Review Exercise 5

Q. 1 Choose the correct option.

i. In the following, linear equation is:

(a) $5x > 7$ (b) $4x - 2 < 1$ (c) $2x + 1 = 1$ (d) $4 = 1 + 3$

ii. Solution of $5x - 10 = 10$ is:

(a) 0 (b) 50 (c) 4 (d) -4

iii. If $7x + 4 < 6x + 6$, then x belongs to the interval:

(a) $(2, \infty)$ (b) $[2, \infty)$ (c) $(-\infty, 2)$ (d) $(-\infty, 2]$

iv. A vertical line divides the plane into

(a) left half plane (b) right half plane (c) full plane (d) two half plane

v. The linear equation formed out of the linear inequality is called:

(a) Linear equation (b) Associated equation (c) Quadratic equal (d) None of these

vi. $3x + 4 < 0$ is:

(a) Equation (b) Inequality (c) Not inequality (d) identity

vii. Corner point is also called:

(a) Code (b) Vertex (c) Curve (d) Region

viii. $(0,0)$ is solution of inequality:

(a) $4x + 5y > 8$ (b) $3x + y > 6$ (c) $-2x + 3y < b$ (d) $x + y > 4$

ix. The solution region restricted to the first quadrant is called:

(a) Objective region (b) Feasible region (c) Solution region (d) Constraints region

x. A function that is to be maximized or minimized is called:

(a) Solution function (b) Objective function (c) Feasible functioned) (d) None of these

Answers Key

i	c	Ii	c	iii	c	iv	d	v	b
vi	b	vii	b	viii	c	ix	b	x	b

Multiple Choice Questions (Additional)

1. A statement involving any of the symbols $<\ >$ or $\leq$ or $\geq$ is called?
(a) Equation (b) Identity (c) Inequality (d) Linear equation
2. The degree of linear inequality is
(a) 1 (b) 2 (c) 3 (d) 4
3. Which of the following is the solution of the inequality $3 - 4x \leq 19$?
(a) $x \geq -8$ (b) $x \geq -4$ (c) $x \geq \frac{-22}{4}$ (d) 16
4. $x =$………………. is not a solution of the inequality $x < -\frac{3}{2}$
(a) -15 (b) -2.5 (c) -3 (d) -2
5. $x = 0$ is a solution of the inequality
(a) $x > 0$ (b) $3x < 0$ (c) $x + 2 < 0$ (d) $x - 2 < 0$
6. The solution of inequality $x > 1$
(a) $(1, \infty)$ (b) $(-\infty, 1)$ (c) $[1, \infty)$ (d) $(-\infty, 1]$
7. The solution of inequality $x < 1$ is
(a) $[1, \infty)$ (b) $(-\infty, 1)$ (c) $[1, \infty)$ (d) $(-\infty, 1]$

8. The solution of inequality $x \leq 1$ is
(a) $(1, \infty)$ (b) $(-1, \infty)$ (c) $[1, \infty)$ (d) $(-\infty, 1]$
9. The solution of inequality $x \geq 1$ is :
(a) $(1, \infty)$ (b) $(-1, \infty)$ (c) $[1, \infty)$ (d) $(-\infty, 1]$
10. The graph of inequality on is half plane :
(a) lower (b) upper (c) right (d) left
11. The graph of incquality yen is half plane :
(a) lower (b) upper (c)right (d) left
12. The graph of inequality $x < 0$ is half plane
(a) lower (b) upper (c) right (d) left
13. The graph of inequality y<0 is half plane
(a) lower (b) upper (c) right (d) left
14. Which of the following line does pass through the origin?
(a) $y = -1$ (b) $y = 4x$ (c) $y = 4x + 5$ (d) $y \geq -2$
15. The solution region of inequality $x < 1$ is half plane :

(a) Closed right (b) Closed left (c) open right (d) open left
16. The solution region of inequality $x > 1$ is half plane:
(a)Closed right (b) closed left (C) open right (d) open left
17. The solution region of inequality $x \geq 1$ is half plane:

(a) Closed right (b) Closed left (c) open right (d) open left
18. The solution region of inequality $x \leq 2$ is half plane:
(a) Closed right (b) Closed left (c) open right (d) open left
19. The solution region of inequality $y \leq 3$ is half plane:
(a) closed lower (b) closed upper (c) open lower (d) open upper
20. The solution region of inequality $y \geq 3$ is half plane:
(a) closed lower (b) closed upper (c) open lower (d) open upper

Answer Key

1	c	2	a	3	b	4	a	5	d	6	a	7	b	8	d	9	c	10	c
11	b	12	d	13	a	14	b	15	d	16	c	17	a	18	b	19	a	20	b

Q. 2 Solve and represent their solutions on real line.
(i) $\frac{x+5}{3} = 1 - x$
Solution:

$$\frac{x+5}{3} = 1 - x$$

$$3 \times \frac{x+5}{3} = 3(1 - x)$$

$$x + 5 = 3 - 3x$$

$$x + 3x = 3 - 5$$

$$4x = -2$$

$$x = -\frac{2}{4}$$

$$x = -\frac{1}{2}$$

Check: put $x = \frac{-1}{2}$ in equation (i)

$$\frac{-\frac{1}{2}+5}{3} = 1-\left(-\frac{1}{2}\right)$$
$$\frac{-1+10}{2} = 1+\frac{1}{2}$$
$$\frac{9}{2\times 3} = \frac{2+1}{2}$$
$$\frac{9}{6} = \frac{3}{2}$$
$$\frac{3}{2} = \frac{3}{2}$$

(it is true)

So $x = -\frac{1}{2}$ is solution of given equation.

Solution on number line:

Point p on the number line represents $-\frac{1}{2}$.

(ii) $\frac{2x+1}{3}+\frac{1}{2} = 1-\frac{x-1}{3}$

Solution:

$$\frac{2x+1}{3}+\frac{1}{2} = 1-\frac{x-1}{3}$$
$$\frac{2x+1}{3}+\frac{1}{2} = 1-\frac{x-1}{3}$$
$$\frac{2(2x+1)+3}{6} = \frac{3-(x-1)}{3}$$
$$4x+2+3 = \frac{(3-x+1)^3}{3}\times 6$$
$$4x+5 = (-x+4)\times 2$$
$$4x+5 = -2x+8$$
$$4x+2x = 8-5$$
$$6x = 3$$
$$x = \frac{3}{6}x = \frac{1}{2}$$

Check put $x = \frac{1}{2}$ in equation (i)

$$\frac{2\left(\frac{1}{2}\right)+1}{3} + \frac{1}{2} = 1 - \frac{\frac{1}{2}-1}{3}$$

$$\frac{1+1}{3} + \frac{1}{2} = 1 - \frac{\frac{1-2}{2}}{3}$$

$$\frac{2}{3} + \frac{1}{2} = 1 - \frac{-\frac{1}{2}}{3}$$

$$\frac{4+3}{6} = 1 + \frac{1}{2 \times 3}$$

$$\frac{7}{6} = 1 + \frac{1}{6}$$

$$\frac{?}{6} = \frac{6+1}{6}$$

$$\frac{7}{6} - \frac{7}{6}$$

(It is true)

So $x = \frac{1}{2}$ is solution of given equation.

Solution on number line:

Point p on the number line represents $x = \frac{1}{2}$

(iii) $3x + 7 < 16$

Solution

$3x + 7 < 16$

$3x < 16 - 7$

$3x < 9$

$x < \frac{9}{3}$

$x < 3$

The solution of given equality is $(-\infty, 3)$ or $-\infty < x < 3$

Solution on number line

A empty circle on 3 shows that 3 is not included in the solution.

(iv) $5(x-3) \geq 26x - (10x + 4)$

Solution

$$5(x-3) \geq 26x-(10x+4)$$
$$5x-15 \geq 26x-10x-4$$
$$5x-15 \geq 16x-4$$
$$-15+4 \geq 16x-5x$$
$$11 \geq 11x$$
$$\frac{-11}{11} \geq x.$$
$$-1 \geq x \Rightarrow x \leq -1$$

The given inequality is $(-\infty,-1]$ or $\infty < x-1$
Solution on number line

A filled circle on -1 shows that -1 is included in the solution.
Q. 3 Find the solution region of the following linear equalities:
(i) $3x-4y \leq 12; 3x+2y \geq 3$

Solution:

$$3x-4y \leq 12 \quad \text{(i)}$$
$$3x+2y \geq 3 \quad \text{(ii)}$$

The sketch of inequality (i) is shown as shaded region in the figure 3(0) (a) by joining the points $A(4,0)$ and $B(0,-3)$
The sketch of mequality (ii) is shown as shaded region in the figure 3(i) (b) by joining the points $C(1.9)$ and $D(0,3/2)$.
Now the solution region of the given syatems of inequalities is the intersection of the graphis indicated in the figure $3(i)$ (a), and 3 (i) (b) is shown at shaded region in the figure 3(i) (c).

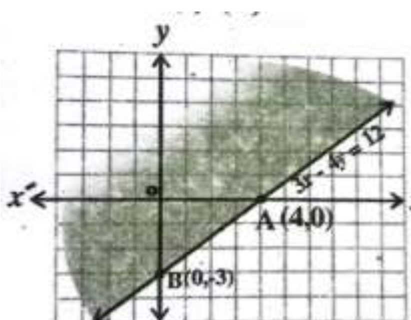


Fig. 3(i)(a)

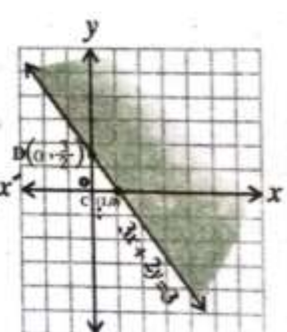


Fig. 3(i)(b)

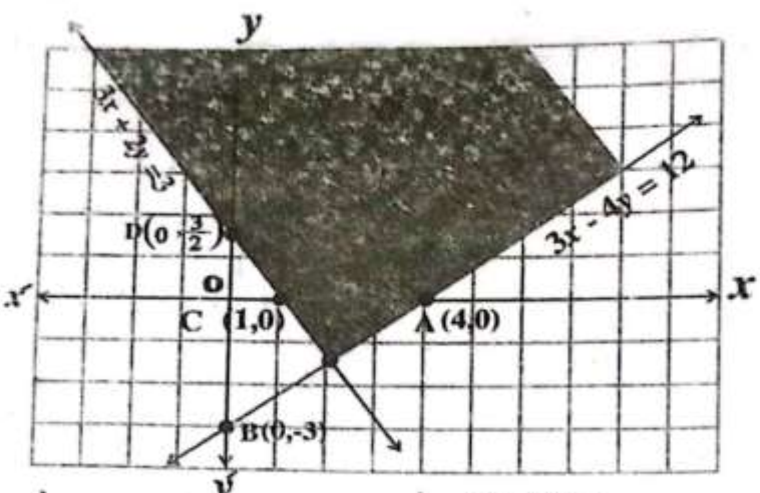


Fig. 3(i)(c)

(ii) $2x + y \leq 4; x + 2y \leq 6$

Solution:
$2x + y \leq 4$ (i)

$x + 2y \leq 6$ (ii)

The sketch of inequality (i) is shown as shaded region in the figure 3 (ii) (a) by joining the points $A(2,0)$ and $B(0,4)$.
The sketch of inequality (ii) is shown as shaded region in the figure 3 (ii) (b) by joining the points $C(6,0)$ and $D(0,3)$.
Now the solution region of the given systems of inequalities is the infersection of the graphs indicated in the figure 3(iii) (a), and 3 (iii)(b) and is shown as shaded region in the figure 3(ii)(c).

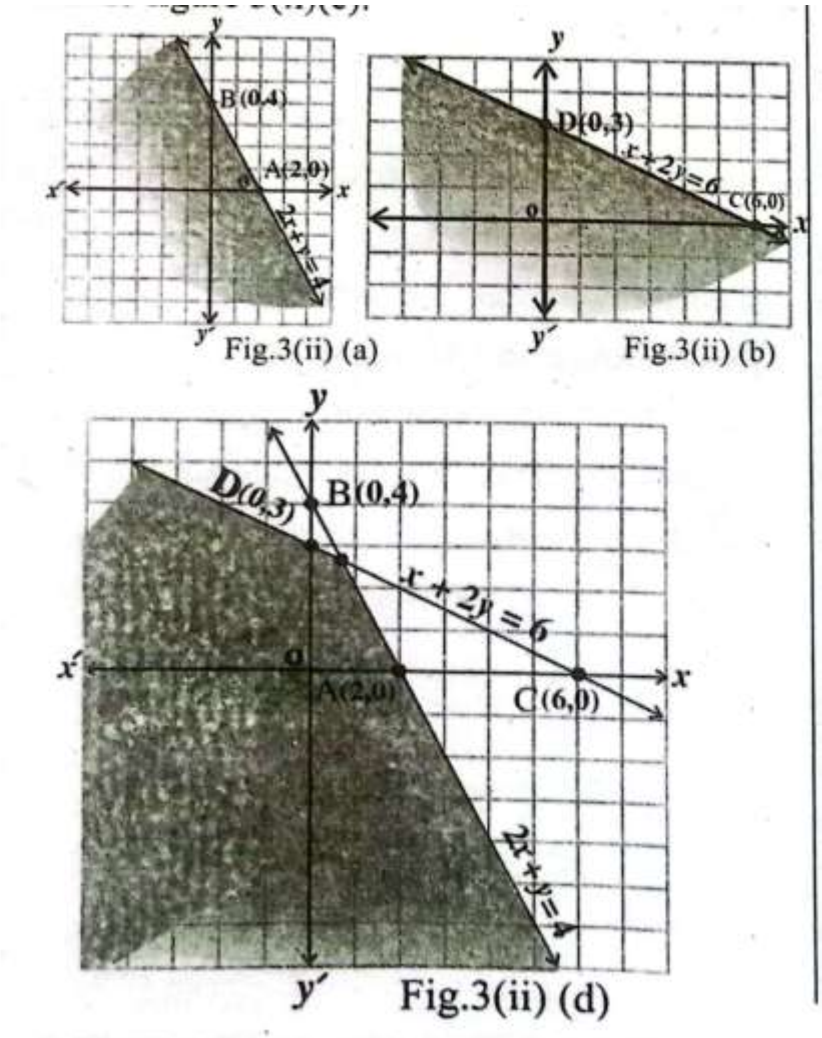


Fig.3(ii) (a)

Fig.3(ii) (b)

Fig.3(ii) (d)

Q. 4 Find the maximum value of $g(x, y) = x + 4y$ subject to constraints.

$$x + y \leq 4, x \geq 0 \text{ and } y \geq 0.$$

Solution:

$$g(x, y) = x + 4y$$

The constraints

$$x + y \leq 4 \quad \text{(i)}$$
$$x \geq 0 \quad \text{(ii)}$$
$$y \geq 0 \quad \text{(iii)}$$

The associated equation of inequality(i) is $x + y = 4$ (iv)

Draw the line of equation (iv) using x-intercept and y-intercept $A(4,0)$ and $B(0,4)$. The sketch of graph of inequality (i) is shown as shaded region in figure $4a$

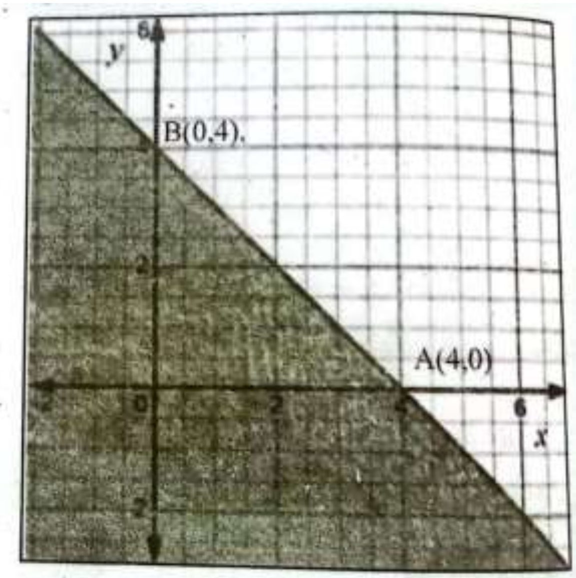


Fig.4(a)

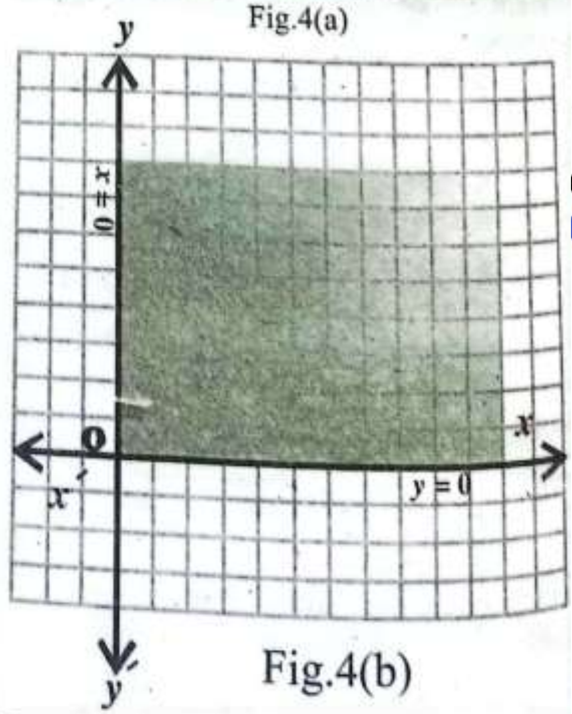


Fig.4(b)

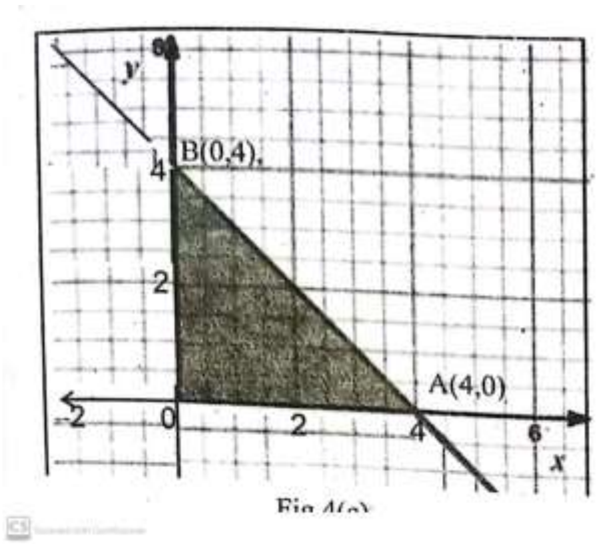


Fig. 4(c)
The sketch of inequalities (ii) and (iii) is shown as shaded region in figure it Now, the solution region of given systems of inequalities is the intersection of the graphs indicated in figure 4a and 4 b , which is shown as shaded region in figure to We observe that the comer points of solution region are $O(0,0), A(4,0), B(0,4)$.
Now, we find the value of $\varepsilon(x, y)$ at the corner points.

Corner points	f(x,y)=x+4y
O(0,0)	$f(0,0) = 0 + 4(0) = 0$
A(4,0)	$f(4,0) = 4 + 4(0) = 4$
B(0,4)	$(0,4) - 01 + (4) = 16$

Thus the given function is maximum at the comer point $(0,4)$
Q. 5 Find the minimum value of $f(x, y) = 3x + 5y$ subject to constraints. $x + 3y \geq 3, x + y \geq 2, x \geq 0, y \geq 0$. maser Solution:

$$
\begin{gathered}
x + 3y \geq 3 \\
x + y \geq 2 \\
x \geq 0, y \geq 0
\end{gathered}
$$

The associated equation of inequality (i) is $x + 3y = 3$ and it can be written in the form
$\frac{x}{3}, \frac{y}{1} = 1$ This line imersect the x axis and y-avis at point $A(3,0)$ and

$B(0,1)$ repectively. The sketch of ingualty (i) is shown as shaded region in the Fig 5(a)

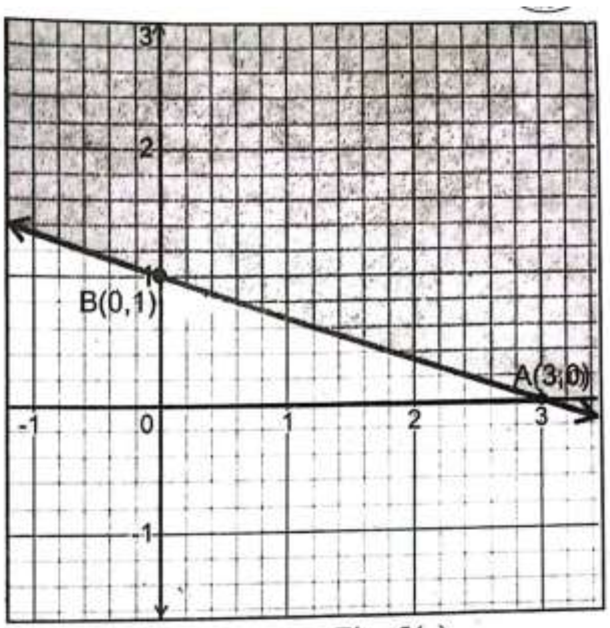
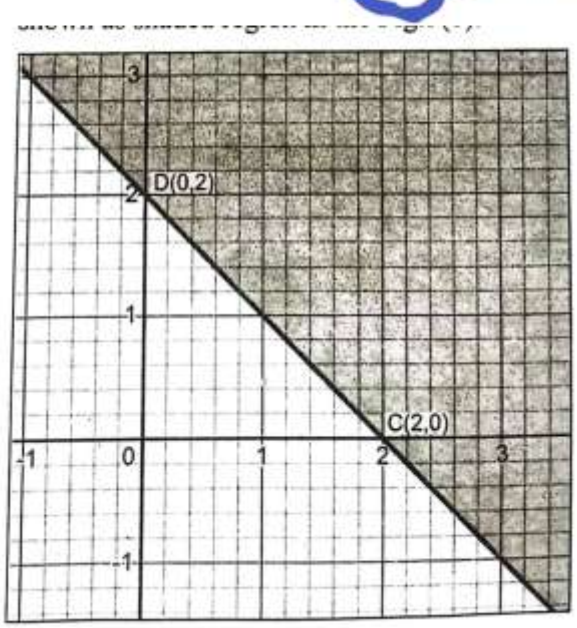


Fig. 5(a)

The associated equation of inequality (ii) $x + y = 2$ and it can be written in the for $\frac{x}{2} + \frac{y}{2} = 1$ This line intersects the x-axis and y-axis at point $\mathrm{C}(2,0)$ and $\mathrm{D}(0,2$ respectively. The sketch of inequality (i) lhown as shaded region in the Fig 5(b)



The graph of the inequalities in (iii) is the intersection region of graphs of $x \geq 0$ and

$y \geq 0$ which is the first quadrant shown as shaded region in Fig.6(c)

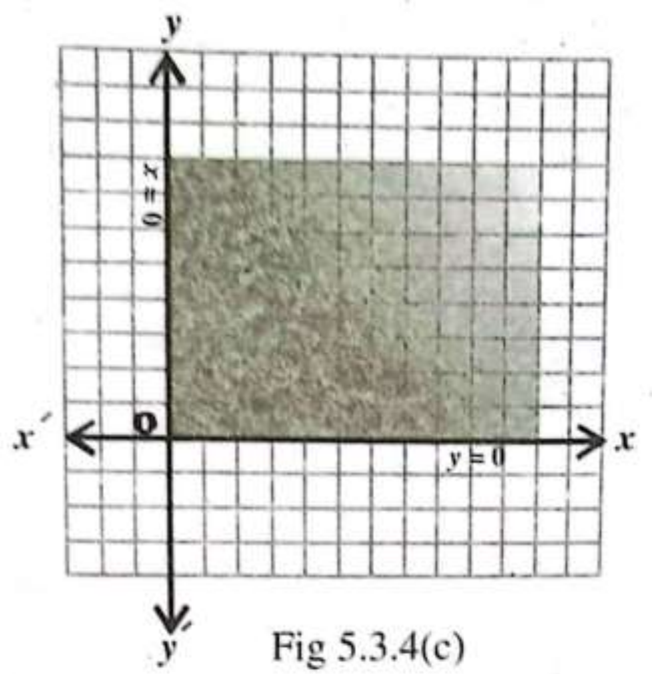


Fig 5.3.4(c)

Now the solution region of the given system of inequalities is the intersection of the graphs indicated in the Fig 5(a) 5(b) and 5(c) which is shown as shaded region in the Fig.(d)

We observe that solution region is unbounded and it comer points are $A(3,0)$, $\mathrm{D}(0,2)$ and $\mathrm{E}(1,5,0.5)$

Point E is the point of intersection of corresponding lines of inequalities (i) and (ii) which can be obtained by solving the corresponding equations simultaneously.

$x + 3\mathrm{y} = 3$ (iv)

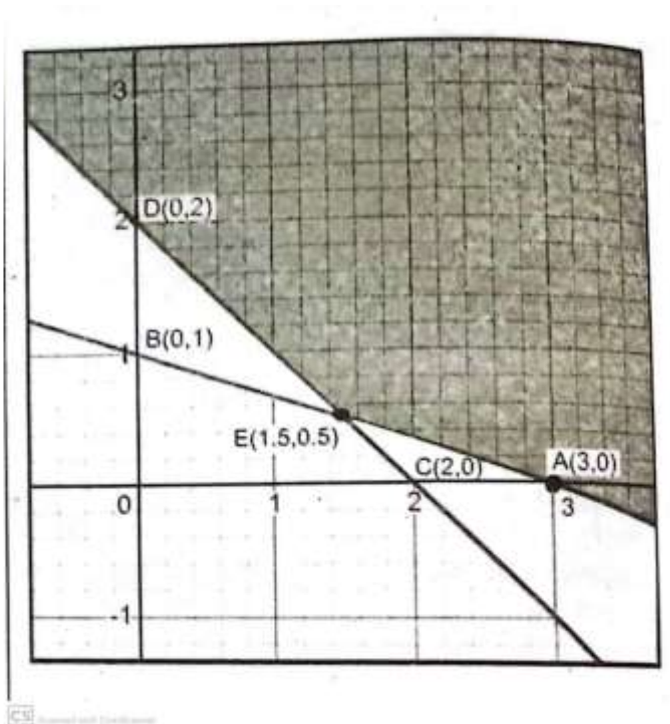
$x + y = 2$ (v)

Subtracting equation (v) from (iv)

$$(x + 3y) - (x + y) = 3 - 2$$
$$x + 3y - x - y = 1$$
$$3y = 1 \Rightarrow y = \frac{1}{2} = 0.5 \Rightarrow y = 0.5$$

Put it in eq (v)

$$x + 3(0.5) = 3$$
$$x + 1.5 - 3$$
$$\Rightarrow x = 3 - 1.5 = 1.5 \Rightarrow x = 1.5$$



Thus point of intersection is $E(1.5,0.5)$
Now; we find the values of $f(x,y) = 3x + 5y$ at comer points

Comer points	$f(x,y) = 3x + 5y$
A(3,0)	$f(3,0) = 3(3) + 5(0)$ $= 910 = 9$
D(0,2)	$f(0,2) = 0(3) + 5(2)$ $= 0 + 10 - 10$

E(1.5.0.5)	$f(1.5{,}0.5) = 3(1.5) + 5(0.5)$ $= 4.5 + 2.5 = 7$

We observe that the value of function is minimum 7 at comer point E(1.5,0.5).