

9th Math Exercise No.4.2

Q. 1 Factorize each of the following expressions:

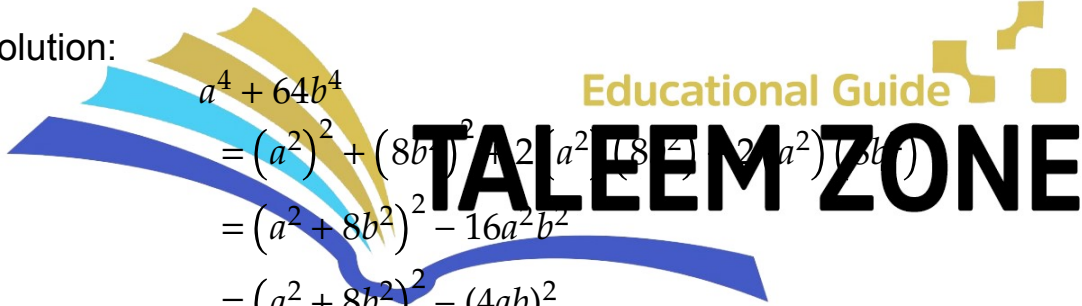
(i) $4x^4 + 81y^4$

Solution:

$$\begin{aligned} &4x^4 + 81y^4 \\ &= (2x^2)^2 + (9y^2)^2 + 2(2x^2)(9y^2) - 2(2x^2)(9y^2) \\ &= (2x^2 + 9y^2)^2 - 36x^2y^2 \\ &= (2x^2 + 9y^2)^2 - (6xy)^2 \\ &= (2x^2 + 9y^2 + 6xy)(2x^2 + 9y^2 - 6xy) \\ &= (2x^2 + 6xy + 9y^2)(2x^2 - 6xy + 9y^2) \end{aligned}$$

(ii) $a^4 + 64b^4$

Solution:



$$\begin{aligned} &a^4 + 64b^4 \\ &= (a^2)^2 + (8b^2)^2 + 2(a^2)(8b^2) - 2(a^2)(8b^2) \\ &= (a^2 + 8b^2)^2 - 16a^2b^2 \\ &= (a^2 + 8b^2)^2 - (4ab)^2 \\ &\quad [\because a^2 - b^2 = (a + b)(a - b)] \\ &= (a^2 + 8b^2 + 4ab)(a^2 + 8b^2 - 4ab) \\ &= (a^2 + 4ab + 8b^2)(a^2 - 4ab + 8b^2) \end{aligned}$$

(iii) $x^4 + 4x^2 + 16$

Solution:

$$x^4 + 4x^2 + 16$$

Rearrange

$$\begin{aligned} &= x^4 + 16 + 4x^2 \\ &= (x^2)^2 + (4)^2 + 4x^2 \end{aligned}$$

By adding and subtracting $2(x^2)(4)$

$$\begin{aligned}
&= (x^2)^2 + (4)^2 + 2(x^2)(4) - 2(x^2)(4) + 4x^2 \\
&= (x^2 + 4)^2 - 8x^2 + 4x^2 \\
&= (x^2 + 4)^2 - 4x^2 \\
&= (x^2 + 4)^2 - (2x)^2 \\
&= (x^2 + 4 + 2x)(x^2 + 4 - 2x) \\
&= (x^2 + 2x + 4)(x^2 - 2x + 4) \\
&\quad \text{(iv) } x^4 - 14x^2 + 1
\end{aligned}$$

Solution:

$$x^4 - 14x^2 + 1$$

Rearrange

$$\begin{aligned}
&= x^4 + 1 - 14x^2 \\
&= (x^2)^2 + (1)^2 - 14x^2
\end{aligned}$$

By adding and subtracting $2(x^2)(1)$

$$= (x^2)^2 + (1)^2 + 2(x^2)(1) - 2(x^2)(1) - 14x^2$$

$$= (x^2 + 1)^2 - 2x^2 - 14x^2$$

$$= (x^2 + 1)^2 - 16x^2$$

$$= (x^2 + 1)^2 - (4x)^2$$

$$\therefore a^2 - b^2 = (a + b)(a - b)$$

$$= (x^2 + 1 + 4x)(x^2 + 1 - 4x)$$

$$= (x^2 + 4x + 1)(x^2 - 4x + 1)$$

$$\text{(v) } x^4 - 30x^2y^2 + 9y^4$$

Solution:

$$x^4 - 30x^2y^2 + 9y^4$$

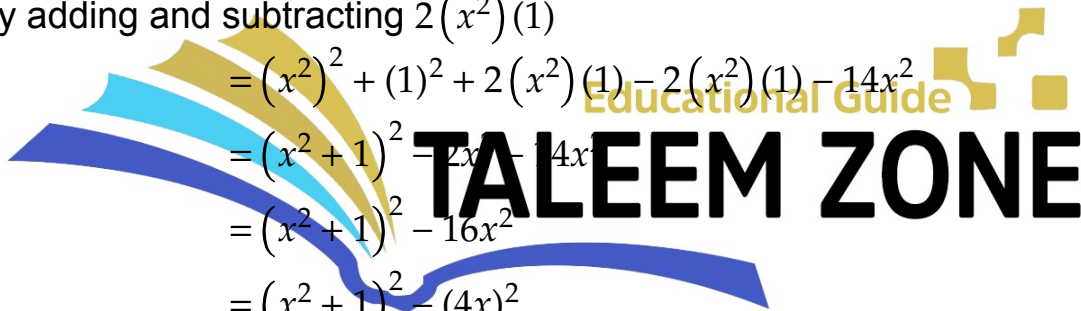
$$= x^4 + 9y^4 - 30x^2y^2$$

$$= (x^2)^2 + (3y^2)^2 + 2(x^2)(3y^2) - 2(x^2)(3y^2) - 30x^2y^2$$

$$= (x^2 + 3y^2)^2 - 6x^2y^2 - 30x^2y^2$$

$$= (x^2 + 3y^2)^2 - 36x^2y^2$$

$$= (x^2 + 3y^2)^2 - (6xy)^2$$



$$\begin{aligned}
 & [\because a^2 - b^2 = (a + b)(a - b)] \\
 & = (x^2 + 3y^2 + 6xy)(x^2 + 3y^2 - 6xy) \\
 & = (x^2 + 6xy + 3y^2)(x^2 - 6xy + 3y^2)
 \end{aligned}$$

(vi) $x^4 - 7x^2y^2 + y^4$

(Correction)

Solution:

$$\begin{aligned}
 & \text{Add and subtract } 2(x^2)(y^2) \\
 & \quad x^4 - 7x^2y^2 + y^4 \\
 & = (x^2)^2 + (y^2)^2 + 2(x^2)(y^2) - 2(x^2)(y^2) - 7x^2y^2 \\
 & = (x^2 + y^2)^2 - 2x^2y^2 - 7x^2y^2 \\
 & = (x^2 + y^2)^2 - 9x^2y^2 \\
 & = (x^2 + y^2)^2 - (3xy)^2
 \end{aligned}$$

$$\because a^2 - b^2 = (a + b)(a - b)$$

$$= (x^2 + y^2 - 3xy)(x^2 + y^2 + 3xy)$$

$$= (x^2 - 3xy + y^2)(x^2 + 3xy + y^2)$$

Q. 2 Factorize each of the following expressions

(i) $(x + 1)(x + 2)(x + 3)(x + 4) + 1$,

Solution:

$$(x + 1)(x + 2)(x + 3)(x + 4) + 1$$

By Rearranging the expressions

$$= (x - 1)(x - 4)(x + 2)(x + 3) + 1 (\because 1 + 4 = 2 + 3)$$

$$= (x^2 + 4x + x + 4)(x^2 + 3x + 2x + 6) + 1$$

$$= (x^2 + 5x + 4)(x^2 + 5x + 6) + 1$$

$$\text{Let } x^2 + 5x = y$$

Now, expression (i) can be written as

$$= (y + 4)(y + 6) + 1$$

$$= y^2 + 6y + 4y + 24 + 1$$

$$= y^2 + 10y + 25$$

$$= (y)^2 + 2(y)(5) + (5)^2$$

$$\because a^2 + 2ab + b^2 = (a + b)^2$$

$$= (y + 5)^2$$

$$= (x^2 + 5x + 5)^2 (\because y = x^2 + 5x)$$

(ii) $(x + 2)(x - 7)(x - 4)(x - 1) + 1$

Solution:

$$\begin{aligned} & (x + 2)(x - 7)(x - 4)(x - 1) + 17 \\ & \because 2 - 7 = -4 - \\ & = (x^2 - 7x + 2x - 14)(x^2 - x - 4x + 4) + 17... \\ & (x^2 - 5x - 14)(x^2 - 5x + 4) + 17 \end{aligned}$$

Let $x^2 - 5x = y$ (ii)

Now, expression (i) can be written as:

$$\begin{aligned} & = (y - 14)(y + 4) + 17 \\ & = y^2 + 4y - 14y - 56 + 17 \\ & = y^2 - 10y - 39 \\ & = y^2 - 13y + 3y - 39 \\ & = y(y - 13) + 3(y - 13) \\ & = (y - 13)(y + 3) \end{aligned}$$

$(\because y = x^2 + 5x)$

$= (x^2 - 5x - 13)(x^2 - 5x + 3)$

(iii) $(2x^2 + 7x + 3)(2x^2 + 7x + 5) + 1$

Solution:

$(2x^2 + 7x + 3)(2x^2 + 7x + 5) + 1$

Let $2x^2 + 7x = y$ (ii)

Now, expression (i) can be written as:

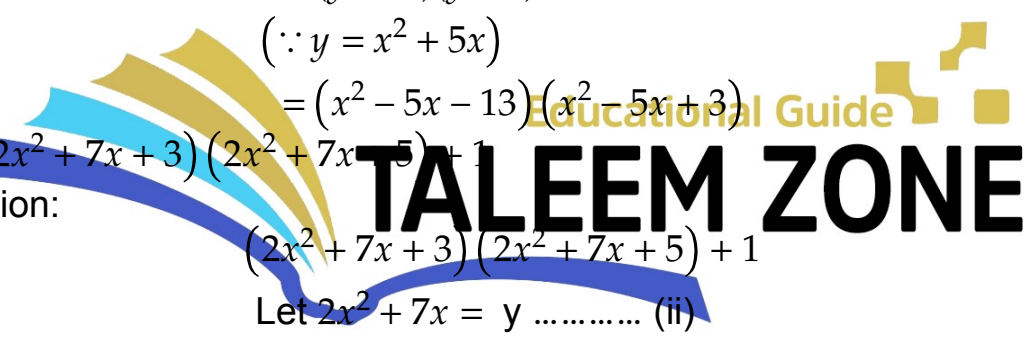
$$\begin{aligned} & = (y + 3)(y + 5) + 1 \\ & = y^2 + 5y + 3y + 15 + 1 \\ & = y^2 + 8y + 16 \\ & = (y)^2 + 2(y)(4) + (4)^2 \\ & = (y + 4)^2 \because a^2 + 2ab + b^2 = (a + b)^2 \end{aligned}$$

Now,

$$\begin{aligned} & = (2x^2 + 7x + 4)^2 (\because y = 2x^2 + 7x) \\ & (iv) (3x^2 + 5x + 3)(3x^2 + 5x + 5) - 3 \end{aligned}$$

Solution:

$(3x^2 + 5x + 3)(3x^2 + 5x + 5) - 3$



Let $3x^2 + 5x = y$

(ii)

Now, expression (i) can be written as:

$$\begin{aligned} &= (y + 3)(y + 5) - 3 \\ &= y^2 + 5y + 3y + 15 - 3 \\ &= y^2 + 8y + 12 \\ &= y^2 + 6y + 2y + 12 \\ &= y(y + 6) + 2(y + 6) \\ &= (y + 6)(y + 2) \\ &\therefore y = 3x^2 + 5x \end{aligned}$$

Now,

$$= (3x^2 + 5x + 6)(3x^2 + 5x + 2)$$

(v) $(x + 1)(x + 2)(x + 3)(x + 6) - 3x^2$

Solution:

$$(x + 1)(x + 2)(x + 3)(x + 6) - 3x^2$$

By rearranging the terms,

$$\begin{aligned} &= (x + 1)(x + 6)(x + 2)(x + 3) - 3x^2 \\ &= (x^2 + 6x + 6)(x^2 + 3x + 6) - 3x^2 \\ &= (x^2 + 7x + 6)(x^2 + 5x + 6) - 3x^2 \\ &= (x^2 + 6 + 7x)(x^2 + 6 + 5x) - 3x^2 \dots\dots(i) \end{aligned}$$

let $x^2 + 6 = y \dots\dots(ii)$

$$\begin{aligned} &= (y + 7x)(y + 5x) - 3x^2 \\ &= y^2 + 5xy + 7xy + 35x^2 - 3x^2 \\ &= y^2 + 12xy + 32x^2 \\ &= y^2 + 8xy + 4xy + 32x^2 \\ &= y(y + 8x) + 4x(y + 8x) \\ &= (y + 8x)(y + 4x) \therefore y = x^2 + 6 \end{aligned}$$

Now,

$$\begin{aligned} &= (x^2 + 6 + 8x)(x^2 + 6 + 4x) \\ &= (x^2 + 8x + 6)(x^2 + 4x + 6) \end{aligned}$$

(vi) $(x + 1)(x - 1)(x + 2)(x - 2) + 5x^2$

Solution: (Excepted correction)

$$\begin{aligned}
& (x+1)(x-1)(x+2)(x-2) + 5x^2 \\
&= (x^2 - 1^2)(x^2 - 2^2) + 5x^2 \\
&= (x^2 - 1)(x^2 - 4) + 5x^2 \\
&= x^4 - 4x^2 - x^2 + 4 + 5x^2 \\
&= x^4 - 5x^2 + 4 + 5x^2 \\
&= x^4 + 4 \\
&= (x^2)^2 + (2)^2 + 2(x^2)(2) - 2(x^2)(2) \\
&= (x^2 + 2)^2 - 4x^2 \\
&= (x^2 + 2)^2 - (2x)^2 \\
&= (x^2 + 2 + 2x)(x^2 + 2 - 2x) \\
&= (x^2 + 2x + 2)(x^2 - 2x + 2)
\end{aligned}$$

Q. 3 Factorize:

(i) $8x^3 + 12x^2 + 6x + 1$

$$\begin{aligned}
& 8x^3 + 12x^2 + 6x + 1 \\
&= (2x)^3 + 3(2x)^2(1) + 3(2x)(1)^2 + (1)^3 \\
&\because a^3 + 3a^2b + 3a^2b + a^2 + b^3 = (a+b)^3 \\
&= (2x + 1)^3
\end{aligned}$$

(ii) $27a^3 + 108a^2b + 144ab^2 + 64b^3$

$$\begin{aligned}
& 27a^3 + 108a^2b + 144ab^2 + 64b^3 \\
&= (3a)^3 + 3(3a)^2(4b) + 3(3a)(4b)^2 + (4b)^3 \\
&= (3a + 4b)^3
\end{aligned}$$

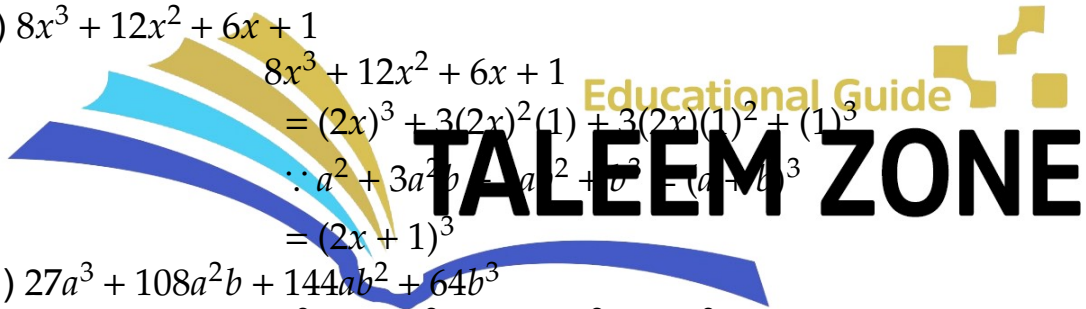
(iii) $x^3 + 48x^2y + 108xy^2 + 216y^3$

(Wrong question, expected correction)

Solution

$$\begin{aligned}
& x^3 + 18x^2y + 108xy^2 + 216y^3 \\
&= (x)^3 + 3(x)^2(6y) + 3(x)(6y)^2 + (6y)^3 \\
&= (x + 6y)^3
\end{aligned}$$

(iv) $8x^3 - 125y^3 + 150xy^2 - 60x^2y$



$$\begin{aligned}
 &8x^3 - 125y^3 + 150xy^2 - 60x^2y \\
 &= 8x^3 - 60x^2y + 150xy^2 - 125y^3 \\
 &= (2x)^3 - 3(2x)^2(5y) + 3(2x)(5y)^2 - (5y)^3 \\
 &= (2x - 5y)^3
 \end{aligned}$$

Q4. Factorize:

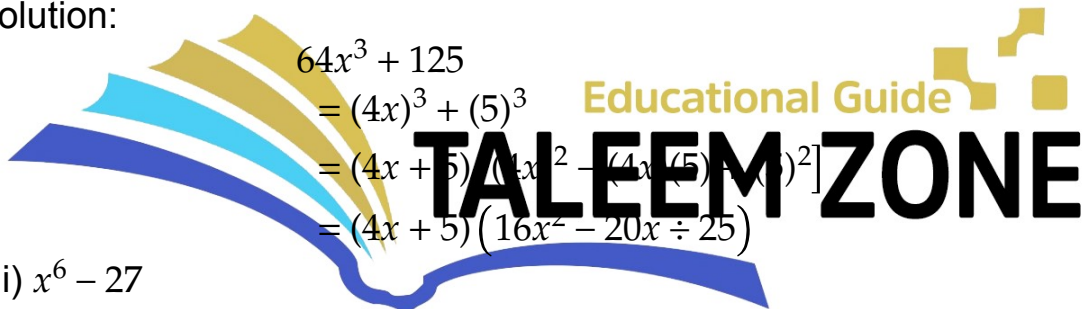
(i) $125a^3 - 1$

Solution:

$$\begin{aligned}
 &125a^3 - 1 \\
 &= (5a)^3 - (1)^3 \\
 &= (5a - 1) [(5a)^2 + (5a)(1) + (1)^2] \\
 &= (5a - 1) (25a^2 + 5a + 1)
 \end{aligned}$$

(ii) $64x^3 + 125$

Solution:



$$\begin{aligned}
 &64x^3 + 125 \\
 &= (4x)^3 + (5)^3 \\
 &= (4x + 5) [(4x)^2 - (4x)(5) + (5)^2] \\
 &= (4x + 5) (16x^2 - 20x + 25)
 \end{aligned}$$

(iii) $x^6 - 27$

Solution:

$$\begin{aligned}
 &x^6 - 27 \\
 &= (x^2)^3 - (3)^3 \\
 &= (x^2 - 3) [(x^2)^2 + (x^2)(3) + (3)^2] \\
 &= (x^2 - 3) (x^4 + 3x^2 + 9)
 \end{aligned}$$

(iv) $1000a^3 + 1$

Solution:

$$\begin{aligned}
 &1000a^3 + 1 \\
 &= (10a)^3 + (1)^3 \\
 &= (10a + 1) [(10a)^2 - (10a)(1) + (1)^2] \\
 &= (10a + 1) (100a^2 - 10a + 1)
 \end{aligned}$$

(v) $343x^3 + 216$

Solution:

$$\begin{aligned} & 343x^3 + 216 \\ &= (7x)^3 + (6)^3 \\ &= (7x + 6) [(7x)^2 - (7x)(6) + (6)^2] \\ &= (7x + 6) (49x^2 - 42x + 36) \end{aligned}$$

(vi) $27 - 512y^3$

Solution

$$\begin{aligned} & 27 - 512y^3 \\ &= (3)^3 - (8y)^3 \\ &= (3 - 8y) [(3)^2 + (3)(8y) + (8y)^2] \\ &= (3 - 8y) (9 + 24y + 64y^2) \end{aligned}$$

