

EXERCISE 7.2

Solution of Triangles and Area

Complete Step-by-Step Worked Solutions

Sine Rule • Cosine Rule • Heron's Formula • Triangle Area

Prepared & Distributed by TaleemZone.com

KEY FORMULAS USED**Sine Rule:**

$$a / \sin \alpha = b / \sin \beta = c / \sin \gamma$$

Cosine Rule:

$$a^2 = b^2 + c^2 - 2bc \cos \alpha$$

$$b^2 = a^2 + c^2 - 2ac \cos \beta$$

$$c^2 = a^2 + b^2 - 2ab \cos \gamma$$

Angle Sum Property:

$$\alpha + \beta + \gamma = 180^\circ$$

Area Formula (SAS):

$$\Delta = \frac{1}{2} ab \sin \gamma = \frac{1}{2} ac \sin \beta = \frac{1}{2} bc \sin \alpha$$

Area Formula (SSS - Heron's Formula):

$$s = (a + b + c) / 2 \quad , \quad \Delta = \sqrt{[s(s-a)(s-b)(s-c)]}$$

QUESTION 1 — Solve the Triangle**(i) Given: $a = 6.1$ cm, $b = 8.4$ cm, $\alpha = 42^\circ$** **Step 1: Use Sine Rule to find β**

$$a / \sin \alpha = b / \sin \beta$$

$$6.1 / \sin 42^\circ = 8.4 / \sin \beta$$

$$\sin \beta = (8.4 \times \sin 42^\circ) / 6.1$$

$$\sin 42^\circ = 0.6691$$

$$\sin \beta = (8.4 \times 0.6691) / 6.1 = 5.6204 / 6.1 = 0.9214$$

$$\beta = \sin^{-1}(0.9214) = 67.15^\circ$$

Step 2: Find γ

$$\gamma = 180^\circ - 42^\circ - 67.15^\circ = 70.85^\circ$$

Step 3: Find c using Sine Rule

$$c / \sin \gamma = a / \sin \alpha$$

$$c = (a \sin \gamma) / \sin \alpha = (6.1 \times \sin 70.85^\circ) / \sin 42^\circ$$

$$\sin 70.85^\circ = 0.9450$$

$$c = (6.1 \times 0.9450) / 0.6691 = 5.7645 / 0.6691 = 8.62 \text{ cm}$$

Answer: $\beta = 67.15^\circ$, $\gamma = 70.85^\circ$, $c = 8.62$ cm**(ii) Given: $a = 12.2$ cm, $c = 15.8$ cm, $\gamma = 50^\circ$** **Step 1: Use Sine Rule**

$$a / \sin \alpha = c / \sin \gamma$$

$$12.2 / \sin \alpha = 15.8 / \sin 50^\circ$$

$$\sin \alpha = (12.2 \times \sin 50^\circ) / 15.8$$

$$\sin 50^\circ = 0.7660$$

$$\sin \alpha = (12.2 \times 0.7660) / 15.8 = 9.3452 / 15.8 = 0.5915$$

$$\alpha = \sin^{-1}(0.5915) = 36.27^\circ$$

Step 2: Find β

$$\beta = 180^\circ - 36.27^\circ - 50^\circ = 93.73^\circ$$

Step 3: Find b

$$b = (a \sin \beta) / \sin \alpha = (12.2 \times \sin 93.73^\circ) / \sin 36.27^\circ$$

$$\sin 93.73^\circ = 0.9982, \quad \sin 36.27^\circ = 0.5915$$

$$b = (12.2 \times 0.9982) / 0.5915 = 12.1780 / 0.5915 = 20.59 \text{ cm}$$

Answer: $\alpha = 36.27^\circ$, $\beta = 93.73^\circ$, $b = 20.59$ cm**(iii) Given: $b = 5.2$ cm, $c = 5$ cm, $\gamma = 48^\circ$** **Step 1: Use Sine Rule**

$$b / \sin \beta = c / \sin \gamma$$

$$5.2 / \sin \beta = 5 / \sin 48^\circ$$

$$\sin \beta = (5.2 \times \sin 48^\circ) / 5$$

$$\sin 48^\circ = 0.7431$$

$$\sin \beta = (5.2 \times 0.7431) / 5 = 3.8641 / 5 = 0.7728$$

$$\beta = \sin^{-1}(0.7728) = 50.61^\circ$$

Step 2: Find α

$$\alpha = 180^\circ - 50.61^\circ - 48^\circ = 81.39^\circ$$

Step 3: Find a

$$a = (c \sin \alpha) / \sin \gamma = (5 \times \sin 81.39^\circ) / \sin 48^\circ$$

$$\sin 81.39^\circ = 0.9889$$

$$a = (5 \times 0.9889) / 0.7431 = 4.9445 / 0.7431 = 6.65 \text{ cm}$$

Answer: $\alpha = 81.39^\circ$, $\beta = 50.61^\circ$, $a = 6.65 \text{ cm}$

(iv) Given: $b = 4.8 \text{ cm}$, $a = 4 \text{ cm}$, $\beta = 71^\circ$

Step 1: Use Sine Rule

$$a / \sin \alpha = b / \sin \beta$$

$$4 / \sin \alpha = 4.8 / \sin 71^\circ$$

$$\sin \alpha = (4 \times \sin 71^\circ) / 4.8$$

$$\sin 71^\circ = 0.9455$$

$$\sin \alpha = (4 \times 0.9455) / 4.8 = 3.7820 / 4.8 = 0.7879$$

$$\alpha = \sin^{-1}(0.7879) = 51.98^\circ$$

Step 2: Find γ

$$\gamma = 180^\circ - 51.98^\circ - 71^\circ = 57.02^\circ$$

Step 3: Find c

$$c = (b \sin \gamma) / \sin \beta = (4.8 \times \sin 57.02^\circ) / \sin 71^\circ$$

$$\sin 57.02^\circ = 0.8389$$

$$c = (4.8 \times 0.8389) / 0.9455 = 4.0267 / 0.9455 = 4.26 \text{ cm}$$

Answer: $\alpha = 51.98^\circ$, $\gamma = 57.02^\circ$, $c = 4.26 \text{ cm}$

(v) Given: $\beta = 70^\circ$, $b = 8 \text{ cm}$, $\alpha = 100^\circ$

Step 1: Find γ

$$\gamma = 180^\circ - 100^\circ - 70^\circ = 10^\circ$$

Step 2: Find a using Sine Rule

$$a = (b \sin \alpha) / \sin \beta = (8 \times \sin 100^\circ) / \sin 70^\circ$$

$$\sin 100^\circ = 0.9848, \quad \sin 70^\circ = 0.9397$$

$$a = (8 \times 0.9848) / 0.9397 = 7.8784 / 0.9397 = 8.38 \text{ cm}$$

Step 3: Find c

$$c = (b \sin \gamma) / \sin \beta = (8 \times \sin 10^\circ) / \sin 70^\circ$$

$$\sin 10^\circ = 0.17365$$

$$c = (8 \times 0.17365) / 0.9397 = 1.3892 / 0.9397 = 1.48 \text{ cm}$$

Answer: $\gamma = 10^\circ$, $a = 8.38 \text{ cm}$, $c = 1.48 \text{ cm}$

(vi) Given: $a = 6$ cm, $\alpha = 55^\circ$, $\gamma = 60^\circ$

Step 1: Find β

$$\beta = 180^\circ - 55^\circ - 60^\circ = 65^\circ$$

Step 2: Find b

$$b = (a \sin \beta) / \sin \alpha = (6 \times \sin 65^\circ) / \sin 55^\circ$$

$$\sin 65^\circ = 0.9063, \quad \sin 55^\circ = 0.8192$$

$$b = (6 \times 0.9063) / 0.8192 = 5.4378 / 0.8192 = 6.64 \text{ cm}$$

Step 3: Find c

$$c = (a \sin \gamma) / \sin \alpha = (6 \times \sin 60^\circ) / \sin 55^\circ$$

$$\sin 60^\circ = 0.8660$$

$$c = (6 \times 0.8660) / 0.8192 = 5.196 / 0.8192 = 6.34 \text{ cm}$$

Answer: $\beta = 65^\circ$, $b = 6.64$ cm, $c = 6.34$ cm

(vii) Given: $c = 7$ cm, $\beta = 34^\circ$, $\gamma = 64^\circ$

Step 1: Find α

$$\alpha = 180^\circ - 34^\circ - 64^\circ = 82^\circ$$

Step 2: Find a

$$a = (c \sin \alpha) / \sin \gamma = (7 \times \sin 82^\circ) / \sin 64^\circ$$

$$\sin 82^\circ = 0.9903, \quad \sin 64^\circ = 0.8988$$

$$a = (7 \times 0.9903) / 0.8988 = 6.9321 / 0.8988 = 7.71 \text{ cm}$$

Step 3: Find b

$$b = (c \sin \beta) / \sin \gamma = (7 \times \sin 34^\circ) / \sin 64^\circ$$

$$\sin 34^\circ = 0.5592$$

$$b = (7 \times 0.5592) / 0.8988 = 3.9144 / 0.8988 = 4.35 \text{ cm}$$

Answer: $\alpha = 82^\circ$, $a = 7.71$ cm, $b = 4.35$ cm

(viii) Given: $b = 12$ cm, $\alpha = 92^\circ$, $\beta = 77^\circ$

Step 1: Find γ

$$\gamma = 180^\circ - 92^\circ - 77^\circ = 11^\circ$$

Step 2: Find a

$$a = (b \sin \alpha) / \sin \beta = (12 \times \sin 92^\circ) / \sin 77^\circ$$

$$\sin 92^\circ = 0.9994, \quad \sin 77^\circ = 0.9744$$

$$a = (12 \times 0.9994) / 0.9744 = 11.9928 / 0.9744 = 12.31 \text{ cm}$$

Step 3: Find c

$$c = (b \sin \gamma) / \sin \beta = (12 \times \sin 11^\circ) / \sin 77^\circ$$

$$\sin 11^\circ = 0.1908$$

$$c = (12 \times 0.1908) / 0.9744 = 2.2896 / 0.9744 = 2.35 \text{ cm}$$

Answer: $\gamma = 11^\circ$, $a = 12.31$ cm, $c = 2.35$ cm

QUESTION 2 — Calculate Area of Each Triangle**(i) Given: $a = 7$ cm, $b = 8$ cm, $\gamma = 38^\circ$**

$$\Delta = \frac{1}{2} ab \sin \gamma$$

$$\Delta = \frac{1}{2} (7)(8) \sin 38^\circ$$

$$\sin 38^\circ = 0.6157$$

$$\Delta = 28 \times 0.6157 = 17.24 \text{ cm}^2$$

Answer: 17.24 cm²**(ii) Given: $a = 11$ cm, $c = 14$ cm, $\beta = 51^\circ$**

$$\Delta = \frac{1}{2} ac \sin \beta$$

$$\Delta = \frac{1}{2} (11)(14) \sin 51^\circ$$

$$\sin 51^\circ = 0.7771$$

$$\Delta = 77 \times 0.7771 = 59.84 \text{ cm}^2$$

Answer: 59.84 cm²**(iii) Given: $b = 3$ cm, $c = 9$ cm, $\alpha = 78^\circ$**

$$\Delta = \frac{1}{2} bc \sin \alpha$$

$$\Delta = \frac{1}{2} (3)(9) \sin 78^\circ$$

$$\sin 78^\circ = 0.9781$$

$$\Delta = 13.5 \times 0.9781 = 13.20 \text{ cm}^2$$

Answer: 13.20 cm²**(iv) Given: $a = 10$ cm, $\alpha = 62^\circ$, $\beta = 69^\circ$**

$$\text{Step 1: } \gamma = 180^\circ - 62^\circ - 69^\circ = 49^\circ$$

$$\text{Step 2: } b = (a \sin \beta) / \sin \alpha = (10 \times \sin 69^\circ) / \sin 62^\circ$$

$$\sin 69^\circ = 0.9336, \quad \sin 62^\circ = 0.8829$$

$$b = (10 \times 0.9336) / 0.8829 = 9.336 / 0.8829 = 10.57 \text{ cm}$$

$$\text{Step 3: } \Delta = \frac{1}{2} ab \sin \gamma = \frac{1}{2} (10)(10.57) \sin 49^\circ$$

$$\sin 49^\circ = 0.7547$$

$$\Delta = 52.85 \times 0.7547 = 39.89 \text{ cm}^2$$

Answer: 39.89 cm²**(v) Given: $c = 4$ cm, $\beta = 36^\circ$, $\gamma = 80^\circ$**

$$\text{Step 1: } \alpha = 180^\circ - 36^\circ - 80^\circ = 64^\circ$$

$$\text{Step 2: } a = (c \sin \alpha) / \sin \gamma = (4 \times \sin 64^\circ) / \sin 80^\circ$$

$$\sin 64^\circ = 0.8988, \quad \sin 80^\circ = 0.9848$$

$$a = (4 \times 0.8988)/0.9848 = 3.5952/0.9848 = 3.65 \text{ cm}$$

$$\text{Step 3: } \Delta = \frac{1}{2} ac \sin \beta = \frac{1}{2} (3.65)(4) \sin 36^\circ$$

$$\sin 36^\circ = 0.5878$$

$$\Delta = 7.3 \times 0.5878 = 4.29 \text{ cm}^2$$

Answer: 4.29 cm²

(vi) Given: c = 6.6 cm, $\alpha = 23^\circ$, $\gamma = 89^\circ$

$$\text{Step 1: } \beta = 180^\circ - 23^\circ - 89^\circ = 68^\circ$$

$$\text{Step 2: } a = (c \sin \alpha)/\sin \gamma = (6.6 \times \sin 23^\circ)/\sin 89^\circ$$

$$\sin 23^\circ = 0.3907, \quad \sin 89^\circ = 0.9998$$

$$a = (6.6 \times 0.3907)/0.9998 = 2.5786/0.9998 = 2.58 \text{ cm}$$

$$\text{Step 3: } \Delta = \frac{1}{2} ac \sin \beta = \frac{1}{2} (2.58)(6.6) \sin 68^\circ$$

$$\sin 68^\circ = 0.9272$$

$$\Delta = 8.514 \times 0.9272 = 7.89 \text{ cm}^2$$

Answer: 7.89 cm²

(vii) Given: a = 5.3 cm, b = 4.7 cm, c = 8.2 cm (SSS)

$$\text{Step 1: } s = (a+b+c)/2 = (5.3+4.7+8.2)/2 = 18.2/2 = 9.1 \text{ cm}$$

$$\text{Step 2: } \Delta = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\Delta = \sqrt{9.1(9.1-5.3)(9.1-4.7)(9.1-8.2)}$$

$$\Delta = \sqrt{9.1(3.8)(4.4)(0.9)}$$

$$\Delta = \sqrt{9.1 \times 3.8 \times 4.4 \times 0.9}$$

$$\Delta = \sqrt{136.9368} = 11.70 \text{ cm}^2$$

Answer: 11.70 cm²

(viii) Given: a = 6.12 cm, b = 8.34 cm, c = 7.12 cm (SSS)

$$\text{Step 1: } s = (6.12+8.34+7.12)/2 = 21.58/2 = 10.79 \text{ cm}$$

$$\text{Step 2: } \Delta = \sqrt{10.79(10.79-6.12)(10.79-8.34)(10.79-7.12)}$$

$$\Delta = \sqrt{10.79(4.67)(2.45)(3.67)}$$

$$\Delta = \sqrt{10.79 \times 4.67 \times 2.45 \times 3.67}$$

$$\Delta = \sqrt{453.45} = 21.29 \text{ cm}^2$$

Answer: 21.29 cm²

SUMMARY TABLE

Question 1: Triangle Solutions

Part	Given	α	β	γ	a	b	c
(i)	$a=6.1, b=8.4, \alpha=42^\circ$	42°	67.15°	70.85°	6.1	8.4	8.62
(ii)	$a=12.2, c=15.8, \gamma=50^\circ$	36.27°	93.73°	50°	12.2	20.59	15.8
(iii)	$b=5.2, c=5, \gamma=48^\circ$	81.39°	50.61°	48°	6.65	5.2	5
(iv)	$b=4.8, a=4, \beta=71^\circ$	51.98°	71°	57.02°	4	4.8	4.26
(v)	$\beta=70^\circ, b=8, \alpha=100^\circ$	100°	70°	10°	8.38	8	1.48
(vi)	$a=6, \alpha=55^\circ, \gamma=60^\circ$	55°	65°	60°	6	6.64	6.34
(vii)	$c=7, \beta=34^\circ, \gamma=64^\circ$	82°	34°	64°	7.71	4.35	7
(viii)	$b=12, \alpha=92^\circ, \beta=77^\circ$	92°	77°	11°	12.31	12	2.35

Question 2: Areas

Part	Given	Area
(i)	$a=7, b=8, \gamma=38^\circ$	17.24 cm^2
(ii)	$a=11, c=14, \beta=51^\circ$	59.84 cm^2
(iii)	$b=3, c=9, \alpha=78^\circ$	13.20 cm^2
(iv)	$a=10, \alpha=62^\circ, \beta=69^\circ$	39.89 cm^2
(v)	$c=4, \beta=36^\circ, \gamma=80^\circ$	4.29 cm^2
(vi)	$c=6.6, \alpha=23^\circ, \gamma=89^\circ$	7.89 cm^2
(vii)	$a=5.3, b=4.7, c=8.2$	11.70 cm^2
(viii)	$a=6.12, b=8.34, c=7.12$	21.29 cm^2

This resource is prepared and distributed by TaleemZone.com for educational use. Visit www.taleemzone.com for more free notes, past papers and solved exercises.