(4i - 3j)$

Solution:

Step 1: Expand RHS:

$$5i - 3j = mi - 10mj + 4ni - 3nj \quad 5i - 3j = (m + 4n)i + (-10m - 3n)j$$

Step 2: Equate components:

$$m + 4n = 5 \text{ (Equation 1)} \quad -10m - 3n = -3 \text{ (Equation 2)}$$

Step 3: Solve the system:

From Equation 1: $m = 5 - 4n$

Substitute into Equation 2:

$$-10(5 - 4n) - 3n = -3 \quad -50 + 40n - 3n = -3 \quad -50 + 37n = -3 \quad 37n = 47 \quad n = \frac{47}{37}$$

Now find m :

$$m = 5 - 4\left(\frac{47}{37}\right) = \frac{185}{37} - \frac{188}{37} = -\frac{3}{37}$$

Answer: $m = -\frac{3}{37}$, $n = \frac{47}{37}$

Summary Table

1(i)	$8i + 2j$
1(ii)	$10i + 12j$
1(iii)	$\frac{43}{2}i - \frac{13}{2}j$
1(iv)	$-6i + 8j$
1(v)	$-31i + 35j$
1(vi)	$\frac{19}{2}i - \frac{19}{2}j$
2(i)	2
2(ii)	$\sqrt{85}$
2(iii)	$\sqrt{17}$
3(i)	$x = 5, y = 3$
3(ii)	$x = -11, y = 0$
3(iii)	$x = 2, y = 14$
4	$\frac{\sqrt{5}}{2}$
5	$13i - 18j$

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$$m = -\frac{3}{37}, n = \frac{47}{37}$$

Key Formulas Used:

1. **Vector Addition:** Add corresponding components
2. **Vector Subtraction:** Subtract corresponding components
3. **Scalar Multiplication:** Multiply each component by scalar
4. **Magnitude:** $|v| = \sqrt{x^2 + y^2}$
5. **Equating Vectors:** If two vectors are equal, their corresponding components are equal