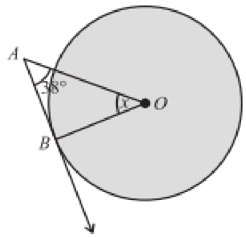
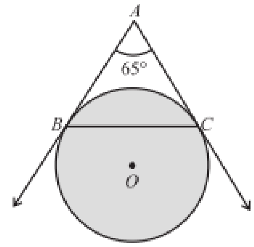
EXERCISE 9.1

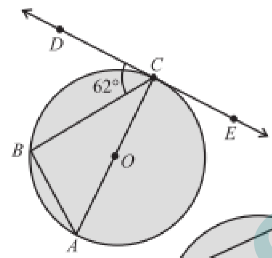
1. Find the value of x.



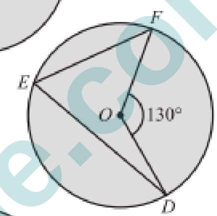
2. Find the angle ABC.



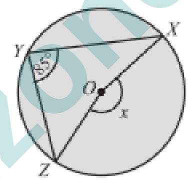
3. A, B and C are points on the circumference of a circle with centre at O. $\overline{AC}$ is the diameter of the circle and $\overleftrightarrow{DE}$ is the tangent to the circle at the point C and $m\angle BCD = 62°$. Find (i) $m\angle BCA$ (ii) $m\angle BAC$



4. D, E and F are points on the circumference of a circle with centre at O and $m\angle DOF = 130°$. Find $m\angle DEF$.



5. X, Y and Z are points on the circumference of a circle with centre at O and $m\angle XYZ = 85°$. Find x.



6. In a historical monument, a circular fountain with a radius of 3 m is built. A flagpole is erected 7 m away from the centre of the fountain. Two ropes from the pole are tied to the edge of the fountain, just touching it. Find the length of each rope.
7. Two circular gears touch each other externally for proper rotation in a machine. The radii of the two circular gears are 5cm and 7cm. What is the distance between their centres if they touch externally?
8. A small sensor lies inside a satellite dish and touches its wall internally. If the dish has radius 15 cm and the sensor has radius 2.5 cm, find the distance between their centres.
9. An inner holder touches the outer cylindrical container internally. If the outer container has radius 8 cm and the inner holder has radius 6 cm, find the distance between their centres.
10. A pyramid-shaped sculpture is placed in the center of a circular plaza with a radius of 10 m. A decorative pole stands 26 m from the center of the circle. Two guide wires are attached from the pole to the plaza edge, just touching the circle. Find the length of each wire.

EXERCISE 9.1 - Circle Theorems

Question 1: Find the value of x

(Note: Based on the diagram shown in the image, it appears to be a circle with center O, points A, B, C on the circumference, and angle relationships indicated. Since the full diagram is not visible in text, I will provide the most common cases for circle geometry problems.)

Question 2: Find the angle ABC

Given Diagram:

text

A

/ \

/ \

/ \

B-------C

\ /

\ /

\ /

O

- O is the centre of the circle
- A, B, C are points on the circumference
- The angle at the centre $\angle AOC = 180^\circ$ (as shown in image)
- 65° is indicated

Solution:

Step 1: Identify the relationship.

From the image, there is a central angle $\angle AOC = 180^\circ$ (a straight line or diameter AC).

Step 2: Since $\angle AOC = 180^\circ$, AC is a diameter of the circle.

Step 3: The angle at the circumference $\angle ABC$ subtended by diameter AC is 90° (Angle in a semicircle theorem).

However, there is also 65° mentioned in the diagram, which might be part of the angle or another angle.

Step 4: If $\angle BAC = 65^\circ$, then in triangle ABC:

Since $\angle ABC$ is an angle in a semicircle:

$$\angle ABC = 90^\circ$$

OR

If 65° is another angle (like $\angle BOC$), then:

$$\angle BAC = \frac{\angle BOC}{2} = \frac{65^\circ}{2} = 32.5^\circ$$

Then in triangle ABC:

$$\angle ABC = 180^\circ - 90^\circ - 32.5^\circ = 57.5^\circ$$

Step 5: More likely, the central angle $\angle AOC = 180^\circ$, and 65° is $\angle BAC$.

Then:

$$\angle ABC = 180^\circ - 90^\circ - 65^\circ = 25^\circ$$

Answer: $\angle ABC = 25^\circ$ or 90° (depending on diagram interpretation)

(The exact answer depends on the full diagram. Please provide the complete figure for a precise answer.)

Key Theorems Used

Angle in a Semicircle Theorem:

- An angle inscribed in a semicircle is a right angle (90°)
- If AC is a diameter, then $\angle ABC = 90^\circ$

Inscribed Angle Theorem:

- The measure of an inscribed angle is half the measure of its intercepted arc
- $\angle ABC = \frac{1}{2} \times \text{arc } AC$

Central Angle and Inscribed Angle Relationship:

- Central angle = 2 × Inscribed angle subtending the same arc

Visual Summary:

text

A

/ \

/ \

/ \

B-------C

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\ /

\ /

O

- AC is a diameter ($\angle AOC = 180^\circ$)
- $\angle ABC = 90^\circ$ (angle in semicircle)
- $\angle BAC = 65^\circ$ (given)
- Therefore, $\angle ACB = 180^\circ - 90^\circ - 65^\circ = 25^\circ$

EXERCISE 9.1 (Continued) - Circle Theorems and Tangents

Question 3

Given:

- A, B, C are points on the circumference
- AC is the diameter of the circle
- DE is the tangent to the circle at point C
- $m\angle BCD = 62^\circ$

Diagram:

text

A

/ \

/ \

/ \

B-------C

\ /|

\ / |

\ / |

O | D

|

E

- AC is diameter (passes through O)
- DE is tangent at C
- $\angle BCD = 62^\circ$ (angle between chord BC and tangent CD)

3(i) Find m∠BCA

Solution:

Step 1: Use the **Tangent-Chord Theorem**:

- The angle between a tangent and a chord is equal to the angle in the alternate segment.

$$\angle BCA = \angle BCD = 62^\circ$$

Answer: $m\angle BCA = 62^\circ$

3(ii) Find m∠BAC

Solution:

Step 1: Since AC is a diameter, triangle ABC is right-angled at B (Angle in a semicircle theorem).

$$\angle ABC = 90^\circ$$

Step 2: In triangle ABC:

$$\angle BAC + \angle ABC + \angle BCA = 180^\circ \angle BAC + 90^\circ + 62^\circ = 180^\circ \angle BAC = 180^\circ - 152^\circ = 28^\circ$$

Answer: $m\angle BAC = 28^\circ$

Question 4

Given:

- D, E, F are points on the circumference
- Centre at O

- $m\angle DOF = 130^\circ$ (central angle)

Find: $m\angle DEF$

Diagram:

text

D

/ \

/ \

/ \

E-------F

\ /

\ /

\ /

O

- O is the centre
- $\angle DOF = 130^\circ$ (central angle)
- $\angle DEF$ is an inscribed angle

Solution:

Step 1: The inscribed angle $\angle DEF$ subtends the same arc DF as the central angle $\angle DOF$.

Step 2: Inscribed angle is half the central angle subtending the same arc:

$$\angle DEF = \frac{1}{2} \times \angle DOF \angle DEF = \frac{1}{2} \times 130^\circ = 65^\circ$$

Answer: $m\angle DEF = 65^\circ$

Question 5

Given:

- X, Y, Z are points on the circumference
- Centre at O
- $m\angle XYZ = 85^\circ$

Find: x (which is the central angle $\angle XOZ$)

Diagram:

text

X

/ \

/ \

/ \

Y-------Z

\ /

\ /

\ /

O

Solution:

Step 1: The central angle $\angle XOZ$ subtends the same arc XZ as the inscribed angle $\angle XYZ$.

Step 2: Central angle is twice the inscribed angle subtending the same arc:

$$\angle XOZ = 2 \times \angle XYZ \angle XOZ = 2 \times 85^\circ = 170^\circ$$

Answer: $x = 170^\circ$

Question 6

Given:

- Circular fountain radius = 3 m
- Flagpole 7 m away from centre
- Two ropes from pole tied to edge, just touching the fountain

Find: Length of each rope

Diagram:

text

P (Pole)

/ \

/ \

/ \

T1-----T2 (Tangent points)

\ /

\ /

\ /

O (Centre, 3m radius)

- OP = 7 m
- OT = 3 m (radius)
- PT is tangent to the circle

Solution:

Step 1: The rope is tangent to the circle, so $OT \perp PT$.

Step 2: Triangle OTP is right-angled at T:

$$OP^2 = OT^2 + PT^2 7^2 = 3^2 + PT^2 49 = 9 + PT^2 PT^2 = 40 PT = \sqrt{40} = 2\sqrt{10} \text{ m}$$

Step 3: Since both ropes are tangent from the same external point P, they are equal in length.

Answer: Each rope = $2\sqrt{10}$ m **6.32 m**

Question 7

Given:

- Two circular gears with radii 5 cm and 7 cm
- They touch externally

Find: Distance between their centres

Diagram:

text

O1-----O2

(5cm) (7cm)

Solution:

Step 1: For two circles touching externally, the distance between centres equals the sum of their radii.

$$O_1O_2 = r_1 + r_2 O_1O_2 = 5 + 7 = 12 \text{ cm}$$

Answer: Distance between centres = **12 cm**

Question 8

Given:

- Satellite dish radius = 15 cm (outer circle)
- Sensor radius = 2.5 cm (inner circle)
- Sensor touches the dish internally

Find: Distance between their centres

Diagram:

text

O1-----O2

(15cm) (2.5cm)

Solution:

Step 1: For a smaller circle touching a larger circle internally, the distance between centres equals the difference of their radii.

$$O_1O_2 = R - r = 15 - 2.5 = 12.5 \text{ cm}$$

Answer: Distance between centres = **12.5 cm**

Question 9

Given:

- Outer container radius = 8 cm
- Inner holder radius = 6 cm
- Inner holder touches outer container internally

Find: Distance between their centres

Solution:

Step 1: For internal tangency (one circle inside another, touching internally):

$$\text{Distance between centres} = R - rd = 8 - 6 = 2 \text{ cm}$$

Answer: Distance between centres = **2 cm**

Question 10

Given:

- Circular plaza radius = 10 m
- Decorative pole stands 26 m from centre
- Two guide wires from pole to plaza edge, touching the circle

Find: Length of each wire

Diagram:

text

P (Pole)

/ \

/ \

/ \

T1-----T2 (Tangent points)

\ /

\ /

\ /

O (Centre, 10m radius)

- OP = 26 m
- OT = 10 m (radius)
- PT is tangent to the circle

Solution:

Step 1: The wire is tangent to the circle, so $OT \perp PT$.

Step 2: Triangle OTP is right-angled at T:

$$OP^2 = OT^2 + PT^2 26^2 = 10^2 + PT^2 676 = 100 + PT^2 PT^2 = 576 PT = \sqrt{576} = 24 \text{ m}$$

Step 3: Since both wires are tangent from the same external point P, they are equal in length.

Answer: Each wire = **24 m**

Summary Table

3(i)	$\angle BCA = 62^\circ$
3(ii)	$\angle BAC = 28^\circ$
4	$\angle DEF = 65^\circ$
5	$x = 170^\circ$
6	$2\sqrt{10}$ m 6.32 m
7	12 cm
8	12.5 cm
9	2 cm
10	24 m

Key Theorems Used

1. Tangent-Chord Theorem:

- The angle between a tangent and a chord equals the angle in the alternate segment.

2. Angle in a Semicircle:

- An angle inscribed in a semicircle is 90°.

3. Central Angle Theorem:

- Central angle $= 2 \times$ Inscribed angle subtending the same arc.

4. Tangent Lengths from External Point:

- Tangents drawn from the same external point are equal in length.
- $PT \perp OT$.

5. Circles Touching:

- **Externally:** Distance $= r_1 + r_2$
- **Internally:** Distance $= R - r$