Class 10th Math — Exercise 9.2

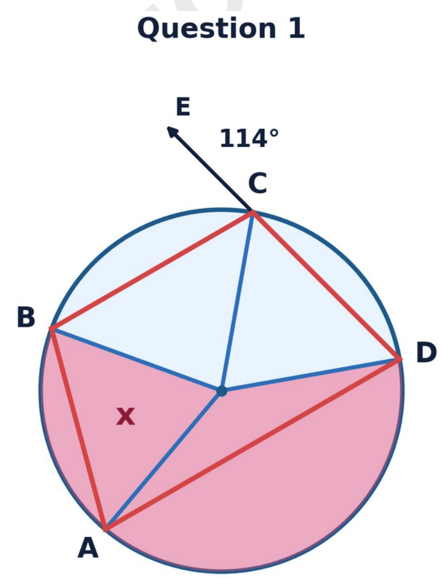
Circle Theorems, Cyclic Quadrilaterals, Arc Length & Sector Area — with corrected, colour-coded diagrams

Key Theorems Used

- Angles in the Same Segment — angles subtended by the same arc at the circumference are equal.
- Central Angle Theorem — the central angle subtending an arc is twice the inscribed angle subtending the same arc.
- Cyclic Quadrilateral Property — opposite angles of a cyclic quadrilateral are supplementary (sum to 180°).
- Exterior Angle of a Cyclic Quadrilateral — equals the interior opposite angle.
- Angle in a Semicircle — an angle inscribed in a semicircle (subtending a diameter) is always 90°.
- Tangent–Chord Angle — the angle between a tangent and a chord equals the inscribed angle in the alternate segment.
- Tangent–Radius — a tangent to a circle is always perpendicular to the radius at the point of contact.
- Two Tangents from an External Point — are equal in length, and the quadrilateral formed with the two radii has angle sum 360°.

Question 1

Given: A, B, C, D are points on a circle with centre O. Side DC is extended beyond C to a point E, so that the exterior angle ∠BCE = 114°. Find the reflex angle x = ∠BOD (the central angle for arc BCD, measured through C).



Step 1 — Exterior angle of a cyclic quadrilateral equals the interior opposite angle: ∠BAD = ∠BCE = 114°.

Step 2 — ∠BAD is the inscribed angle standing on arc BCD (the arc from B to D that passes through C, i.e. the arc NOT containing A).

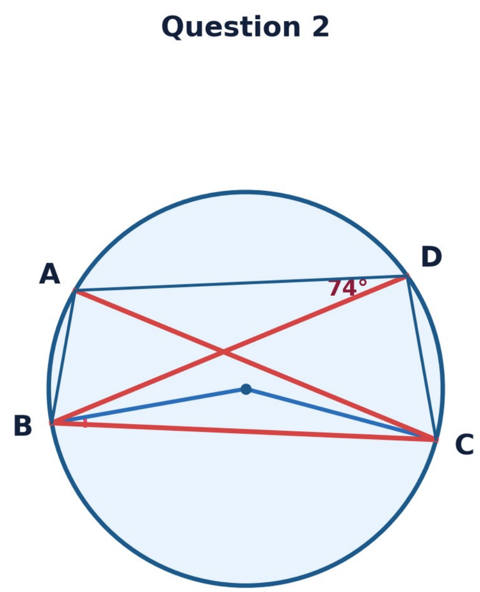
Step 3 — The central angle standing on the same arc is twice the inscribed angle: $x = 2 \times \angle BAD = 2 \times 114°$.

Step 4 — Solving: $x = 228°$.

Answer: $x = 228°$

Question 2

Given: A, B, C, D lie on a circle with centre O; $m\angle BDC = 74°$. Find $m\angle BAC$, $m\angle BOC$ and $m\angle OBC$.



Part (i): $m\angle BAC$ — $\angle BAC$ and $\angle BDC$ both stand on the same arc BC, so angles in the same segment are equal: $\angle BAC = \angle BDC = 74°$.

Part (ii): $m\angle BOC$ — $\angle BOC$ is the central angle on arc BC, and the central angle is twice the inscribed angle on the same arc: $\angle BOC = 2 \times \angle BAC = 2 \times 74° = 148°$.

Part (iii): $m\angle OBC$ — Triangle OBC is isosceles since OB = OC (both radii), so $\angle OBC = \angle OCB$. The angles of triangle OBC sum to 180°: $148° + 2\times\angle OBC = 180° \Rightarrow \angle OBC = 16°$.

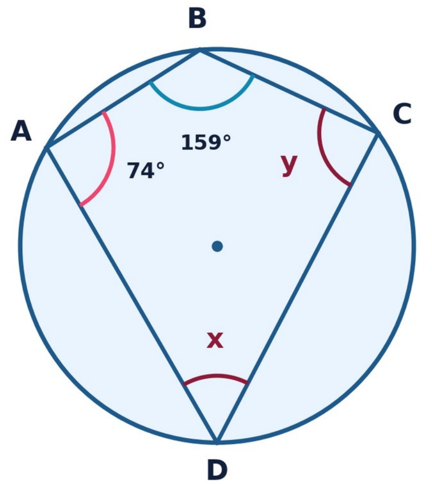
Answer: $\angle BAC = 74°$, $\angle BOC = 148°$, $\angle OBC = 16°$

Question 3

Find the angles x and y in each figure.

(i) Cyclic quadrilateral ABCD with $\angle A = 74°$, $\angle B = 159°$, $\angle C = y$, $\angle D = x$.

Question 3 (i)

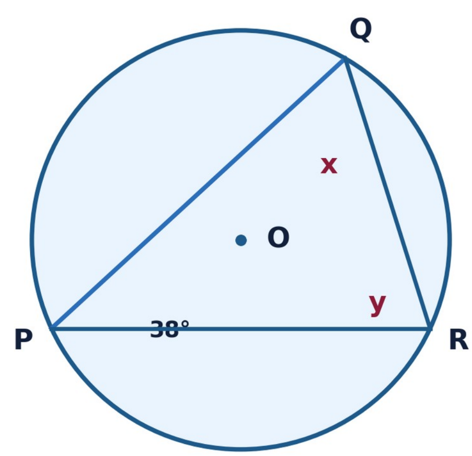


Opposite angles of a cyclic quadrilateral are supplementary. ∠A and ∠C are opposite: y = 180° − 74° = 106°. ∠B and ∠D are opposite: x = 180° − 159° = 21°.

Answer: x = 21°, y = 106°

(ii) PQ is a diameter through centre O; R lies on the circle. ∠QPR = 38°, ∠PQR = x, ∠PRQ = y.

Question 3 (ii)



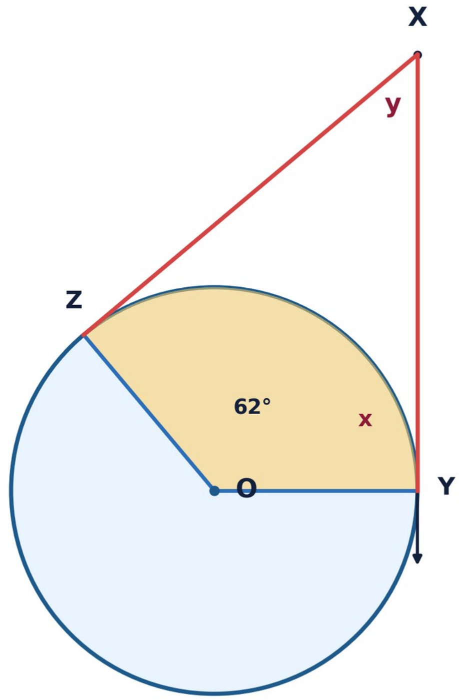
Since PQ is a diameter, the angle inscribed in the semicircle at R is a right angle: y = ∠PRQ = 90°.

The angles of triangle PQR sum to 180°: $38° + x + 90° = 180° \Rightarrow x = 52°$.

Answer: $x = 52°$, $y = 90°$

(iii) X is an external point; XZ and XY are tangents touching the circle at Z and Y. $\angle ZOY = 62°$, $\angle OYX = x$, $\angle ZXY = y$.

Question 3 (iii)



A tangent is perpendicular to the radius at the point of contact, so $x = \angle OYX = 90°$ (and $\angle OZX = 90°$ too).

Quadrilateral OZXY has angle sum 360°: $\angle ZOY + \angle OZX + \angle ZXY + \angle OYX = 360° \Rightarrow 62° + 90° + y + 90° = 360°$.

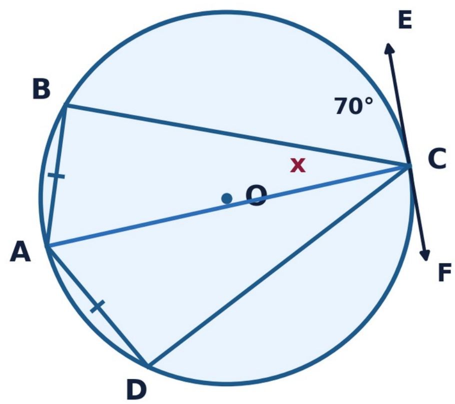
Solving: $y = 360° - 242° = 118°$.

Answer: $x = 90°$, $y = 118°$

Question 4

Given: AC is a diameter through centre O; B and D lie on the circle with chords AB = AD (equal, tick-marked). EF is tangent to the circle at C, and the tangent–chord angle $\angle BCE = 70°$. Find $x = \angle ACD$.

Question 4



Step 1 — Tangent–chord angle theorem: the angle between tangent CE and chord CB equals the inscribed angle in the alternate segment: $\angle BAC = \angle BCE = 70°$.

Step 2 — Since AC is a diameter, the angle inscribed in the semicircle at B is 90°: $\angle ABC = 90°$.

Step 3 — In triangle ABC: $\angle BCA = 180° - \angle BAC - \angle ABC = 180° - 70° - 90° = 20°$.

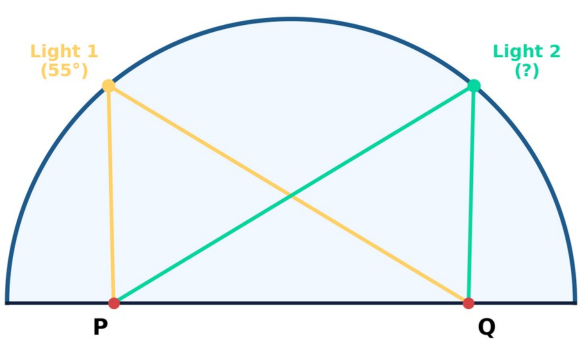
Step 4 — Chords AB and AD are equal (tick marks), so they cut off equal arcs, and equal arcs give equal inscribed angles from C: $\angle ACD = \angle ACB = 20°$.

Answer: x = 20°

Question 5

Two spotlights on a circular arch shine from two different points on the arch toward the same chord PQ on the base. The angle formed at one light is 55°. Find the angle at the second light, on the same side of chord PQ.

Question 5 — Arch spotlights on chord PQ



Step 1 — Both lights sit on the same arc and look at the same chord PQ from the same side, so their angles are angles in the same segment.

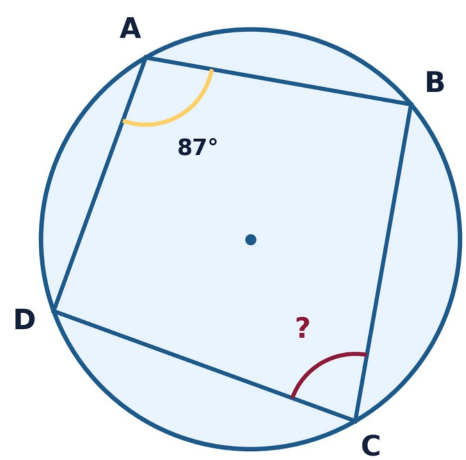
Step 2 — Angles in the same segment (subtending the same chord, from the same side) are always equal.

Answer: Angle at the second light = 55°

Question 6

A circular garden has a walking path forming a quadrilateral inscribed in it. One angle is 87°. Find the opposite angle.

Question 6 — Circular garden path



Step 1 — Any quadrilateral inscribed in a circle is a cyclic quadrilateral, and its opposite angles are supplementary.

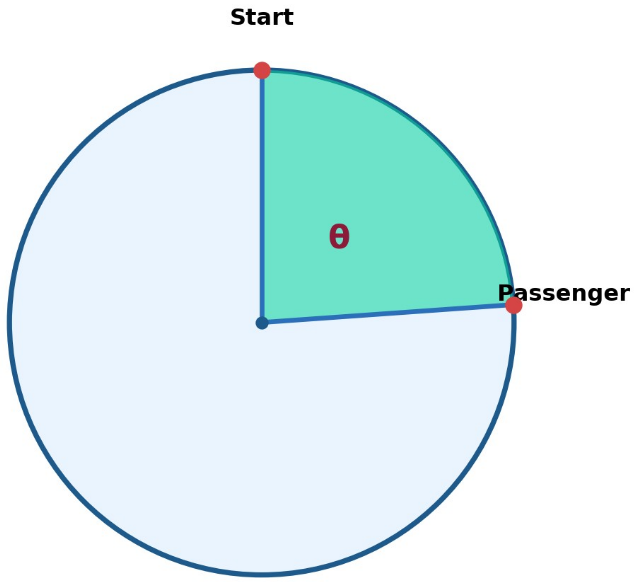
Step 2 — Opposite angle = 180° − 87° = 93°.

Answer: Opposite angle = 93°

Question 7

A Ferris wheel has radius 12 m. A passenger travels 18 m along the circumference. Find the angle (in radians) swept from the starting position.

Question 7 — Ferris wheel



radius = 12 m, arc travelled = 18 m

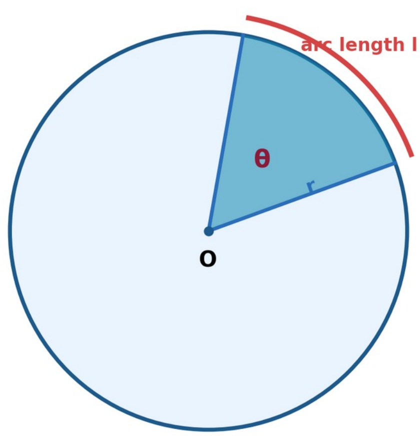
Step 1 — Arc length formula: $l = r\theta$.

Step 2 — Substitute: $18 = 12\theta \Rightarrow \theta = 18/12 = 1.5$.

Answer: $\theta = 1.5$ radians

Question 8 — Find θ

Sector: arc length & area



Formula: $\theta = l / r$ (θ in radians).

(i) $l = 3$ cm, $r = 2.2$ cm: $\theta = 3 / 2.2 = 1.3636... \approx 1.36$ rad.

(ii) $l = 5.6$ cm, $r = 2$ cm: $\theta = 5.6 / 2 = 2.8$ rad.

Answer: (i) $\theta \approx 1.36$ rad (ii) $\theta = 2.8$ rad

Question 9 — Find r

Formula: $r = l / \theta$ (θ converted to radians first).

(i) $l = 5.5$ cm, $\theta = 40°20'$. Convert: $40°20' = 40 + 20/60 = 40.333°$, and $40.333° \times \pi/180° \approx 0.7039$ rad. Then $r = 5.5 / 0.7039 \approx 7.81$ cm.

(ii) $l = 13$ cm, $\theta = 70°$. Convert: $70° \times \pi/180° \approx 1.2217$ rad. Then $r = 13 / 1.2217 \approx 10.64$ cm.

Answer: (i) $r \approx 7.81$ cm (ii) $r \approx 10.64$ cm

Question 10 — Find l and Area of Sector

Formulas: arc length $l = r\theta$, sector area $A = \frac{1}{2}r^2\theta$ (θ in radians).

(i) $r = 1.7$ cm, $\theta = 0.25$ rad: $l = 1.7 \times 0.25 = 0.425$ cm. $A = \frac{1}{2} \times (1.7)^2 \times 0.25 = \frac{1}{2} \times 2.89 \times 0.25 = 0.36125$ cm².

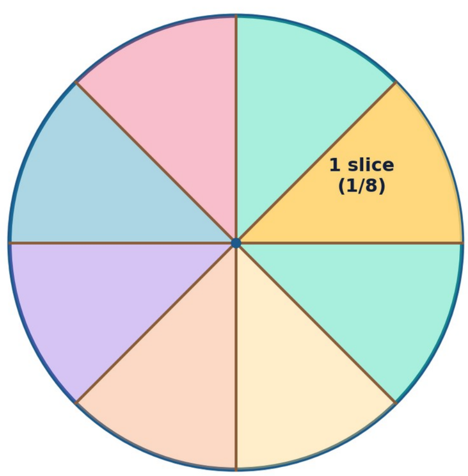
(ii) $r = 3$ cm, $\theta = 45° = \pi/4 \approx 0.7854$ rad: $l = 3 \times 0.7854 \approx 2.356$ cm. $A = \frac{1}{2} \times 9 \times 0.7854 \approx 3.534$ cm².

Answer: (i) $l = 0.425$ cm, $A \approx 0.361$ cm² (ii) $l \approx 2.36$ cm, $A \approx 3.53$ cm²

Question 11

Uzma cuts a pizza of radius 14 cm into 8 equal slices. Find the area of one slice (sector).

Question 11 — Pizza cut into 8 slices (r = 14 cm)



Step 1 — Central angle of one slice: $\theta = 360° / 8 = 45° = \pi/4$ rad.

Step 2 — Sector area: $A = \frac{1}{2}r^2\theta = \frac{1}{2} \times (14)^2 \times \pi/4 = \frac{1}{2} \times 196 \times \pi/4 = 24.5\pi$ cm².

Step 3 — Numerically: $A \approx 24.5 \times 3.1416 \approx 76.97$ cm².

Answer: Area of one slice = 24.5π cm² ≈ 76.97 cm²

Question 12

The perimeter and area of a sector are 14 cm and 10 cm² respectively. Find the radius r and the central angle θ.

Step 1 — Perimeter of a sector: $P = 2r + l = 2r + r\theta = 14 \Rightarrow r(2 + \theta) = 14 \Rightarrow \theta = 14/r - 2$.

Step 2 — Area of a sector: $A = \frac{1}{2}r^2\theta = 10$. Substitute θ from Step 1: $\frac{1}{2}r^2(14/r - 2) = 10 \Rightarrow 7r - r^2 = 10 \Rightarrow r^2 - 7r + 10 = 0$.

Step 3 — Factorise: $(r - 2)(r - 5) = 0 \Rightarrow r = 2$ cm or $r = 5$ cm.

Step 4 — Find θ for each case: If $r = 2$: $\theta = 14/2 - 2 = 5$ rad. If $r = 5$: $\theta = 14/5 - 2 = 0.8$ rad.

Answer: $r = 2$ cm, $\theta = 5$ rad OR $r = 5$ cm, $\theta = 0.8$ rad

Summary Table

Question	Answer
1	$x = 228°$
2	$\angle BAC = 74°$, $\angle BOC = 148°$, $\angle OBC = 16°$
3(i)	$x = 21°$, $y = 106°$
3(ii)	$x = 52°$, $y = 90°$
3(iii)	$x = 90°$, $y = 118°$
4	$x = 20°$
5	55°
6	93°
7	1.5 rad
8(i)	1.36 rad
8(ii)	2.8 rad
9(i)	7.81 cm
9(ii)	10.64 cm
10(i)	$l = 0.425$ cm, $A \approx 0.361$ cm²
10(ii)	$l \approx 2.36$ cm, $A \approx 3.53$ cm²
11	24.5π cm² ≈ 76.97 cm²
12	$r = 2$ cm, $\theta = 5$ rad OR $r = 5$ cm, $\theta = 0.8$ rad

Key Formulas Used

Formula	Application
$l = r\theta$	Arc length (θ in radians)
$A = \frac{1}{2}r^2\theta$	Sector area (θ in radians)
$P = 2r + r\theta$	Sector perimeter
$\theta(\text{radians}) = \theta(\text{degrees}) \times \pi/180°$	Angle conversion
$\angle A + \angle C = 180°$, $\angle B + \angle D = 180°$	Opposite angles of a cyclic quadrilateral
Angles in the same segment are equal	Circle theorem