

EXERCISE 7.3 - PERMUTATIONS WITH REPETITIONS & ROUND TABLE

Complete Solutions

Question 1: Arrangements of letters of words taken all together

(i) PAKISTAN

Step 1: Count letters: P=1, A=2, K=1, I=1, S=1, T=1, N=1

- Total letters: 8
- Repetitions: A appears 2 times

Step 2: Number of arrangements

$$\frac{8!}{2!} = \frac{40320}{2} = 20160$$

Answer: 20160

(ii) CURRICULUM

Step 1: Count letters: C=2, U=2, R=2, I=1, L=1, M=1

- Total letters: 9
- Repetitions: C(2), U(2), R(2)

Step 2: Number of arrangements

$$\frac{9!}{2! \cdot 2! \cdot 2!} = \frac{362880}{8} = 45360$$

Answer: 45360

(iii) PROBABILITY

Step 1: Count letters: P=1, R=1, O=1, B=2, A=1, I=2, L=1, T=1, Y=1

- Total letters: 11
- Repetitions: B(2), I(2)

Step 2: Number of arrangements

$$\frac{11!}{2! \cdot 2!} = \frac{39916800}{4} = 9979200$$

Answer: 9979200

Question 2: Permutations of "BANANA" with B first

Step 1: B fixed at first position

- Remaining letters: A, N, A, N, A (5 letters with A=3, N=2)

Step 2: Number of arrangements

$$\frac{5!}{3! \cdot 2!} = \frac{120}{6 \times 2} = \frac{120}{12} = 10$$

Answer: 10

Question 3: Arrangements of TRIGONOMETRY beginning with T and ending with Y

Step 1: T fixed at first, Y fixed at last

- Remaining letters: R, I, G, O, N, O, M, E, T, R
- Total: 10 letters with R=2, O=2, T=1, I=1, G=1, N=1, M=1, E=1

Step 2: Number of arrangements

$$\frac{10!}{2! \cdot 2!} = \frac{3628800}{4} = 907200$$

Answer: 907200

Question 4: Arrangements of 9 marbles (4 red, 3 blue, 2 green)

Step 1: Total marbles = 9

- Red = 4, Blue = 3, Green = 2

Step 2: Number of distinct arrangements

$$\frac{9!}{4! \cdot 3! \cdot 2!} = \frac{362880}{24 \times 6 \times 2} = \frac{362880}{288} = 1260$$

Answer: 1260

Question 5: Persons sitting around a round table

(a) 8 persons

- Number of arrangements = $(8 - 1)! = 7! = 5040$

Answer: 5040

(b) 7 persons

- Number of arrangements = $(7 - 1)! = 6! = 720$

Answer: 720

(c) 6 persons

- Number of arrangements = $(6 - 1)! = 5! = 120$

Answer: 120

Question 6: 3 couples around round table, no two women together

Step 1: Arrange 3 men around round table first

- Number of ways = $(3 - 1)! = 2! = 2$

Step 2: There are 3 gaps between men (including between last and first)

- Women must sit in these 3 gaps to ensure no two women are together
- 3 women in 3 gaps: $3! = 6$ ways

Step 3: Total = $2 \times 6 = 12$

Answer: 12

Question 7: 6-digit numbers from 7,7,8,8,9,9

Step 1: Total digits = 6

- Repetitions: 7 appears 2 times, 8 appears 2 times, 9 appears 2 times

Step 2: Number of arrangements

$$\frac{6!}{2! \cdot 2! \cdot 2!} = \frac{720}{8} = 90$$

Answer: 90

Question 8: 15 members into committees of 3, 5, 4, 3

Step 1: Choose members sequentially

- Committee 1 (3 members): $^{15}C_3$
- Committee 2 (5 members): $^{12}C_5$
- Committee 3 (4 members): $^7C_4$
- Committee 4 (3 members): $^3C_3$

Step 2: Total

$$\begin{aligned}
 & ^{15}C_3 \times ^{12}C_5 \times ^7C_4 \times ^3C_3 \\
 &= \frac{15!}{3!12!} \times \frac{12!}{5!7!} \times \frac{7!}{4!3!} \times \frac{3!}{3!0!} \\
 &= \frac{15!}{3! \cdot 5! \cdot 4! \cdot 3!}
 \end{aligned}$$

Step 3: Calculate

$$\frac{15!}{6 \times 120 \times 24 \times 6} = \frac{1307674368000}{103680} = 12612600$$

Answer: 12,612,600

Question 9: 11 D.C.Os at round table, two particular sit together

Step 1: Treat the two particular D.C.Os as one unit

- Total units = 10

Step 2: Arrange 10 units around round table

- Number = $(10 - 1)! = 9!$

Step 3: Arrange the two particular D.C.Os within their unit

- Number = $2! = 2$

Step 4: Total = $9! \times 2 = 362880 \times 2 = 725760$

Answer: 725760

Question 10: 14 officers at round table

Step 1: Number of arrangements at round table

- $(14 - 1)! = 13! = 6227020800$

Answer: 6,227,020,800

Question 11: 14 guests (9 males, 5 females) at two round tables

Step 1: One table for 9 males, one table for 5 females

Step 2: Arrange 9 males at round table

- $(9 - 1)! = 8! = 40320$

Step 3: Arrange 5 females at round table

- $(5 - 1)! = 4! = 24$

Step 4: Total = $8! \times 4! = 40320 \times 24 = 967680$

Answer: 967680

Summary Table

Question	Answer
1(i)	20160

Question	Answer
1(ii)	45360
1(iii)	9979200
2	10
3	907200
4	1260
5(a)	5040
5(b)	720
5(c)	120
6	12
7	90
8	12,612,600
9	725760
10	6,227,020,800
11	967680

EXERCISE 7.3 - ROUND TABLE ARRANGEMENTS (Continued)

Question 12: 5 men and 5 women at round table, no two same sex together

Step 1: To have no two persons of same sex together, men and women must alternate

- Pattern: M W M W M W M W M W

Step 2: Arrange 5 men around round table first

- Number of ways = $(5 - 1)! = 4! = 24$

Step 3: Arrange 5 women in the 5 gaps between men

- Number of ways = $5! = 120$

Step 4: Total = $24 \times 120 = 2880$

Answer: 2880

Question 13: 8 keys arranged in a circular key ring

Step 1: For a circular arrangement of n distinct objects, number = $(n - 1)!$

Step 2: However, a key ring can be flipped over (no distinction between clockwise and counterclockwise)

- Number = $\frac{(n-1)!}{2}$

Step 3: For $n = 8$

$$\frac{(8-1)!}{2} = \frac{7!}{2} = \frac{5040}{2} = 2520$$

Answer: 2520

Question 14: Necklaces from 10 beads of different colours

Step 1: For a necklace (circular arrangement that can be flipped), number = $\frac{(n-1)!}{2}$

Step 2: For $n = 10$

$$\frac{(10-1)!}{2} = \frac{9!}{2} = \frac{362880}{2} = 181440$$

Answer: 181440

Summary Table

Question	Answer
12	2880
13	2520
14	181440

Key Formula Review

Scenario	Formula
Linear arrangement of n distinct objects	$n!$
Circular arrangement of n distinct objects	$(n-1)!$
Circular arrangement with flip (necklace/key ring)	$\frac{(n-1)!}{2}$
Arrangements with repetitions	$\frac{n!}{p!q!r!\dots}$
Treating objects as one unit	Reduce total objects, multiply by internal arrangements