EXERCISE 7.3

3D Geometry and Trigonometry Applications

Prisms • Pyramids • Angles of Elevation • Cosine Rule

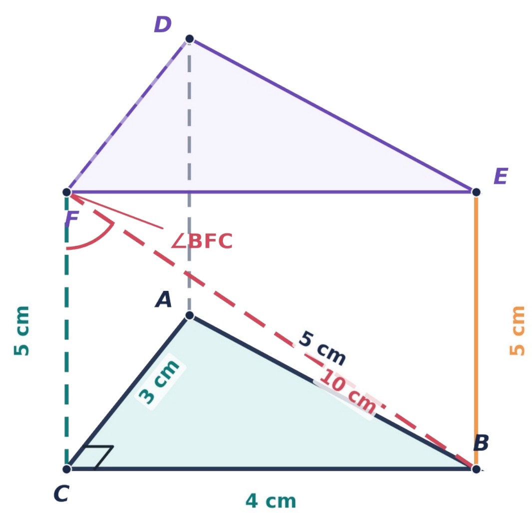
Step-by-step worked solutions with colorful diagrams for every question.

Question 1: Triangular Prism

In the triangular prism, find:

(i) the length CF.
(ii) the length BF.
(iii) the angle BFC, correct to one decimal place.

Right Triangular Prism — Question 1



(Note: Based on the dimensions given: AB = 5 cm, BC = 4 cm, AC = 3 cm, and height AD = 5 cm, BE = 10 cm)

From the diagram, we have a right triangular prism with:

- Triangular base ABC with right angle at C
- AC = 3 cm, BC = 4 cm, AB = 5 cm (3-4-5 right triangle)
- Height of prism = 5 cm (AD = 5 cm, BE = 5 cm, CF = 5 cm)
- BF = 10 cm (given in diagram)

1(i) Find the length CF

Solution:

In a right triangular prism, all vertical edges are equal to the height of the prism.

From the diagram:

- CF is a vertical edge of the prism
- The height of the prism is given as 5 cm

CF = 5 cm

Answer: CF = 5 cm

1(ii) Find the length BF

Solution:

From the diagram, BF is given directly as 10 cm.

BF = 10 cm

Answer: BF = 10 cm

1(iii) Find the angle ∠BFC, correct to one decimal place

Solution:

Step 1: Identify triangle BFC:

- BF = 10 cm (from part ii)
- CF = 5 cm (from part i)
- BC = 4 cm (base of triangle)

Step 2: Note that triangle BFC is a right triangle because:

- CF is perpendicular to the base plane
- BC lies in the base plane
- Therefore, CF ⊥ BC, so ∠BCF = 90°

Step 3: In right triangle BFC:

- Hypotenuse = BF = 10 cm
- Opposite side to angle BFC = BC = 4 cm
- Adjacent side to angle BFC = CF = 5 cm

Step 4: Use trigonometric ratio:

sin(∠BFC) = Opposite / Hypotenuse = BC / BF = 4 / 10 = 0.4

OR

Using tangent:

tan(∠BFC) = Opposite / Adjacent = BC / CF = 4 / 5 = 0.8

Step 5: Find the angle:

∠BFC = $\tan^{-1}(0.8)$

∠BFC = 38.6598°

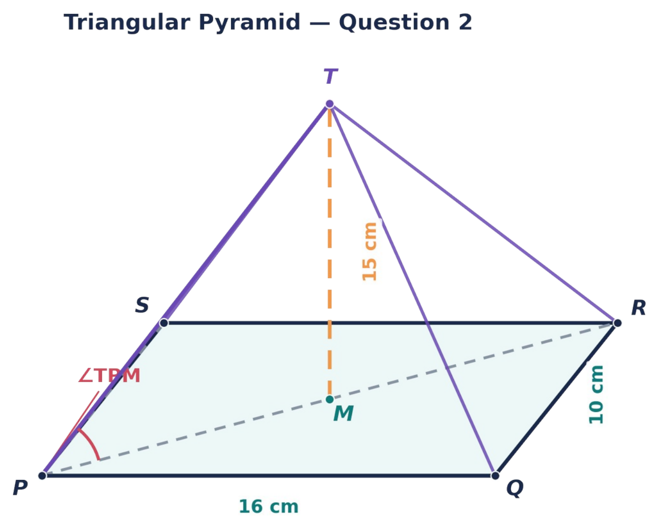
Step 6: Round to one decimal place:

∠BFC ≈ 38.7°

Answer: ∠BFC = 38.7°

Question 2: Triangular Pyramid (Rectangular Base)

The diagram shows a triangular pyramid with a horizontal rectangular base PQRS, in which PQ = 16 cm, QR = 10 cm. M is the midpoint of the line PR. The vertex, T, is vertically above M and MT = 15 cm. Calculate the size of the angle between TP and the base PQRS. Give your answer correct to 1 decimal place.



- PQRS is a horizontal rectangular base
- PQ = 16 cm, QR = 10 cm
- M is the midpoint of PR
- T is vertically above M
- MT = 15 cm (height of pyramid)

Solution:

Step 1: Understand the geometry.

In rectangle PQRS:

- PQ = 16 cm (length)
- QR = 10 cm (width)

PR is a diagonal of the rectangle.

Step 2: Find PR using Pythagoras theorem:

$PR = \sqrt{(PQ^2 + QR^2)}$

$PR = \sqrt{(16^2 + 10^2)} = \sqrt{(256 + 100)} = \sqrt{356}$

$PR = \sqrt{(4 \times 89)} = 2\sqrt{89}$ cm

Step 3: M is the midpoint of PR, so:

$PM = PR / 2 = 2\sqrt{89} / 2 = \sqrt{89}$ cm

Step 4: In triangle TMP:

- TM ⊥ PM (T is vertically above M, so TM is perpendicular to the base plane)
- Therefore, ∠TMP = 90°
- TM = 15 cm (given)
- PM = √89 cm

Step 5: We need the angle between TP and the base PQRS.

The angle between TP and the base is the angle that TP makes with its projection on the base plane. Since PM is the projection of TP on the base plane, the angle is ∠TPM.

In right triangle TMP:

tan(∠TPM) = Opposite / Adjacent = TM / PM

tan(∠TPM) = 15 / √89

Step 6: Calculate the value:

√89 = 9.43398

tan(∠TPM) = 15 / 9.43398 = 1.590

Step 7: Find the angle:

∠TPM = $\tan^{-1}$(1.590)

∠TPM = 57.84°

Step 8: Round to one decimal place:

∠TPM ≈ 57.8°

Answer: The angle between TP and the base PQRS is 57.8°

Summary Table

Question	Part	Answer
1(i)	CF	5 cm
1(ii)	BF	10 cm
1(iii)	$\angle BFC$	38.7°
2	Angle between TP and base	57.8°

Key Formulas Used

Formula	Application
$\tan\theta$ = Opposite / Adjacent	Finding angles in right triangles
$a^2 + b^2 = c^2$ (Pythagoras)	Finding diagonal of rectangle
Right triangular prism properties	Vertical edges are equal to height
Projection of a line on a plane	Angle between line and plane

Visual Summary

Triangular Prism (Question 1):

- Vertical edges: AD = BE = CF = 5 cm
- Base triangle: ABC (3-4-5 right triangle)
- BF = 10 cm (diagonal of rectangular face)
- Angle BFC is in right triangle BCF

Triangular Pyramid (Question 2):

- Base: Rectangle PQRS (16 × 10)
- Diagonal PR = $2\sqrt{89}$ cm
- M is midpoint of PR → PM = $\sqrt{89}$ cm
- Height MT = 15 cm
- Angle between TP and base = $\angle TPM$ = 57.8°

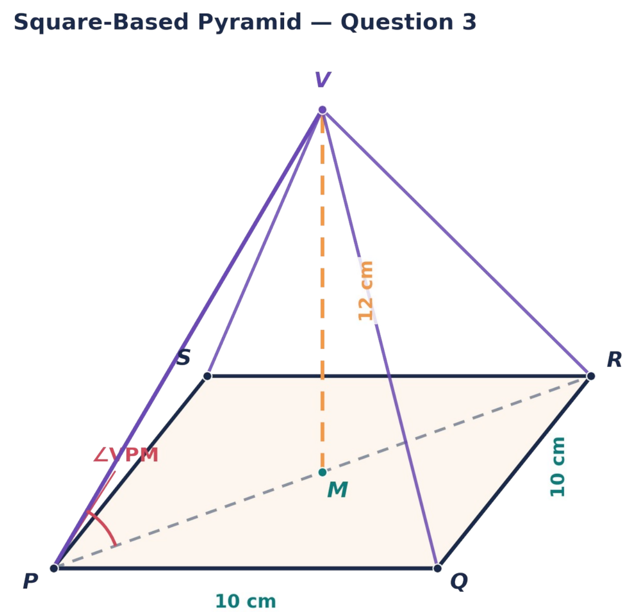
Solve these questions by using step by step approach, add proper graphics and diagrams wherever needed in question, diagrams and graphics must incorporate in the solution and provide formatted document in printable format

EXERCISE — 3D Geometry and Trigonometry Applications

Question 3: Square-Based Pyramid

Given:

- Base PQRS is a horizontal square of side 10 cm
- V is vertically above M (center of square)
- MV = 12 cm (height of pyramid)



- M is the center of square PQRS
- P, Q, R, S are vertices of the square

Solution:

Step 1: Identify triangle VPM.

In square PQRS with side 10 cm:

- Diagonal PR = $10\sqrt{2}$ cm
- M is the midpoint of PR
- PM = PR / 2 = $10\sqrt{2} / 2 = 5\sqrt{2}$ cm

Step 2: Triangle VPM is right-angled at M (since VM is perpendicular to the base):

- VM = 12 cm (height)

- PM = $5\sqrt{2}$ cm (half diagonal)

Step 3: Find angle VPM (angle at P between PV and PM):

In right triangle VPM:

$\tan(\angle VPM)$ = Opposite / Adjacent = VM / PM

$\tan(\angle VPM) = 12 / 5\sqrt{2}$

Step 4: Calculate:

$5\sqrt{2} = 7.071$

$\tan(\angle VPM) = 12 / 7.071 = 1.697$

$\angle VPM = \tan^{-1}(1.697) = 59.5°$

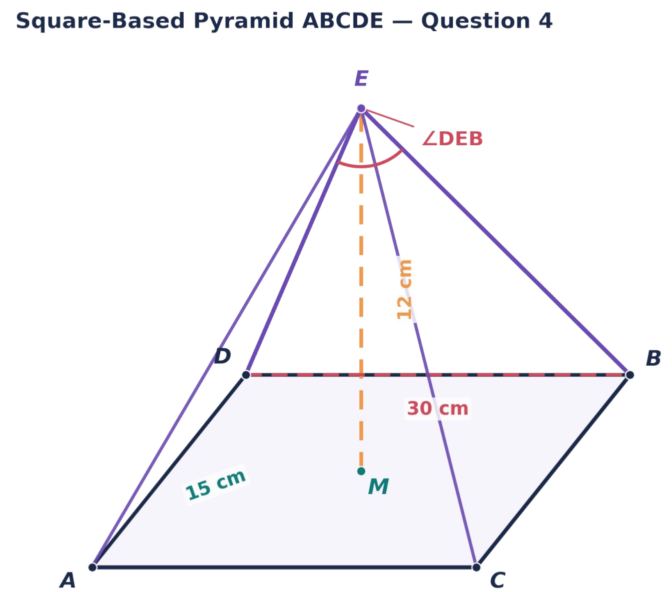
Answer: $\angle VPM = 59.5°$

Question 4: Square-Based Pyramid ABCDE

Given:

ABCDE is a square based pyramid, in which AE = BE = CE = DE = 12 cm and AB = 15 cm.

Calculate the size of angle DEB. Give your answer in degree (whole numbers).



- M is the center of square base
- E is the apex vertically above M
- A, B, C, D are vertices of square base

Solution:

Step 1: Understand the given information.

From the diagram, we can deduce:

- MA = 15 cm (distance from center to vertex)
- ME = 12 cm (height of pyramid)
- AE = BE = CE = DE = ? (not directly given, but can be found)

Step 2: Find AE using Pythagoras in triangle AME:

$AE^2 = AM^2 + ME^2$

$AE^2 = 15^2 + 12^2 = 225 + 144 = 369$

$AE = \sqrt{369} = 19.235$ cm

Step 3: In square ABCD, MA = MB = MC = MD = 15 cm.

For angle DEB (angle at E between ED and EB):

Step 4: Find DB (diagonal of square):

$DB = 2 \times MA = 2 \times 15 = 30$ cm

Step 5: In triangle DEB:

- $ED = EB = AE = 19.235$ cm
- $DB = 30$ cm

Using cosine rule:

$\cos(\angle DEB) = (ED^2 + EB^2 - DB^2) / (2 \times ED \times EB)$

$\cos(\angle DEB) = (19.235^2 + 19.235^2 - 30^2) / (2 \times 19.235 \times 19.235)$

$\cos(\angle DEB) = (369 + 369 - 900) / (2 \times 369)$

$\cos(\angle DEB) = (738 - 900) / 738 = -162 / 738 = -0.2195$

Step 6: Find the angle:

$\angle DEB = \cos^{-1}(-0.2195) = 102.7°$

Rounding to whole numbers:

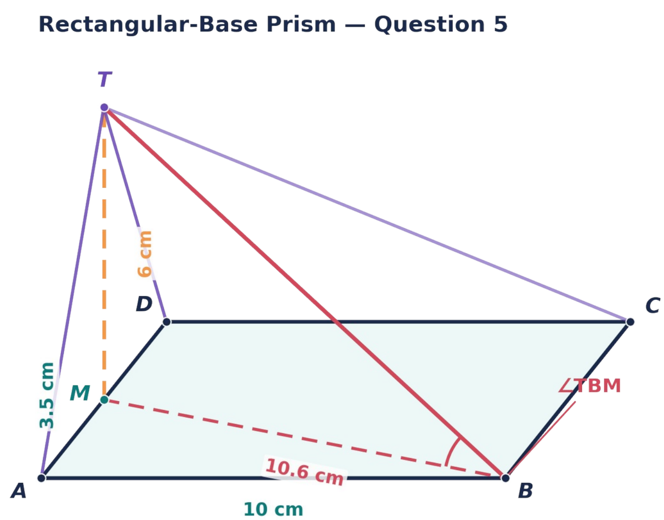
$\angle DEB = 103°$

Answer: $\angle DEB = 103°$

Question 5: Triangular Prism

Given:

The diagram shows a triangular prism with a horizontal rectangular base ABCD. AB = 10 cm, BC = 7 cm, M is the midpoint of AD. The vertex T is vertically above M and MT = 6 cm. Calculate the size of the angle between TB and the base.



- ABCD is a horizontal rectangle
- AB = 10 cm, BC = 7 cm
- M is the midpoint of AD
- T is vertically above M

Solution:

Step 1: Identify the relevant triangle.

We need the angle between TB and the base ABCD.

The projection of TB on the base is MB, so the angle is ∠TBM (angle at B in triangle TBM).

Step 2: Find BM.

M is the midpoint of AD, so AM = MD = 3.5 cm.

In rectangle ABCD:

- A to M: horizontal distance = 3.5 cm (half of AD)
- M to B: horizontal distance = AB = 10 cm

Using Pythagoras in triangle ABM:

$BM^2 = AB^2 + AM^2 = 10^2 + 3.5^2 = 100 + 12.25 = 112.25$

$BM = \sqrt{112.25} = 10.595$ cm

Step 3: Triangle TBM is right-angled at M (TM ⊥ base):

- TM = 6 cm
- BM = 10.595 cm

Step 4: Find angle between TB and base (∠TBM):

tan(∠TBM) = TM / BM = 6 / 10.595 = 0.5663

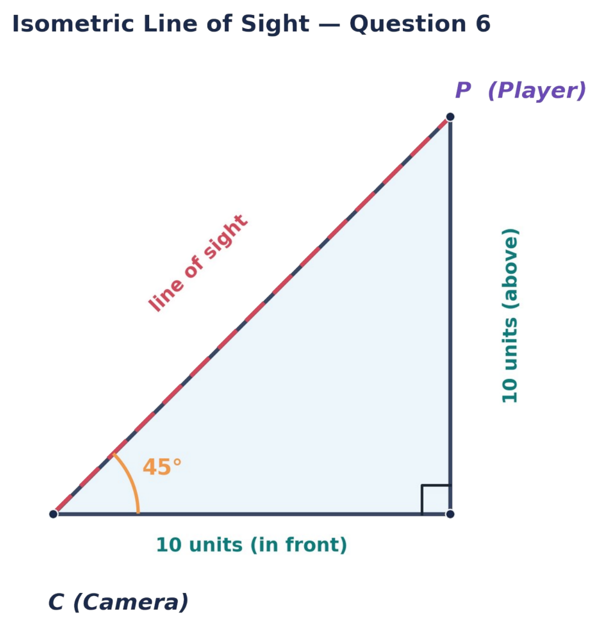
∠TBM = $\tan^{-1}$(0.5663) = 29.5°

Answer: Angle between TB and base = 29.5°

Question 6: Isometric Game — Line of Sight

Given:

- Camera at 45° angle from ground
- Player is 10 units in front and 10 units above



Solution:

Step 1: Use Pythagoras theorem in 3D.

Horizontal distance = 10 units

Vertical distance = 10 units

Direct line of sight:

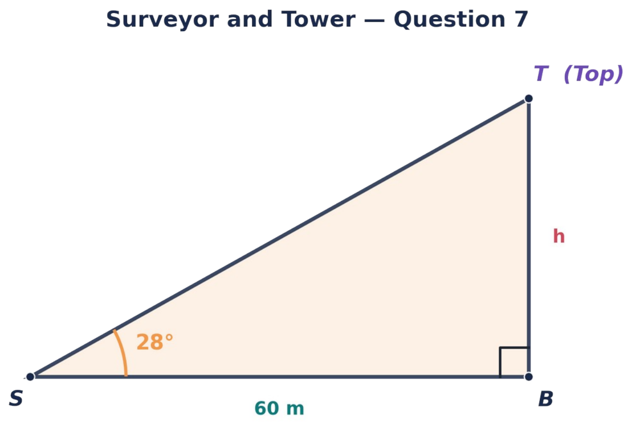
Distance = $\sqrt{(10^2 + 10^2)} = \sqrt{(100 + 100)} = \sqrt{200} = 10\sqrt{2}$ units

Answer: $10\sqrt{2}$ units ≈ 14.14 units

Question 7: Surveyor and Tower

Given:

- Elevation angle = 28°
- Distance from base = 60 m



Solution:

tan 28° = h / 60

h = 60 × tan 28°

tan 28° = 0.5317

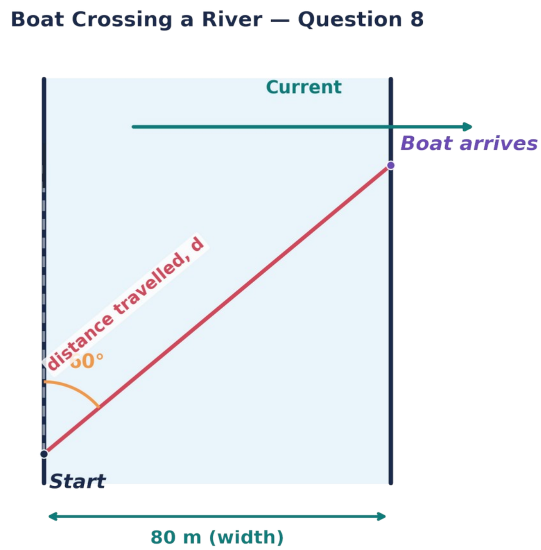
h = 60 × 0.5317 = 31.90 m

Answer: Height of tower = 31.90 m

Question 8: Boat Crossing River

Given:

- River width = 80 m
- Boat travels at 60° angle to current



Solution:

The angle between the boat's path and the current is 60°.

The perpendicular distance (width) is 80 m.

Let actual distance be d:

sin 60° = Width / d = 80 / d

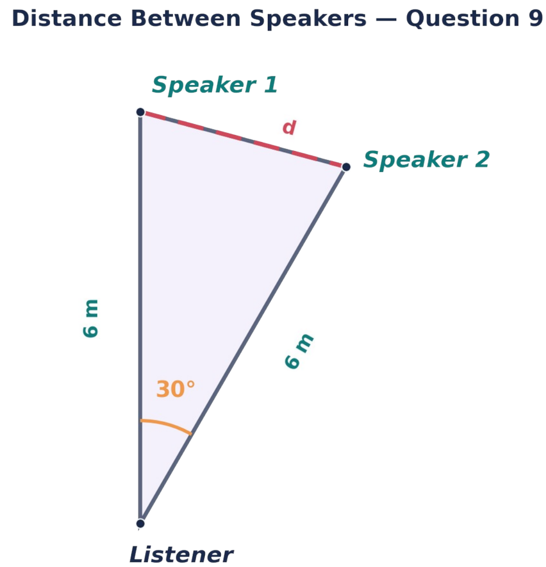
d = 80 / sin 60° = 80 / 0.8660 = 92.38 m

Answer: Boat travels 92.38 m

Question 9: Distance Between Speakers

Given:

- One speaker 6 m directly ahead
- Other speaker 6 m away at 30° to the side



Solution:

Using cosine rule:

$d^2 = 6^2 + 6^2 - 2(6)(6)\cos 30°$

$d^2 = 36 + 36 - 72 \times 0.8660$

$d^2 = 72 - 62.352 = 9.648$

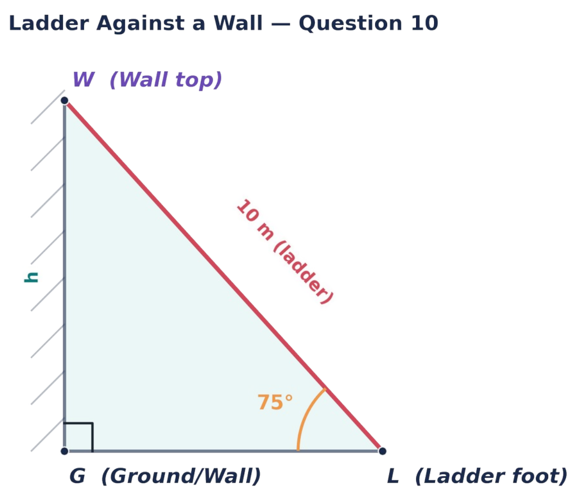
$d = \sqrt{9.648} = 3.106$ m

Answer: Distance between speakers = 3.11 m

Question 10: Ladder Against Wall

Given:

- Ladder length = 10 m
- Angle with ground = 75°



Solution:

sin 75° = h / 10

h = 10 × sin 75°

sin 75° = 0.9659

h = 10 × 0.9659 = 9.659 m

Answer: Ladder reaches 9.66 m up the wall

Summary Table

Question	Answer
3	∠VPM = 59.5°
4	∠DEB = 103°
5	Angle between TB and base = 29.5°
6	Line of sight = $10\sqrt{2}$ units
7	Tower height = 31.90 m
8	Boat distance = 92.38 m
9	Speaker distance = 3.11 m
10	Ladder height = 9.66 m

Key Formulas Used

Formula	Application
$\tan \theta$ = Opposite / Adjacent	Angles in right triangles
$\cos \theta$ = Adjacent / Hypotenuse	Cosine rule applications
$\sin \theta$ = Opposite / Hypotenuse	Finding heights and distances
$a^2 + b^2 = c^2$	Pythagoras theorem
$c^2 = a^2 + b^2 - 2ab \cdot \cos C$	Cosine rule for non-right triangles
Distance = $\sqrt{(x^2 + y^2)}$	2D and 3D distances

— End of Booklet —