

## Exercise 3.2

Q. 1 Consider the universal set

$U = \{x: x \text{ is multiple of 2 and } 0 < x \leq 30\}$ .

$A = \{x: x \text{ is a multiple of 6}\}$  and

$B = \{x: x \text{ is a multiple of 8}\}$

(i) List all elements of sets A and B in tabular form.

(i) Tabular form:

Solution:

$$U = \{2, 4, 6, 8, \dots, 30\}$$

$$A = \{6, 12, 18, 24, 30\}$$

$$B = \{8, 16, 24\}$$

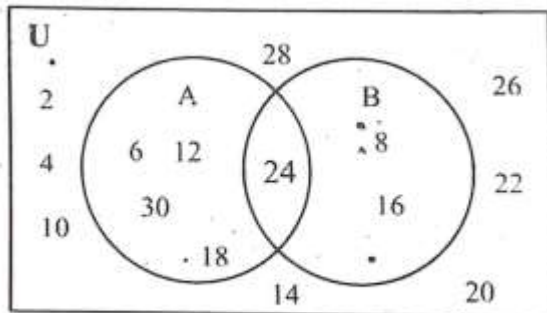
(ii) Find  $A \cap B$

Solution:

$$\begin{aligned} A \cap B &= \{6, 12, 18, 24, 30\} \cap \{8, 16, 24\} \\ &= \{24\} \end{aligned}$$

(iii) Draw a Venn diagram

Venn diagram:



Q. 2 Let,  $U = \{x: x \text{ is an integer and } 0 < x \leq 150\}$ ,  $G = \{x: x = 2^m \text{ for integer } m \text{ and } 0 \leq m \leq 12\}$  and  $H = \{x: x \text{ is a square}\}$

(i) List all elements of sets G and H in tabular form Solution:

(i) Tabular form:

$$U = \{1,2,3,4, \dots, 150\}$$

$$G = \{1,2,4,8,16,32,64,128\}$$

$$H = \{1,4,9,16,25,36,49,64,81,100,121,144\}$$

(ii) Find  $G \cup H$

Solution

$$G \cup H = \{1,2,4,8,16,32,64,128\} \cup \{1,4,9,16,25,36,49,64,81,100,121,144\}$$

$$G \cup H = \{1,2,4,8,9,16,25,32,36,49,64,81,100,121,128,144\}$$

(iii) Find  $G \cap H$

Solution:

$$G \cap H = \{1,2,4,8,16,32,64,128\} \cap \{1,4,9,16,25,36,49,64,81,100,121,144\}$$

$$G \cap H = \{1,4,16,64\}$$

Q. 3 Consider the sets  $P = \{x : x \text{ is a prime number and } 0 < x < 20\}$  and  $Q = \{x : x \text{ is a divisor of } 210 \text{ and } 0 < x < 20\}$

Solution:



$P = \{2,3,5,7,11,13,17,19\}$   
 $Q = \{1,2,3,5,6,7,10,14,15\}$

(i) Find  $P \cap Q$

Solution:

$$P \cap Q = \{2,3,5,7,11,13,17,19\} \cap \{1,2,3,5,6,7,10,14,15\}$$

$$P \cap Q = \{2,3,5,7\}$$

(ii) Find  $P \cup Q$

Solution:

$$P \cup Q = \{2,3,5,7,11,13,17,19\} \cup \{1,2,3,5,6,7,10,14,15\}$$

$$P \cup Q = \{1,2,3,5,6,7,10,11,13,14,15,17,19\}$$

Q. 4 Verify the commutative properties of union and intersection for the following pairs of sets:

Solution:

(i)  $A = \{1,2,3,4,5\}$ ,  $B = \{4,6,8,10\}$

Solution

(a) Commutative property of union

$$A \cup B = B \cup A$$

$$\text{L.H.S} = A \cup B$$

$$= \{1,2,3,4,5\} \cup \{4,6,8,10\}$$

$$= \{1,2,3,4,5,6,8,10\}$$

$$\text{R.H.S} = B \cup A$$

$$= \{4,6,8,10\} \cup \{1,2,3,4,5\}$$

$$= \{1,2,3,4,5,6,8,10\}$$

From (i) and (ii), L.H.S = R.H.S.

Hence,  $A \cup B = B \cup A$

(b) Commutative property of intersection

$$A \cap B = B \cap A$$

$$\text{L.H. S} = A \cap B$$

$$\text{L.H.S} = \{1,2,3,4,5\} \cap \{4,6,8,10\}$$

$$= \{4\}$$

$$\text{Now, R.H.S} = B \cap A$$

$$= \{4,6,8,10\} \cap \{1,2,3,4,5\}$$

$$= \{4\} \text{ (ii)}$$

From (i) and (ii) L.H.S = R.H.S

Hence  $(A \cap B) = (B \cap A)$

(ii) N, Z

Solution

For N, Z We know that

$$N \subset Z$$

(a) Commutative property of union:

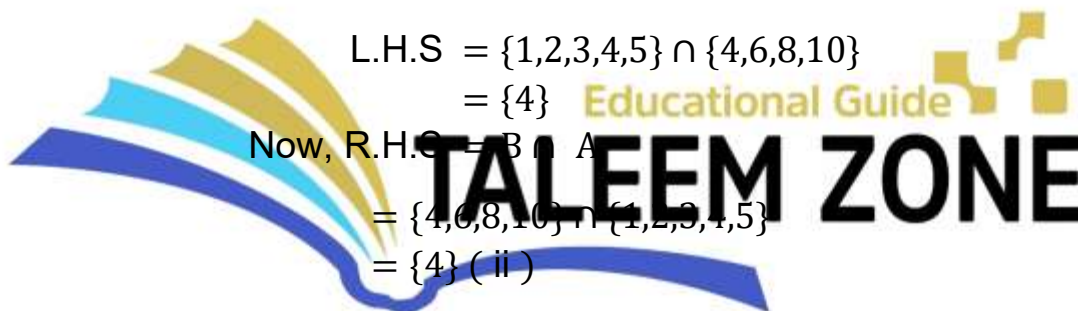
$$N \cup Z = Z \cup N$$

$$\text{L.H.S} = N \cup Z = Z$$

$$\text{R.H.S} = Z \cup N = Z$$

From (i) and (ii) L.H.S = R.H.S

(b) Commutative property of intersection:



$$\begin{aligned}
 N \cap Z &= Z \cap N \\
 L \cdot H \cdot S &= N \cap Z = N \\
 R \cdot H \cdot S &= Z \cap N = N
 \end{aligned}$$

From (i), & (ii) L.H.S = R.H.S

$$(iii) A = \{x \mid x \in \mathbb{R} \wedge x \geq 0\}, B = \mathbb{R}$$

Solution

$$\begin{aligned}
 A &= \{x \mid x \in \mathbb{R} \wedge x \geq 0\}, B = \mathbb{R} \\
 \Rightarrow A &\subset B
 \end{aligned}$$

(a) Commutative property of union:

$$A \cup B = B \cup A$$

$$L \cdot H \cdot S = A \cup B = B$$

$$R \cdot H \cdot S = B \cup A = B$$

From (i) and (ii) L.H.S = R.H.S

(b) Commutative property of intersection:

$$\begin{aligned}
 A \cap B &= B \cap A \\
 \text{L.H.S} &= A \cap B = A \\
 \text{R.H.S} &= B \cap A = A
 \end{aligned}$$

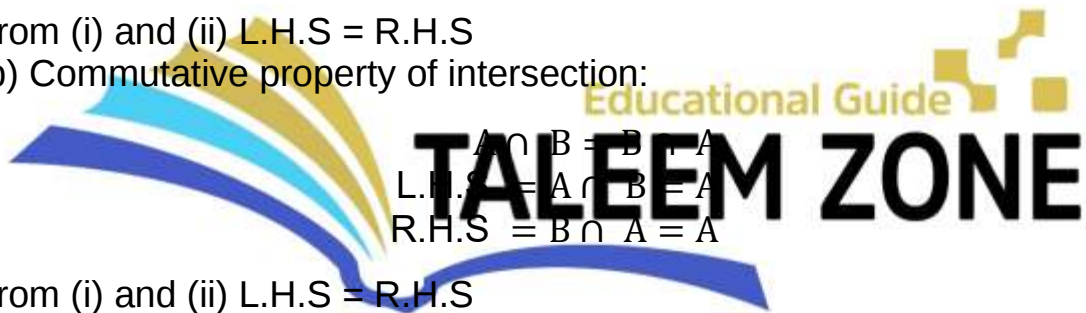
From (i) and (ii) L.H.S = R.H.S

Q. 5

$$\begin{aligned}
 \text{Let } U &= \{a, b, c, d, e, f, g, h, i, j\} \\
 A &= \{a, b, c, d, g, h\}, B = \{c, d, e, f, j\}
 \end{aligned}$$

Verify De Morgan's Laws for these sets, Draw Venn diagram.

Solution:



$$U = \{a, b, c, d, e, f, g, h, i, j\}$$

$$A = \{a, b, c, d, g, h\}$$

$$B = \{c, d, e, f, j\}$$

$$(i) \text{ De Morgan's law: } (A \cup B)' = A' \cap B'$$

$$\text{L.H.S} = (A \cup B)'$$

$$(A \cup B) = \{a, b, c, d, g, h\} \cup \{c, d, e, f, j\}$$

$$(A \cup B) = \{a, b, c, d, e, f, g, h, j\}$$

$$\text{Now, } (A \cup B)' = U - (A \cup B)$$

$$= \{a, b, c, d, e, f, g, h, i, j\} - \{a, b, c, d, e, f, g, h, j\}$$

$$= \{i\}$$

$$\text{Now, R.H.S} = A' \cap B'$$

$$A' = U - A$$

$$A' = \{a, b, c, d, e, f, g, h, i, j\} - \{a, b, c, d, g, h\}$$

$$A' = \{e, f, i, j\}$$

$$\text{Now, } B' = U - B$$

$$B' = \{a, b, c, d, e, f, g, h, i, j\} - \{c, d, e, f, j\}$$

$$B' = \{a, b, g, h, i\}$$

$$\text{Now, R.H.S} = A' \cap B'$$

$$= \{e, f, i, j\} \cap \{a, b, g, h, i\}$$

$$= \{i\} \quad (ii)$$

From (i) and (ii)

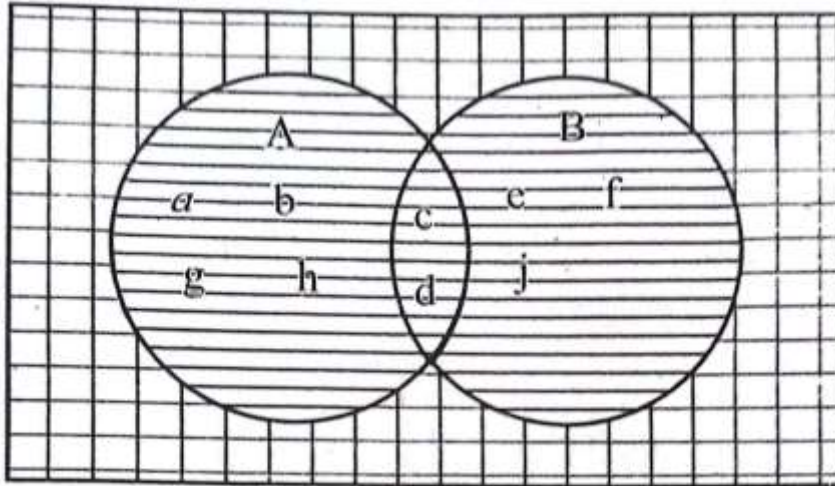
$$\text{L.H.S} = \text{R.H.S}$$

$$(A \cup B)' = A' \cap B'$$

Verification by Venn Diagram:

Since,  $A \cap B = \{c, d\} \neq \phi$ , So,  $A$  and  $B$  are overlapping sets.

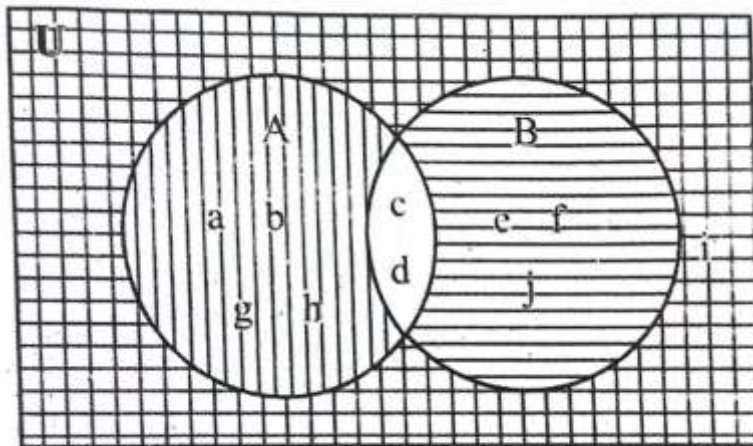
$$\text{L.H.S} = (A \cup B)'$$



**Fig. 1**

$(A \cup B)$  = region of horizontal line segments  $(A \cup B)'$  = region of squares.

$$\text{R.H.S} = A' \cap B'$$



**Fig. 2**

$A' \cap B'$  = Region of squares.

From Fig. 1 and Fig. 2,

Region showing  $(A \cup B)'$  and  $A' \cap B'$  are same which prove that  $(A \cup B)' = A' \cap B'$

(ii) De-Morgan's law:  $(A \cap B)' = A' \cup B'$

$$\text{L.H.S} = (A \cap B)'$$

$$(A \cap B) = \{a, b, c, d, g, h\} \cap \{c, d, e, f, j\}$$

$$(A \cap B) = \{c, d, \}$$

Now,

$$\begin{aligned}(A \cap B)' &= U - (A \cap B) \\ &= \{a, b, c, d, e, f, g, h, i, j\} - \{c, d\} \\ &= \{a, b, e, f, g, h, i, j\}\end{aligned}$$

$$\text{R.H.S} = A' \cup B'$$

$$A' = U - A$$

$$A' = \{a, b, c, d, e, f, g, h, i, j\} - \{a, b, c, d, g, h\}$$

$$A' = \{e, f, i, j\}$$

$$\text{Now, } B' = U - B$$

$$B' = \{a, b, c, d, e, f, g, h, i, j\} - \{c, d, c, f, i\}$$

$$B' = \{a, b, g, h, j\}$$

$$\text{Now, R.H.S} = A' \cup B'$$

$$= \{e, f, i, j\} \cup \{a, b, g, h, j\}$$

$$= \{a, b, e, f, g, h, i, j\}$$

From (i) and (ii), L.H.S = R.H.S

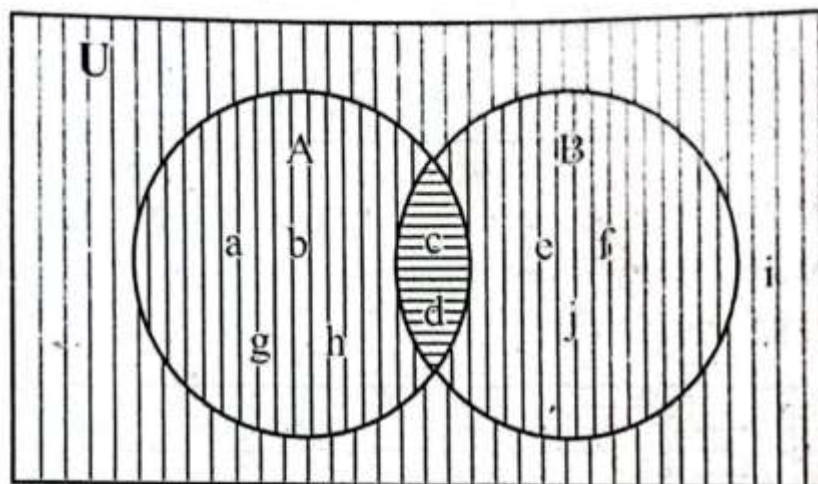
$$(A \cap B)' = A' \cup B'$$

Hence proved

Verification by Venn diagram

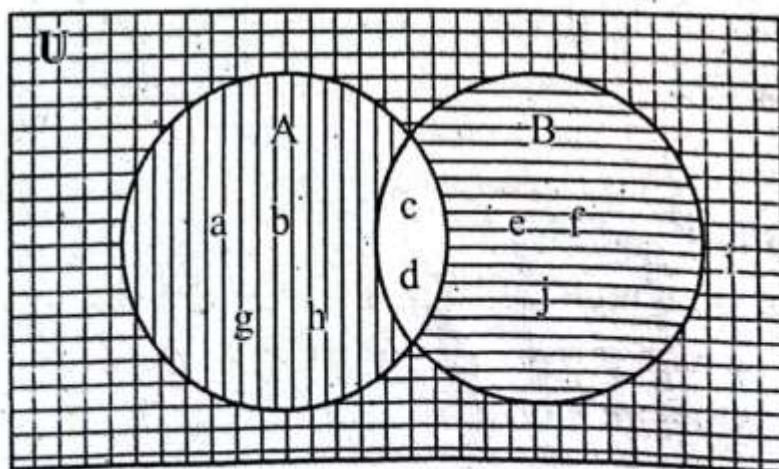
$$\text{L.H.S} = (A \cap B)'$$





**Fig. 1**

$A \cap B$  = Region of horizontal line segments  
 $(A \cap B)'$  = Regions of vertical line segments.  
 R.H.S =  $A' \cup B'$



**Fig. 2**

$A' \cup B'$  = Regions of squares, horizontal and vertical line segments.  
 From Fig. 1 and fig. 2, regions showing  $(A \cap B)'$  and  $A' \cup B'$  are same  
 which prove that  $(A \cap B)' = A' \cup B'$ .

Q. 6 If  $U = \{1, 2, 3, \dots, 20\}$  and  $A = \{1, 3, 5, \dots, 19\}$ , verify the following:  
 Solution:

$$U = \{1, 2, 3, 4, \dots, 20\}$$

$$A = \{1, 3, 5, \dots, 19\}$$

$$(i) A \cup A' = U$$

Solution:

$$A \cup A' = U$$

$$U = \{1,2,3,4, \dots, 20\}$$

$$A' = U - A$$

$$A' = \{1,2,3,4, \dots, 20\} - \{1,3,5, \dots, 19\}$$

$$A' = \{2,4,6,8, \dots, 20\}$$

Now,

$$A \cup A' = \{1,3,5, \dots, 19\} \cup \{2,4,6,8, \dots, 20\}$$

$$A \cup A' = \{1,2,3,4,5, \dots, 20\} \quad (ii)$$

From (i) and (ii)

$$A \cup A' = U \text{ (Hence proved)}$$

$$(ii) A \cap U = A$$

Solution:

$$A \cap U = A$$

Proving  $A \cap U = A$  :

$$A = \{1,3,5, \dots, 19\}$$

Now,

$$A \cap U = \{1,3,5, \dots, 19\} \cap \{1,2,3,4, 5 \dots 20\}$$

$$A \cap U = \{1,3,5, \dots, 19\} \quad (ii)$$

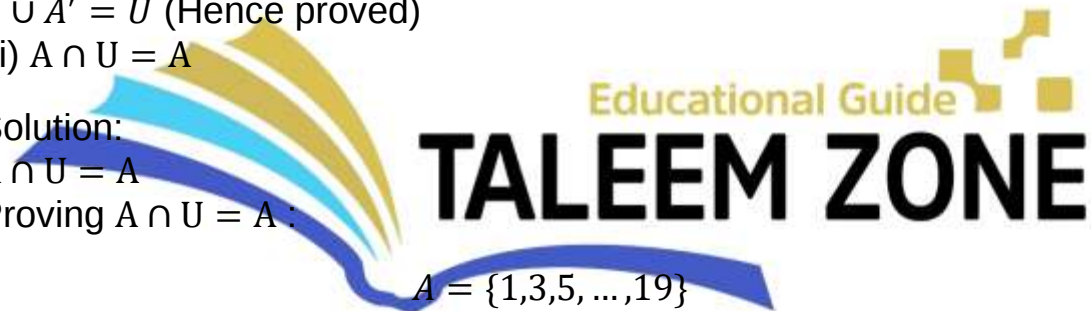
From (i) and (ii)

$$A \cap U = A$$

Hence proved

$$(iii) A \cap A' = \phi :$$

Solution:



$$\begin{aligned}
 A \cap A' &= \phi \\
 A &= \{1,3,5, \dots, 19\} \\
 U &= \{1,2,3,4,5, \dots, 20\} \\
 A' &= U - A \\
 A' &= \{1,2,3,4,5, \dots, 20\} - \{1,3,5, \dots, 9\}
 \end{aligned}$$

$$A' = \{2,4,6,8, \dots, 20\}$$

Now,

$$A \cap A' = \{1,3,5, \dots, 19\} \cap \{2,4,6,8, \dots, 20\}$$

$$A \cap A' = \{\}$$

$A \cap A' = \phi$  Hence proved

Q. 7 In a class of 55 students, 34 like to play cricket and 30 like to play hockey. Also each students likes to play at least one of the two games. How many students like to play both games?

Solution :

Let  $A$  and  $B$  are two sets showing the students playing cricket and hockey respectively.

$$\text{Total student} = n(A \cup B) = 55$$

$$34 \text{ like to play cricket} = n(A) = 34$$

$$30 \text{ like to play hockey} = n(B) = 30$$

Let  $x$  students like to play both games,  $n(A \cap B) = x$

Using principle of inclusion and exclusion for two sets.

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$55 = 34 + 30 - x$$

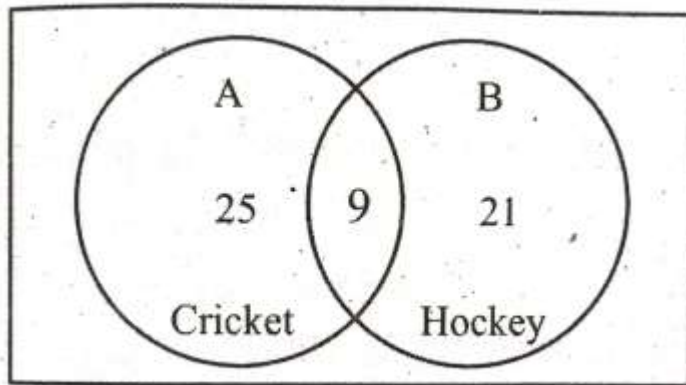
$$55 = 64 - x$$

$$\Rightarrow x = 64 - 55$$

$$x = 9$$

Thus 9 students like to play both games.

Venn diagram:



Q. 8 In a group of 500 employees, 250 can speak Urdu, 150 can speak English, 50 can speak Punjabi, 40 can speak both Urdu and English, 30 can speak both English and Punjabi and 10 can speak both Urdu and Punjabi. How many can speak all three languages?  
Solution:

Let A, B and C be three sets representing the employees who can speak Urdu, English and Punjabi respectively.

Total employees = 500

250 can speak Urdu, i.e.  $n(A) = 250$

150 can speak English, i.e.  $n(B) = 150$

50 can speak Punjabi, i.e.  $n(C) = 50$

40 can speak Urdu and English i.e.

$$n(A \cap B) = 40$$

30 can speak English and Punjabi, i.e.

$$n(B \cap C) = 30$$

10 can speak Urdu, and Punjabi, i.e.

$$n(A \cap C) = 10$$

Let  $x$  can speak all the three languages,

$$n(A \cap B \cap C) = x$$

Using principle of inclusion and exclusion for three sets.

$$\begin{aligned}
n(A \cup B \cup C) &= n(A) + n(B) + n(C) - n(A \cap B) - \\
&n(B \cap C) - n(A \cap C) + n(A \cap B \cap C) \\
500 &= 250 + 150 + 50 - 40 - 30 - 10 + x \\
500 &= 450 - 80 + x \\
500 &= 370 + x \\
500 - 370 &= x \\
130 &= x \\
\Rightarrow x &= 130
\end{aligned}$$

Thus 130 employees can speak all the three languages.

Q.9 In sports events, 19 people wear blue shirts, 15 wear green shirts, 3 wear blue and green shirts, 4 wear a cap and blue shirts, and 2 wear a cap and green shirts. The Total number of people with either a blue or green shirt or cap is 25. How many people are wearing caps?

Solution: (Correction)

Let A, B and C be three showing people who wear green, blue shirts and cap respectively.

15 wear green shirts =  $n(A) = 15$

19 wear blue shirts =  $n(B) = 19$

Let x wear caps =  $n(C) = x$

3 wear blue and green shirts =  $n(A \cap B) = 3$

4 wear cap and blue shirts =  $n(B \cap C) = 4$

2 wear cap and green shirts =  $n(A \cap C) = 2$

34 Wear either blue, green or cap

$n(A \cup B \cup C) = 34$

According to information no one wear all the three items, so  $n(A \cap B \cap C) = 0$

Using principle of inclusion and exclusion:

$$\begin{aligned}
n(A \cup B \cup C) &= n(A) + n(B) + n(C) - n(A \cap B) - n \\
&(B \cap C) - n(A \cap C) + n(A \cap B \cap C) \\
34 &= 15 + 19 + x - 3 - 4 - 2 + 0 \\
34 &= 34 + x - 9 \\
34 - 34 + 9 &= x \\
9 &= x \\
\Rightarrow x &= 9
\end{aligned}$$

Thus 9 people are wearing cap.

Q. 10 In a training session, participants have laptop, 11 have tablet 9 have laptops and tablets, 6 have laptop and books and 4 have both tablets and books. Eight participants have all three items. The total number of participants with laptops, tablets, or books is 35. How many participants have books?

Solution:

Let A, B and C be the sets representing participants having laptops, tablets books respectively.

17 participants have laptops,  $n(A) = 17$

11 participants have tablets,  $n(B) = 11$

$x$  participants have books,  $n(C) = x$

9 have laptops and tablets,  $n(A \cap B) = 9$

6 have laptops and books,  $n(A \cap C) = 6$

4 have tablets and book,  $n(B \cap C) = 4$

8 have all these items =  $n(A \cap B \cap C) = 8$

35 have laptops, tablets or books =  $n(A \cup B \cup C) = 35$

Using principle of inclusion and exclusion for three sets.

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

$$35 = 17 + 11 + x - 9 - 4 - 6 + 8$$

$$35 = 36 - 19 + x$$

$$35 = 17 + x$$

$$35 - 17 = x$$

$$18 = x$$

$$\Rightarrow x = 18$$

Thus 18 participants have books.

Q. 11 A shopping mall has 150 employees, labeled 1 to 150 representing the universal set U. The employees fall into the following categories:

- Set-A: 40 employees with a salary range of 30k – 45k, labeled from 50 to 89
- Set-B: 50 employees with a salary range of 50k – 80k, labeled from 101 to 150.

- Set-C: 60 employees with a salary range of 100k – 150k, labeled from 1 to 49 and 90 to 100 .
  - (a) Find  $(A' \cup B') \cap C$
  - (b) Find  $n\{A \cap (B^c \cap C^c)\}$
- Solution:
  - $U = \{1, 2, 3, 4, \dots, 150\}, n(U) = 150$
  - $A = \{50, 51, 52, \dots, 89\}, n(A) = 40$
  - $B = \{101, 102, 103, \dots, 150\}, n(B) = 50$
  - $C = \{1, 2, 3, 4, \dots, 49, 90, 91, 92, \dots, 100\}, n(C) = 60$
  - (a) Find  $(A' \cup B') \cap C$ 
    - $A' = U - A$
    - $A' = \{1, 2, 3, 4, \dots, 150\} - \{50, 51, 52, \dots, 89\}$
    - $A' = \{1, 2, 3, 4, \dots, 49, 90, 91, 92, 93, \dots, 150\}$
  - Now,  $B' = U - B$ 
    - $= \{1, 2, 3, 4, \dots, 150\} - \{101, 102, 103, \dots, 150\}$
    - $B' = \{1, 2, 3, 4, \dots, 100\}$
  - Now,
    - $A' \cup B' = \{1, 2, 3, 4, \dots, 49, 90, 91, 92, 93, \dots, 150\} \cup \{1, 2, 3, 4, \dots, 100\}$
    - $A' \cup B' = \{1, 2, 3, 4, \dots, 150\}$
  - Now,
    - $(A' \cup B') \cap C = \{1, 2, 3, 4, \dots, 150\} \cap \{1, 2, 3, 4, \dots, 49, 90, 91, 92, \dots, 100\}$
    - $(A' \cup B') \cap C = \{1, 2, 3, 4, \dots, 49, 90, 91, 93, \dots, 100\}$
  - (b) Find  $n\{A \cap (B^c \cap C^c)\}$ 
    - $B^c = U - B$
    - $B^c = \{1, 2, 3, 4, \dots, 150\} - \{101, 102, 103, \dots, 150\}$
    - $B^c = \{1, 2, 3, 4, \dots, 100\}$
  - Now,  $C^c = U - C$ 
    - $C^c = \{1, 2, 3, 4, \dots, 150\} - \{1, 2, 3, 4, \dots, 49, 90, 91, 92, \dots, 100\}$
    - $C^c = \{50, 51, 52, \dots, 89, 101, 102, 103, \dots, 150\}$
  - Now
    - $B^c \cap C^c = \{50, 51, 52, \dots, 89\} = A$
  - Now
    - $A \cap (B^c \cap C^c) = A \cap A$
    - $A \cap (B^c \cap C^c) = A$
    - $n\{A \cap (B^c \cap C^c)\} = n(A) = 40$

Q. 12 In a secondary school 125 students participate in at least one of the following sports: cricket, football, or hockey.

- 60 students play cricket.
  - 70 students play football.
  - 40 students play hockey.
  - 25 students play both cricket and foot ball.
  - 15 students play both football and hockey.
  - 10 students play both cricket and hockey
- (a) How many students play all three sports?  
 (b) Draw a Venn diagram showing the distribution of sports participation in all the games.

Solution:

Let A, B and C be the sets representing the students who play cricket, football and hockey respectively.

60 students play cricket =  $n(A) = 60$

70 students play football =  $n(B) = 70$

40 students play hockey =  $n(C) = 40$

25 students play both cricket and football =  $n(A \cap B) = 25$

15 students play both football and hockey =  $n(B \cap C) = 15$

10 students play both cricket and hockey =  $n(A \cap C) = 10$

(a) Let students playing all three games be  $n(A \cap B \cap C) = x$ ,  
 Using principle of inclusion and exclusion.

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B)$$

$$- (B \cap C) - n(C \cap A) + n(A \cap B \cap C)$$

$$125 = 60 + 70 + 40 - 25 - 15 - 10 + x$$

$$125 = 170 - 50 + x$$

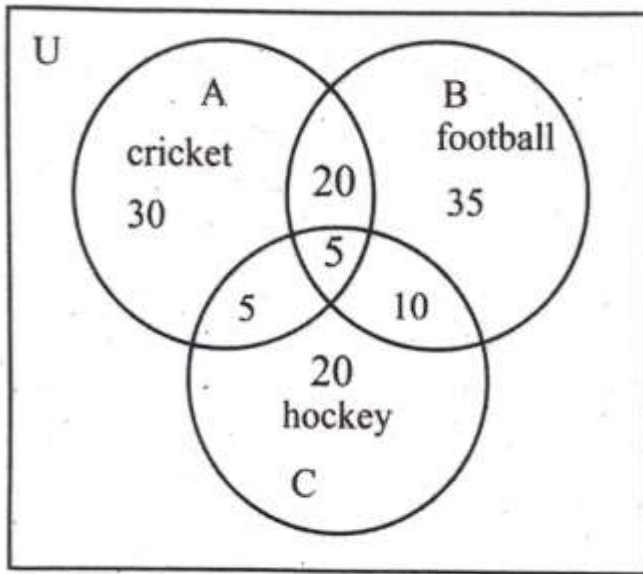
$$125 = 120 + x$$

$$125 - 120 = x$$

$$5 = x \Rightarrow x = 5$$

Thus 5 students play all the three games.

(b) Venn Diagram



Q. 13 A survey was conducted in which 130 people were asked about their favourite foods. The survey results showed the following information:

- 40 people said they like nihari
  - 65 people said they like biryani
  - 50 people said they like korma
  - 20 people said they liked nihari and biryani.
  - 35 people said they like biryani and korma
  - 27 people said they like nihari and korma.
  - 12 people said they like all three foods nihari, biryani and korma.
- (a) At least how many people like nihari, biryani or korma?  
 (b) How many people did not like nihari, biryani or korma?  
 (c) How many people like only one of the following foods: nihari, biryani or korma?  
 (d) Draw a Venn diagram.

**Solution**

Let A, B and C be the sets representing people who like nihari, biryani and korma respectively.

130 total people surveyed,  $n(U) = 130$

40 people liked nihari  $= n(A) = 40$

65 people liked biryani =  $n(B) = 65$

50 people liked korma =  $n(C) = 50$

20 people liked, nihari and biryani  $\Rightarrow n(A \cap B) = 20$

35 people liked biryani, and korma =  $n(B \cap C) = 35$

27 people liked nihari and korma =  $n(A \cap C) = 27$

12 people liked all three foods =  $n(A \cap B \cap C) = 12$

(a) Number of people who liked nihari, biryani or korma is  $n(A \cup B \cup C)$ .

Using principle of inclusion and exclusion.

$$\begin{aligned}n(A \cup B \cup C) &= n(A) + n(B) + n(C) - \\& n(A \cap B) - (B \cap C) - n(A \cap C) + n(A \cap B \cap C) \\&= 40 + 65 + 50 - 20 - 35 - 27 + 12 \\&= 167 - 82 \\&= 85\end{aligned}$$

Thus 85 people like atleast one of foods item.

(b) Number of people who did not like any food item.

$$n(A \cup B \cup C)' = n(U) - n(A \cup B \cup C)$$

$$= 120 - 85 = 45$$

Thus 45 people did not like any food item.

(c) People who like only nihari =

$$\begin{aligned}&= n(A) - n(A \cap B) - n(A \cap C) + n(A \cap B \cap C) \\&= 40 - 20 - 27 + 12 \\&= 52 - 47 = 5\end{aligned}$$

People who like only biryani:

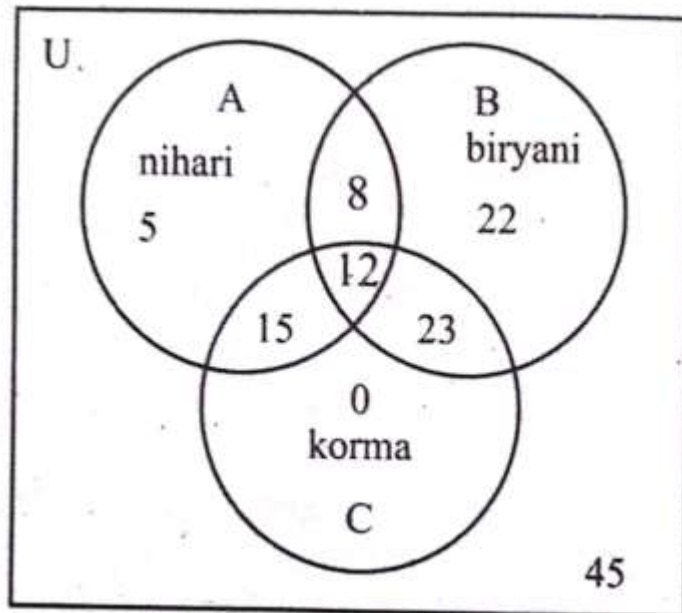
$$\begin{aligned}&= n(B) - n(A \cap B) - n(B \cap C) + n(A \cap B \cap C) \\&= 65 - 20 - 35 + 12 \\&= 77 - 55 \\&= 22\end{aligned}$$

People who like only korma:

$$\begin{aligned}&= n(C) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C) \\&= 50 - 27 - 35 + 12 \\&= 62 - 62 \\&= 0\end{aligned}$$

Thus number of people who like only one of the foods item =  $5 + 22 + 0 = 27$

(d) Venn Diagram



tional Guide

**TALEEM ZONE**