

Class 10th Math — Exercise 8.2

Arcs and Chords of a Circle — with corrected, colour-coded diagrams

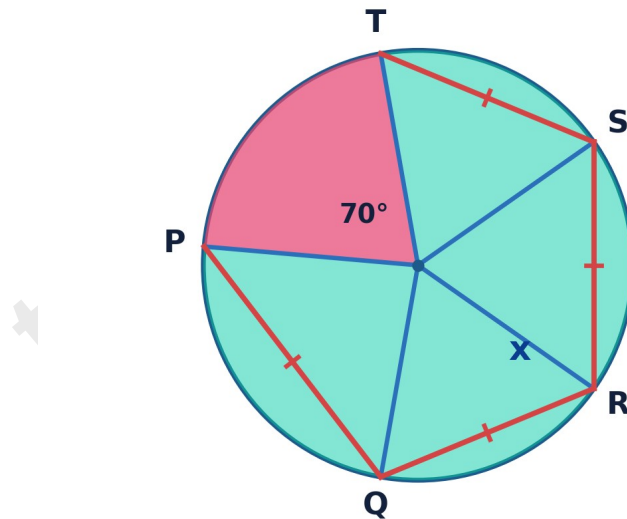
Key Theorems for Arcs and Chords

- Equal Arcs Subtend Equal Angles at the Centre — if two arcs are equal, the angles they subtend at the centre are equal.
- Equal Chords Subtend Equal Arcs — if two chords are equal, the arcs they subtend are equal.
- Equal Angles at the Centre Subtend Equal Arcs — if two central angles are equal, the arcs they subtend are equal.
- Equal Arcs Have Equal Chords — if two arcs are equal, their corresponding chords are equal.
- Congruent Circles — in congruent circles, equal arcs have equal chords and vice versa.

Question 1

Given: arc PQ = arc QR = arc RS = arc ST, and $\angle POT = 70^\circ$. Find the value of x, where x is the reflex angle $\angle POT$ that opens the other way around the circle (through Q, R, S).

Question 1



Step 1 — The four arcs PQ, QR, RS and ST are equal, so they subtend equal central angles. Call each one θ .

Step 2 — The marked angle x is the central angle that spans all four equal arcs together (from P to T going through Q, R, S), so $x = 4\theta$.

Step 3 — The angles x and 70° ($\angle POT$ taken the short way) together make one full turn at O: $x + 70^\circ = 360^\circ$.

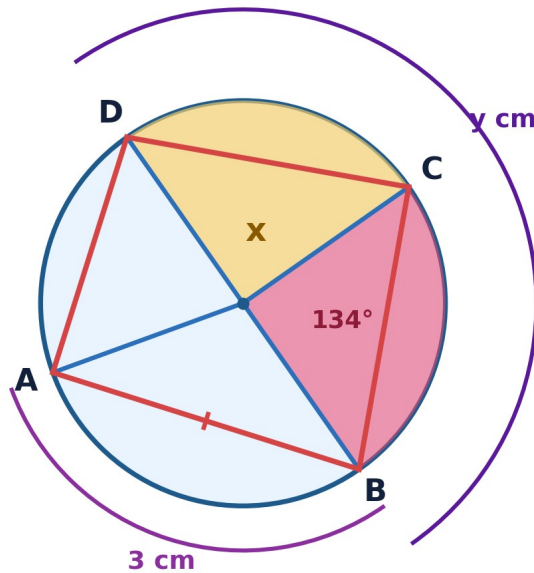
Step 4 — Solving: $x = 360^\circ - 70^\circ = 290^\circ$. (Each individual arc then measures $\theta = 290^\circ/4 = 72.5^\circ$.)

Answer: $x = 290^\circ$

Question 2

Given: quadrilateral ABCD inscribed in a circle with centre O; the diagonals AC and BD are drawn straight through O. $\angle DOC = x$, $\angle COB = 134^\circ$, arc AB = 3 cm, and arc DCB = y cm. Find x and y.

Question 2



Step 1 — Since B, O, D lie on a straight line (BD passes through the centre), $\angle DOC$ and $\angle COB$ lie on a straight line and are supplementary: $x + 134^\circ = 180^\circ \Rightarrow x = 46^\circ$.

Step 2 — Because AC also passes through O, $\angle AOB$ and $\angle DOC$ are vertically opposite, so $\angle AOB = x = 46^\circ$. This matches the arc AB = 3 cm.

Step 3 — Arc DCB is the semicircle on the other side of diameter BD, so it corresponds to $\angle DOC + \angle COB = 46^\circ + 134^\circ = 180^\circ$.

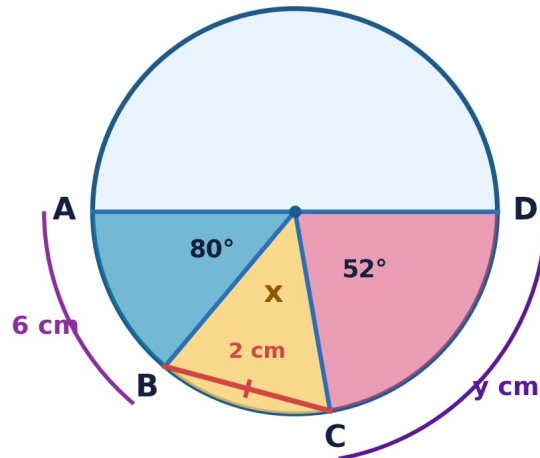
Step 4 — In the same circle, arc length is proportional to its central angle: $y / 180^\circ = 3 \text{ cm} / 46^\circ \Rightarrow y = 3 \times 180 / 46 \approx 11.7 \text{ cm}$.

Answer: $x = 46^\circ$, $y \approx 11.7 \text{ cm}$

Question 3

Given: A, O, D lie on a circle with AD a straight chord through the figure; $\angle AOB = 80^\circ$, $\angle BOC = x$, $\angle COD = 52^\circ$, arc AB = 6 cm, chord BC = 2 cm, arc CD = y cm, with the condition arc AB = arc BD (arc BD = arc BC + arc CD).

Question 3



Step 1 — Equal arcs subtend equal central angles, so arc AB = arc BD gives $\angle AOB = \angle BOC + \angle COD$: $80^\circ = x + 52^\circ$.

Step 2 — Solving: $x = 80^\circ - 52^\circ = 28^\circ$.

Step 3 — Arc length is proportional to central angle in the same circle. Using arc AB (6 cm $\leftrightarrow$ 80°) to scale arc CD ($y \leftrightarrow 52^\circ$): $y / 52^\circ = 6 \text{ cm} / 80^\circ$.

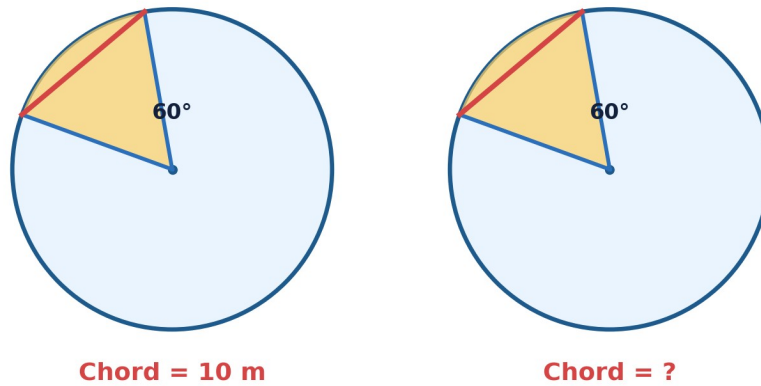
Step 4 — Solving: $y = 6 \times 52 / 80 = 3.9 \text{ cm}$.

Answer: $x = 28^\circ$, $y = 3.9 \text{ cm}$

Question 4

Two congruent arcs in a circular track subtend angles of 60° each at the centre. The length of one chord is 10 metres. What is the length of the other chord?

Question 4 — Two congruent circular tracks



Step 1 — The two arcs are congruent, so they subtend equal (60°) angles at the centre — this is given.

Step 2 — Equal arcs have equal chords (in the same circle, or in congruent circles as here).

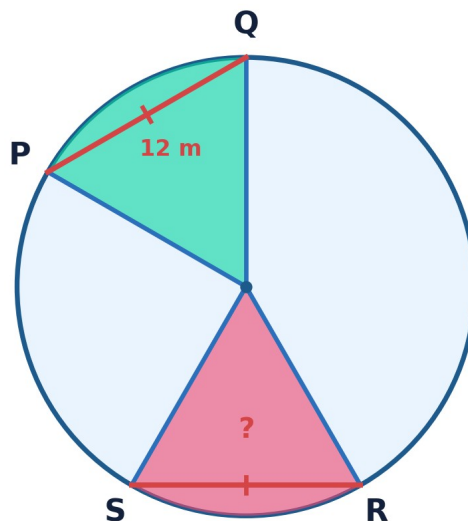
Step 3 — Since the arcs are congruent, their corresponding chords must be equal in length.

Answer: Length of the other chord = 10 metres

Question 5

In a circular fountain, two water jets spray along arcs of equal length: arc PQ and arc RS. If chord PQ = 12 metres, find chord RS.

Question 5 — Circular fountain jets



Step 1 — Equal arcs in the same circle subtend equal chords.

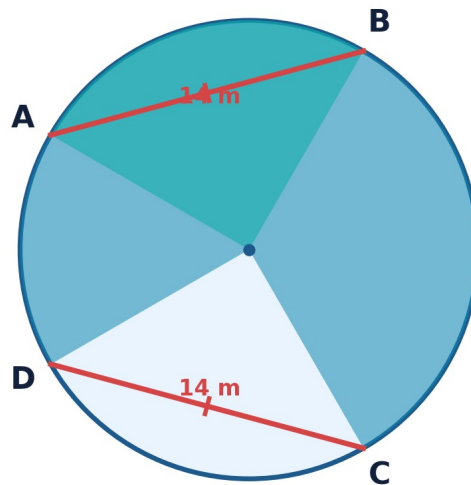
Step 2 — Since arc PQ = arc RS (given as congruent), the chords they subtend must also be equal: RS = PQ.

Answer: Chord RS = 12 metres

Question 6

In a circular park, two walkways AB and CD are chords, each 14 metres long. What can be said about the minor arcs subtended by these walkways?

Question 6 — Circular park walkways



Step 1 — Equal chords subtend equal arcs (in the same circle).

Step 2 — Since AB = CD = 14 m, the minor arcs cut off by AB and by CD must be equal in measure.

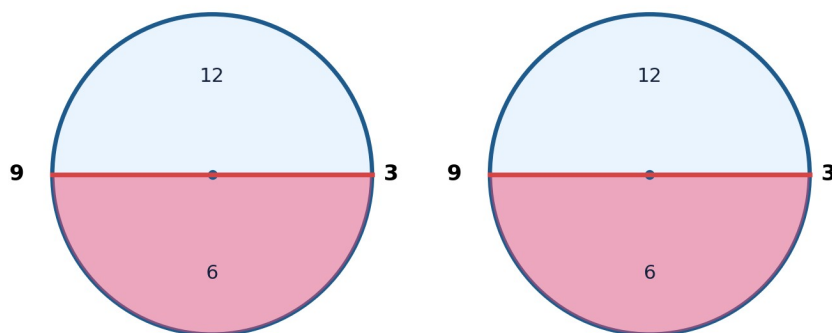
Step 3 — Consequently, the corresponding major arcs are equal too, since each major arc is 360° minus its minor arc.

Answer: The minor arcs subtended by AB and CD are congruent (equal in measure).

Question 7

Two congruent circular clocks each have a decorative string connecting the 3 to the 9 on the clock face. Are the arcs from 3 to 9 on both clocks congruent?

Question 7 — Two congruent clocks



Step 1 — On any standard clock face, the points 3 and 9 are diametrically opposite, so the arc from 3 to 9 always measures 180° (a semicircle).

Step 2 — The two clocks are congruent, meaning they have equal radii.

Step 3 — Equal central angles (180° on both) in congruent circles subtend congruent arcs.

Answer: Yes — the arcs from 3 to 9 are congruent on both clocks.

Summary Table

Question	Answer
1	$x = 290^\circ$
2	$x = 46^\circ$, $y \approx 11.7$ cm
3	$x = 28^\circ$, $y = 3.9$ cm
4	10 metres
5	12 metres
6	The minor arcs are congruent
7	Yes, they are congruent

Key Concepts Summary

Property	Statement
Equal arcs $\rightarrow$ Equal chords	If two arcs are equal, their chords are equal.
Equal chords $\rightarrow$ Equal arcs	If two chords are equal, their arcs are equal.
Equal arcs $\rightarrow$ Equal central angles	If two arcs are equal, they subtend equal angles at the centre.
Equal central angles $\rightarrow$ Equal arcs	If two central angles are equal, their arcs are equal.

Important Notes

- Minor Arc: the smaller arc between two points on a circle (central angle $< 180^\circ$).
- Major Arc: the larger arc between two points on a circle (central angle $> 180^\circ$).
- Congruent Circles: circles that have equal radii.