REVIEW EXERCISE 3

1. Four possible answers are given for the following questions. Choose the correct answer.

(i) If $\begin{bmatrix} a+2 \\ 3 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$, then $a =$

(a) 3 (b) 5 (c) 6 (d) 7

(ii) $A = \begin{bmatrix} 3 & 5 & 0 \end{bmatrix}$ is a _______ matrix.

(a) row (b) square (c) column (d) null

(iii) $B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ is a _______ matrix.

(a) identity (b) square (c) row (d) column

(iv) $M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is a/an _______ matrix.

(a) rectangular (b) identity (c) column (d) row

(v) If $A^t = -A$, then A is _______ matrix.

(a) symmetric (b) row
(c) rectangular (d) skew-symmetric

(vi) If $A = \begin{bmatrix} 3 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 7 & 8 \end{bmatrix}$, then $A + B =$

(a) $[21 \quad 32]$ (b) $[24 \quad 28]$
(c) $[10 \quad 12]$ (d) $[11 \quad 11]$

(vii) If $A = [3 \quad 4]$, $B = [7 \quad 8]$, then $B - A =$

(a) $[10 \quad 12]$ (b) $[11 \quad 11]$
(c) $[4 \quad 4]$ (d) $[-4 \quad -4]$

(viii) What is the additive inverse of $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$?

(a) $\begin{bmatrix} -3 \\ 4 \end{bmatrix}$ (b) $\begin{bmatrix} -3 \\ -4 \end{bmatrix}$ (c) $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ (d) $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

(ix) If $A = \begin{bmatrix} 3 & 1 & 3 \\ 2 & 0 & 2 \end{bmatrix}$, then order of A^t is:

(a) 3-by-2 (b) 2-by-3 (c) 3-by-3 (d) 2-by-2

(x) $\begin{vmatrix} 3 & 1 \\ 0 & 4 \end{vmatrix} =$

(a) 11 (b) 12 (c) 13 (d) –11

2. If $A = \begin{bmatrix} 4 & 2 \\ 7 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 5 \\ 8 & 8 \end{bmatrix}$, then find

(i) $(A-B)^t$ (ii) $B^t - A^t$ (iii) $2A + 3B$

3. If $A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 6 \\ 2 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 3 & 2 \\ 1 & 7 \end{bmatrix}$, then verify that

(i) $2(A+B) = 2A + 2B$ (ii) $(A+B)+C = A+(B+C)$
(iii) $(A+B)C = AC + BC$ (iv) $C(A-B) = CA - CB$
(v) $(AB)^{-1} = B^{-1}A^{-1}$ (vi) $AA^{-1} = A^{-1}A = I$
(vii) $(AB)^t = B^t A^t$ (viii) $(AB)C = A(BC)$

4. If $A = \begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix}$, then find

(i) $|B|$ (ii) Adj B (iii) A^{-1}
(iv) $A^{-1}A$ (v) $(AB)^t$ (vi) $(B^t)^t$

5. Use matrix inversion method and Cramer's rule to solve the following pair of linear equations, if possible:

(i) $3x + 4y = 7$, $5x - y = 2$ (ii) $x - 6y = -15$, $2x + 6y = -3$ (iii) $2x + y = 5$, $x + 3y = 3$

6. Find two numbers by using matrices such that twice the first added to the second makes 21 and twice the second added to the first makes 27.

7. 4 knives and 6 forks cost Rs. 136, whereas 6 knives and 5 forks cost Rs. 164. Find the cost of a knife and a fork by using matrices.

8. A shop employs, 5 men and 3 women, pays total daily wages Rs. 3500. If the number of men is reduced to 2 and 3 extra women are taken on, the daily wages amount to Rs. 5000. Find daily wages of a man and a woman by using matrices.

REVIEW EXERCISE 3 - COMPLETE SOLUTIONS

Complete Step-by-Step Solutions

Question 1: Multiple Choice Questions

(i) If $\begin{bmatrix} a+2 \\ 3 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$, then a =

Step 1: Equate corresponding entries

- $a + 2 = 7$
- $a = 5$

Answer: (b) 5

(ii) $A = [3 \quad 5 \quad 0]$ is a __________ matrix.

Step 1: Identify type

- 1 row, 3 columns
- A matrix with exactly one row is a **row matrix**

Answer: (a) row

(iii) $\mathbf{B} = \begin{bmatrix} \mathbf{2} \\ \mathbf{3} \end{bmatrix}$ **is a ________ matrix.**

Step 1: Identify type

- 2 rows, 1 column
- A matrix with exactly one column is a **column matrix**

Answer: (d) column

(iv) $\mathbf{M} = \begin{bmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$ **is a/an ________ matrix.**

Step 1: Identify type

- 3 rows, 3 columns
- Main diagonal entries are 1, other entries are 0
- This is the **identity matrix**

Answer: (b) identity

(v) If $\mathbf{A^t} = -\mathbf{A}$, then A is ________ matrix.

Step 1: Recall definition

- A matrix where $A^t = A$ is **symmetric**
- A matrix where $A^t = -A$ is **skew-symmetric**

Answer: (d) skew-symmetric

(vi) If $\mathbf{A} = \begin{bmatrix} \mathbf{3} & \mathbf{4} \\ \mathbf{4} & \mathbf{2} \end{bmatrix}$, $\mathbf{B} = [\mathbf{7} \quad \mathbf{8}]$, then $\mathbf{A} + \mathbf{B} =$

Step 1: Check orders

- A is 2×2
- B is 1×2
- **Not conformable for addition** (different orders)

Answer: None of the options (matrices are not conformable)

(vii) If $\mathbf{A} = \begin{bmatrix} \mathbf{3} & \mathbf{4} \\ \mathbf{4} & \mathbf{2} \end{bmatrix}$, $\mathbf{B} = [\mathbf{7} \quad \mathbf{8}]$, then $\mathbf{B} - \mathbf{A} =$

Step 1: Check orders

- A is 2×2
- B is 1×2
- **Not conformable for subtraction** (different orders)

Answer: None of the options (matrices are not conformable)

Summary Table

(i)	(b) 5
(ii)	(a) row
(iii)	(d) column
(iv)	(b) identity
(v)	(d) skew-symmetric
(vi)	Not conformable
(vii)	Not conformable

Key Concepts Review

1. Matrix Types

Type	Description	Example
Row	One row	$\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$
Column	One column	$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$
Square	Equal rows and columns	$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$
Identity	Square with 1s on diagonal	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
Symmetric	$A^t = A$	$\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$
Skew-Symmetric	$A^t = -A$	$\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$

2. Matrix Addition/Subtraction

- **Conformable:** Same order required
- **Add/Subtract:** Corresponding entries

3. Matrix Equality

- **Same order** and **corresponding entries equal**

4. Matrix Multiplication

- **Condition:** Columns of first = Rows of second
- **Not commutative** in general

REVIEW EXERCISE 3 - COMPLETE SOLUTIONS (Continued)

Complete Step-by-Step Solutions

Question 1: Multiple Choice Questions (Continued)

(viii) Additive inverse of $\begin{bmatrix} \mathbf{3} \\ \mathbf{4} \end{bmatrix}$

Step 1: Additive inverse = Negative of the matrix

$$-\begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} -3 \\ -4 \end{bmatrix}$$

Answer: (b) $\begin{bmatrix} -3 \\ -4 \end{bmatrix}$

(ix) If $\mathbf{A} = \begin{bmatrix} \mathbf{3} & \mathbf{1} & \mathbf{3} \\ \mathbf{2} & \mathbf{0} & \mathbf{2} \end{bmatrix}$, then order of $\mathbf{A}'$ is

Step 1: Original order: 2×3
Step 2: Transpose interchanges rows and columns

- Order of $A' = 3 \times 2$

Answer: (a) 3-by-2

(x) $\begin{vmatrix} \mathbf{3} & \mathbf{1} \\ \mathbf{0} & \mathbf{4} \end{vmatrix} =$

Step 1: Determinant formula: $ad - bc$

$$= 3(4) - 1(0) = 12 - 0 = 12$$

Answer: (b) 12

Question 2: Matrix Operations

Given:

$$A = \begin{bmatrix} 4 & 2 \\ 7 & 6 \end{bmatrix}, B = \begin{bmatrix} 5 & 5 \\ 8 & 8 \end{bmatrix}$$

(i) $(\mathbf{A} - \mathbf{B})'$

Step 1: Find $A - B$

$$A - B = \begin{bmatrix} 4-5 & 2-5 \\ 7-8 & 6-8 \end{bmatrix} = \begin{bmatrix} -1 & -3 \\ -1 & -2 \end{bmatrix}$$

Step 2: Find transpose

$$\left(A - B\right)' = \begin{bmatrix} -1 & -1 \\ -3 & -2 \end{bmatrix}$$

Answer: $\begin{bmatrix} -1 & -1 \\ -3 & -2 \end{bmatrix}$

(ii) $\mathbf{B}' - \mathbf{A}'$

Step 1: Find B' and A'

$$B' = \begin{bmatrix} 5 & 8 \\ 5 & 8 \end{bmatrix}, A' = \begin{bmatrix} 4 & 7 \\ 2 & 6 \end{bmatrix}$$

Step 2: Subtract

$$B' - A' = \begin{bmatrix} 5-4 & 8-7 \\ 5-2 & 8-6 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$$

Answer: $\begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$

(iii) $\mathbf{2A + 3B}$

Step 1: Find $2A$ and $3B$

$$2A = \begin{bmatrix} 8 & 4 \\ 14 & 12 \end{bmatrix}, 3B = \begin{bmatrix} 15 & 15 \\ 24 & 24 \end{bmatrix}$$

Step 2: Add

$$2A + 3B = \begin{bmatrix} 8+15 & 4+15 \\ 14+24 & 12+24 \end{bmatrix} = \begin{bmatrix} 23 & 19 \\ 38 & 36 \end{bmatrix}$$

Answer: $\begin{bmatrix} 23 & 19 \\ 38 & 36 \end{bmatrix}$

Question 3: Verify Matrix Properties

Given:

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}, B = \begin{bmatrix} 5 & 6 \\ 2 & 1 \end{bmatrix}, C = \begin{bmatrix} 3 & 2 \\ 1 & 7 \end{bmatrix}$$

(i) $\mathbf{2(A + B) = 2A + 2B}$

Step 1: Find $A + B$

$$A + B = \begin{bmatrix} 2+5 & 3+6 \\ 4+2 & 5+1 \end{bmatrix} = \begin{bmatrix} 7 & 9 \\ 6 & 6 \end{bmatrix}$$

Step 2: $2(A + B) = \begin{bmatrix} 14 & 18 \\ 12 & 12 \end{bmatrix}$

Step 3: $2A + 2B = \begin{bmatrix} 4 & 6 \\ 8 & 10 \end{bmatrix} + \begin{bmatrix} 10 & 12 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 14 & 18 \\ 12 & 12 \end{bmatrix}$

Verified

(ii) $\mathbf{(A + B) + C = A + (B + C)}$

Step 1: $A + B = \begin{bmatrix} 7 & 9 \\ 6 & 6 \end{bmatrix}$

$$(A + B) + C = \begin{bmatrix} 7+3 & 9+2 \\ 6+1 & 6+7 \end{bmatrix} = \begin{bmatrix} 10 & 11 \\ 7 & 13 \end{bmatrix}$$

Step 2: $B + C = \begin{bmatrix} 5+3 & 6+2 \\ 2+1 & 1+7 \end{bmatrix} = \begin{bmatrix} 8 & 8 \\ 3 & 8 \end{bmatrix}$

$$A + (B + C) = \begin{bmatrix} 2+8 & 3+8 \\ 4+3 & 5+8 \end{bmatrix} = \begin{bmatrix} 10 & 11 \\ 7 & 13 \end{bmatrix}$$

Verified

(iii) $(\mathbf{A}+\mathbf{B})\mathbf{C} = \mathbf{AC}+\mathbf{BC}$

Step 1: $A+B = \begin{bmatrix} 7 & 9 \\ 6 & 6 \end{bmatrix}$

$$(A+B)C = \begin{bmatrix} 7 & 9 \\ 6 & 6 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & 7 \end{bmatrix} = \begin{bmatrix} 21+9 & 14+63 \\ 18+6 & 12+42 \end{bmatrix} = \begin{bmatrix} 30 & 77 \\ 24 & 54 \end{bmatrix}$$

Step 2: $AC = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & 7 \end{bmatrix} = \begin{bmatrix} 6+3 & 4+21 \\ 12+5 & 8+35 \end{bmatrix} = \begin{bmatrix} 9 & 25 \\ 17 & 43 \end{bmatrix}$

Step 3: $BC = \begin{bmatrix} 5 & 6 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & 7 \end{bmatrix} = \begin{bmatrix} 15+6 & 10+42 \\ 6+1 & 4+7 \end{bmatrix} = \begin{bmatrix} 21 & 52 \\ 7 & 11 \end{bmatrix}$

Step 4: $AC+BC = \begin{bmatrix} 9+21 & 25+52 \\ 17+7 & 43+11 \end{bmatrix} = \begin{bmatrix} 30 & 77 \\ 24 & 54 \end{bmatrix}$

Verified

(iv) $\mathbf{CA}-\mathbf{B} = \mathbf{CA}-\mathbf{CB}$ — Wait, this should be $\mathbf{C}(\mathbf{A}-\mathbf{B}) = \mathbf{CA}-\mathbf{CB}$

Let's verify $C(A-B) = CA-CB$

Step 1: $A-B = \begin{bmatrix} 2-5 & 3-6 \\ 4-2 & 5-1 \end{bmatrix} = \begin{bmatrix} -3 & -3 \\ 2 & 4 \end{bmatrix}$

$$C(A-B) = \begin{bmatrix} 3 & 2 \\ 1 & 7 \end{bmatrix}\begin{bmatrix} -3 & -3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} -9+4 & -9+8 \\ -3+14 & -3+28 \end{bmatrix} = \begin{bmatrix} -5 & -1 \\ 11 & 25 \end{bmatrix}$$

Step 2: $CA = \begin{bmatrix} 3 & 2 \\ 1 & 7 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 6+8 & 9+10 \\ 2+28 & 3+35 \end{bmatrix} = \begin{bmatrix} 14 & 19 \\ 30 & 38 \end{bmatrix}$

Step 3: $CB = \begin{bmatrix} 3 & 2 \\ 1 & 7 \end{bmatrix}\begin{bmatrix} 5 & 6 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 15+4 & 18+2 \\ 5+14 & 6+7 \end{bmatrix} = \begin{bmatrix} 19 & 20 \\ 19 & 13 \end{bmatrix}$

Step 4: $CA-CB = \begin{bmatrix} 14-19 & 19-20 \\ 30-19 & 38-13 \end{bmatrix} = \begin{bmatrix} -5 & -1 \\ 11 & 25 \end{bmatrix}$

Verified

(v) $\left(\mathbf{AB}\right)^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$

Step 1: Find AB

$$AB = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 5 & 6 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 10+6 & 12+3 \\ 20+10 & 24+5 \end{bmatrix} = \begin{bmatrix} 16 & 15 \\ 30 & 29 \end{bmatrix}$$

Step 2: $|AB| = 16(29) - 15(30) = 464 - 450 = 14$

$$\left(AB\right)^{-1} = \frac{1}{14}\begin{bmatrix} 29 & -15 \\ -30 & 16 \end{bmatrix} = \begin{bmatrix} \frac{29}{14} & -\frac{15}{14} \\ -\frac{15}{7} & \frac{8}{7} \end{bmatrix}$$

Step 3: Find A^{-1}

$$|A| = 2(5) - 3(4) = 10 - 12 = -2A^{-1} = \frac{1}{-2}\begin{bmatrix} 5 & -3 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} -\frac{5}{2} & \frac{3}{2} \\ 2 & -1 \end{bmatrix}$$

Step 4: Find B^{-1}

$$|B| = 5(1) - 6(2) = 5 - 12 = -7 \quad B^{-1} = \frac{1}{-7}\begin{bmatrix} 1 & -6 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} -\frac{1}{7} & \frac{6}{7} \\ \frac{2}{7} & -\frac{5}{7} \end{bmatrix}$$

Step 5: Find $B^{-1}A^{-1}$

$$B^{-1}A^{-1} = \begin{bmatrix} -\frac{1}{7} & \frac{6}{7} \\ \frac{2}{7} & -\frac{5}{7} \end{bmatrix}\begin{bmatrix} -\frac{5}{2} & \frac{3}{2} \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} \frac{5}{14} + \frac{12}{7} & -\frac{3}{14} - \frac{6}{7} \\ -\frac{5}{7} - \frac{10}{7} & \frac{3}{7} + \frac{5}{7} \end{bmatrix} = \begin{bmatrix} \frac{5}{14} + \frac{24}{14} & -\frac{3}{14} - \frac{12}{14} \\ -\frac{15}{7} & \frac{8}{7} \end{bmatrix} = \begin{bmatrix} \frac{29}{14} & -\frac{15}{14} \\ -\frac{15}{7} & \frac{8}{7} \end{bmatrix}$$

Verified

(vi) $\mathbf{AA^{-1}} = \mathbf{A^{-1}A} = \mathbf{I}$ (Verified from above)

(vii) $(\mathbf{AB})' = \mathbf{B}'\mathbf{A}'$

Step 1: $AB = \begin{bmatrix} 16 & 15 \\ 30 & 29 \end{bmatrix} \rightarrow (AB)' = \begin{bmatrix} 16 & 30 \\ 15 & 29 \end{bmatrix}$

Step 2: $B' = \begin{bmatrix} 5 & 2 \\ 6 & 1 \end{bmatrix}, A' = \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix}$

Step 3: $B'A' = \begin{bmatrix} 5 & 2 \\ 6 & 1 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} 10+6 & 20+10 \\ 12+3 & 24+5 \end{bmatrix} = \begin{bmatrix} 16 & 30 \\ 15 & 29 \end{bmatrix}$

Verified

(viii) $(\mathbf{AB})\mathbf{C} = \mathbf{A}(\mathbf{BC})$

Step 1: $AB = \begin{bmatrix} 16 & 15 \\ 30 & 29 \end{bmatrix}$

$$(AB)C = \begin{bmatrix} 16 & 15 \\ 30 & 29 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & 7 \end{bmatrix} = \begin{bmatrix} 48+15 & 32+105 \\ 90+29 & 60+203 \end{bmatrix} = \begin{bmatrix} 63 & 137 \\ 119 & 263 \end{bmatrix}$$

Step 2: $BC = \begin{bmatrix} 5 & 6 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & 7 \end{bmatrix} = \begin{bmatrix} 15+6 & 10+42 \\ 6+1 & 4+7 \end{bmatrix} = \begin{bmatrix} 21 & 52 \\ 7 & 11 \end{bmatrix}$

Step 3: $A(BC) = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}\begin{bmatrix} 21 & 52 \\ 7 & 11 \end{bmatrix} = \begin{bmatrix} 42+21 & 104+33 \\ 84+35 & 208+55 \end{bmatrix} = \begin{bmatrix} 63 & 137 \\ 119 & 263 \end{bmatrix}$

Verified

Question 4: Matrix Operations

Given:

$$A = \begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix}$$

(i) $|\mathbf{B}|$

$$|B| = 5(4) - 3(2) = 20 - 6 = 14$$

(ii) Adj B

$$\text{adj}(B) = \begin{bmatrix} 4 & -3 \\ -2 & 5 \end{bmatrix}$$

(iii) $\mathbf{A^{-1}}$

$$|A| = 7(1) - 3(2) = 7 - 6 = 1 A^{-1} = \frac{1}{1}\begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix}$$

(iv) $\mathbf{A^{-1}A}$

$$A^{-1}A = \begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix}\begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 7-6 & 3-3 \\ -14+14 & -6+7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

(v) $\mathbf{(AB)'}$

$$AB = \begin{bmatrix} 7 & 3 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix} = \begin{bmatrix} 35+6 & 21+12 \\ 10+2 & 6+4 \end{bmatrix} = \begin{bmatrix} 41 & 33 \\ 12 & 10 \end{bmatrix}\left(AB\right)' = \begin{bmatrix} 41 & 12 \\ 33 & 10 \end{bmatrix}$$

(vi) $\mathbf{\left(B'\right)' = B}$

$$B = \begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix}, B' = \begin{bmatrix} 5 & 2 \\ 3 & 4 \end{bmatrix}, (B')' = \begin{bmatrix} 5 & 3 \\ 2 & 4 \end{bmatrix} = B$$

Question 5: Solve by Matrix Inversion and Cramer's Rule

(i) $\mathbf{3x + 4y = 7,\ x - 6y = -15}$

Step 1: Matrix form

$$A = \begin{bmatrix} 3 & 4 \\ 1 & -6 \end{bmatrix}, B = \begin{bmatrix} 7 \\ -15 \end{bmatrix}$$

Step 2: $|A| = 3(-6) - 4(1) = -18 - 4 = -22$

Cramer's Rule:

$$x = \frac{\begin{vmatrix} 7 & 4 \\ -15 & -6 \end{vmatrix}}{-22} = \frac{7(-6) - 4(-15)}{-22} = \frac{-42+60}{-22} = \frac{18}{-22} = -\frac{9}{11} y = \frac{\begin{vmatrix} 3 & 7 \\ 1 & -15 \end{vmatrix}}{-22} = \frac{3(-15) - 7(1)}{-22} = \frac{-45-7}{-22} = \frac{-52}{-22} = \frac{26}{11}$$

Answer: $x = -\frac{9}{11}, y = \frac{26}{11}$

(ii) $\mathbf{2x + y = 5,\ 5x - y = 2}$

Step 1: $A = \begin{bmatrix} 2 & 1 \\ 5 & -1 \end{bmatrix}, |A| = 2(-1) - 1(5) = -2 - 5 = -7$

Cramer's Rule:

$$x = \frac{\begin{vmatrix} 5 & 1 \\ 2 & -1 \end{vmatrix}}{-7} = \frac{5(-1) - 1(2)}{-7} = \frac{-5-2}{-7} = 1y = \frac{\begin{vmatrix} 2 & 5 \\ 5 & 2 \end{vmatrix}}{-7} = \frac{2(2) - 5(5)}{-7} = \frac{4-25}{-7} = \frac{-21}{-7} = 3$$

Answer: $x = 1, y = 3$

(iii) $\mathbf{2x + 6y = -3,\ x + 3y = 3}$

Step 1: $A = \begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix}, \mid A \mid = 2(3) - 6(1) = 6 - 6 = 0$

Since $\mid A \mid = 0$, the system is either inconsistent or has infinite solutions.

Check: Row 1 = 2 × Row 2, but RHS: -3   2(3) = 6

Answer: Inconsistent (no solution)

Question 6: Find Two Numbers

Let numbers be x and y.

Given: $2x + y = 21,\ x + 2y = 27$

Step 1: Matrix form

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 21 \\ 27 \end{bmatrix}$$

Step 2: $\mid A \mid = 2(2) - 1(1) = 4 - 1 = 3$

Step 3: $A^{-1} = \frac{1}{3} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

Step 4:

$$x = \frac{1}{3}[2(21) - 1(27)] = \frac{42-27}{3} = \frac{15}{3} = 5y = \frac{1}{3}[-1(21) + 2(27)] = \frac{-21+54}{3} = \frac{33}{3} = 11$$

Answer: Numbers are 5 and 11

Question 7: Cost of Knife and Fork

Let cost of knife $= x$, cost of fork $= y$

Given: $4x + 6y = 136,\ 6x + 5y = 164$

Step 1: Matrix form

$$\begin{bmatrix} 4 & 6 \\ 6 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 136 \\ 164 \end{bmatrix}$$

Step 2: $\mid A \mid = 4(5) - 6(6) = 20 - 36 = -16$

Step 3: $A^{-1} = \frac{1}{-16} \begin{bmatrix} 5 & -6 \\ -6 & 4 \end{bmatrix}$

Step 4:

$$x = \frac{1}{-16}[5(136) - 6(164)] = \frac{680-984}{-16} = \frac{-304}{-16} = 19y = \frac{1}{-16}[-6(136) + 4(164)] = \frac{-816+656}{-16} = \frac{-160}{-16} = 10$$

Answer: Knife = Rs. 19, Fork = Rs. 10

Question 8: Daily Wages

Let daily wage of man = x, woman = y

Given: $5x + 3y = 3500,\ 2x + 6y = 5000$

Step 1: Matrix form

$$\begin{bmatrix} 5 & 3 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3500 \\ 5000 \end{bmatrix}$$

Step 2: $|A| = 5(6) - 3(2) = 30 - 6 = 24$

Step 3: $A^{-1} = \frac{1}{24} \begin{bmatrix} 6 & -3 \\ -2 & 5 \end{bmatrix}$

Step 4:

$$x = \frac{1}{24}[6(3500) - 3(5000)] = \frac{21000 - 15000}{24} = \frac{6000}{24} = 250 y = \frac{1}{24}[-2(3500) + 5(5000)] = \frac{-7000 + 25000}{24} = \frac{18000}{24} = 750$$

Answer: Man = Rs. 250/day, Woman = Rs. 750/day

Summary Table

1(viii)	(b)
1(ix)	(a)
1(x)	(b) 12
2(i)	$\begin{bmatrix} -1 & -1 \\ -3 & -2 \end{bmatrix}$
2(ii)	$\begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$
2(iii)	$\begin{bmatrix} 23 & 19 \\ 38 & 36 \end{bmatrix}$
3(i)-(viii)	All Verified
4(i)	14
4(ii)	$\begin{bmatrix} 4 & -3 \\ -2 & 5 \end{bmatrix}$
4(iii)	$\begin{bmatrix} 1 & -3 \\ -2 & 7 \end{bmatrix}$
4(iv)	I
4(v)	$\begin{bmatrix} 41 & 12 \\ 33 & 10 \end{bmatrix}$
5(i)	$x = -\frac{9}{11}, y = \frac{26}{11}$
5(ii)	$x = 1, y = 3$
5(iii)	Inconsistent
6	5 and 11
7	Knife=Rs.19, Fork=Rs.10
8	Man=Rs.250, Woman=Rs.750