$$= \frac{-6-3i+8i+4i^2}{2^2-i^2} = \frac{-6+5i-4}{4-(-1)} = \frac{-10+5i}{5} \Rightarrow z = -2+i$$

Taking conjugate

$$\bar{z} = \overline{-2+i} = -2-i$$

$|\bar{z}| = |-2-i|$.

$$|\bar{z}| = \sqrt{(-2)^2+(-1)^2} |\bar{z}| = \sqrt{4+1} \Rightarrow |\bar{z}| = \sqrt{5}$$

EXVERCISE 1.1

Q. 1 Find the multiplicative inverse of each of the following complex numbers.

11301004
(i) $(-4,7)$

Solution:

If $z=(a,b)$ then:

$$z^{-1} = \left(\frac{a}{a^2+b^2}, \frac{-b}{a^2+b^2}\right)$$

Let $z=(-4,7)$, then

$$z^{-1} = \left(\frac{-4}{(-4)^2+7^2}, \frac{-7}{(-4)^2+7^2}\right) z^{-1} = \left(\frac{-4}{16+49}, \frac{-7}{16+49}\right) z^{-1} = \left(\frac{-4}{65}, \frac{-7}{65}\right)$$

(ii) $(\sqrt{2}, -\sqrt{5})$

Solution:

If $z=(a,b)$ then:

$$z^{-1} = \left(\frac{a}{a^2+b^2}, \frac{-b}{a^2+b^2}\right)$$

Let
$z=(\sqrt{2}, -\sqrt{5})$, then

$$z^{-1} = \left(\frac{\sqrt{2}}{(\sqrt{2})^2+(-\sqrt{5})^2}, \frac{-(-\sqrt{5})}{(\sqrt{2})^2+(-\sqrt{5})^2}\right) z^{-1} = \left(\frac{\sqrt{2}}{2+5}, \frac{\sqrt{5}}{2+5}\right) z^{-1} = \left(\frac{\sqrt{2}}{7}, \frac{\sqrt{5}}{7}\right)$$

(iii) $(1,0)$

Solution:

If $z=(a,b)$ then

$$z^{-1} = \left(\frac{a}{a^2+b^2}, \frac{-b}{a^2+b^2}\right)$$

Let $z=(1,0)$, then

$$z^{-1} = \left(\tfrac{1}{(1)^2+(0)^2}, \tfrac{0}{(1)^2+(0)^2}\right)$$
$$z^{-1} = \left(\tfrac{1}{1}, \tfrac{0}{1}\right)$$
$$z^{-1} = (1,0)$$

Q. 2 Separate into real and imaginary parts (write as a simple complex number).
(i) $\frac{2-7i}{4+5i}$

Solution:

$$\begin{aligned}\frac{2-7i}{4+5i} &= \frac{2-7i}{4+5i}\times\frac{4-5i}{4-5i}\\ &= \frac{8-10i-28i+35i^2}{4^2-(5i)^2}\\ &= \frac{8-38i+35(-1)}{16-25i^2}\\ &= \frac{8-38i-35}{16-25(-1)}\end{aligned}$$

UNIQUE NOTES

$$\begin{aligned}&= \frac{-27-38i}{16+25}\\ &= \frac{-27-38i}{41}\\ &= -\frac{27}{41}-\frac{38i}{41}\end{aligned}$$

Real part $= -\frac{27}{41}$
imaginary part $= -\frac{38}{41}$
(ii) $\frac{(-2+3i)^2}{1+i}$

Solution:

$$\begin{aligned}\frac{(-2+3i)^2}{1+i} &= \frac{4-12i+9i^2}{1+i}\\ &= \frac{4-12i+9(-1)}{1+i}\\ &= \frac{4-12i-9}{1+i}\\ &= \frac{-5-12i}{1+i}\\ &= \frac{-5-12i}{1+i}\times\frac{1-i}{1-i}\\ &= \frac{-5+5i-12i+12i^2}{(1)^2-i^2}\\ &= \frac{-5-7i+12(-1)}{1-(-1)}\\ &= \frac{-5-7i-12}{1+1}\\ &= \frac{-17-7i}{2}\\ &= -\frac{17}{2}-\frac{7i}{2}\end{aligned}$$

$$\begin{aligned}\Rightarrow\ \text{Real part } &= -\frac{17}{2}\\ \text{imaginary part } &= -\frac{7}{2}\end{aligned}$$

(iii) $\frac{i}{1+i}$

Solution:

$$\begin{aligned}\frac{i}{1+i} &= \frac{i}{1+i}\times\frac{1-i}{1-i}\\ &= \frac{i-i^2}{1^2-i^2}\\ &= \frac{i-(-1)}{1-(-1)}\\ &= \frac{i+1}{1+1}\\ &= \frac{1+i}{2}\\ &= \frac{1}{2}+\frac{1}{2}i\end{aligned}$$

So, Real part $= \frac{1}{2}$
imaginary part $= \frac{1}{2}$
(iv) $\frac{(4+3i)^2}{4-3i}$

Solution:

$$\frac{(4+3i)^2}{4-3i} = \frac{16+24i+9i^2}{4-3i}$$
$$= \frac{16+24i+9(-1)}{4-3i}$$
$$= \frac{16+24i-9}{4-3i}$$
$$= \frac{7+24i}{4-3i}$$
$$= \frac{7+24i}{4-3i} \times \frac{4+3i}{4+3i}$$
$$= \frac{28+21i+96i+72i}{(4)^2-(3i)^2}$$
$$= \frac{28+117i+72(-1)}{16-9(-1)}$$

$$= \frac{28+117i-72}{16+9}$$
$$= \frac{-44+117i}{25}$$
$$= \frac{-44}{25} + \frac{117i}{25}$$

So, Real part $= -\frac{44}{25}$
imaginary part $= \frac{117}{25}$

Q. 3 Prove that $\bar{z} = z$ iff z is real.

11301006

Solution:

Let $z = x + iy$

Then $\bar{z} = x - iy$

Where $x, y \in R$
Suppose that: $\bar{z} = z$

$$\Rightarrow x - iy = x + iy \Rightarrow x - iy - x - iy = 0 \Rightarrow -2iy = 0 \Rightarrow y = 0 \because -2 \neq 0, i \neq 0$$

Putting $y = 0$ in (i), we get
$z = x + i.0 = x \in R$ i.e. z is real
Conversely suppose that z is real, then imaginary part of $z = 0 \Rightarrow y = 0$
Putting $y = 0$ in (i) and (ii), we get

$$z = x + i0 = x + 0 = x \bar{z} = x - i0 = x - 0 = x \Rightarrow z = \bar{z}$$

Thus $\bar{z} = z$ iff z is real.
Q. 4 For $z \in C$, show that:

11301007
(i) $\frac{z+\bar{z}}{2} = Re(z)$.

Solution:

$$\because z \in C$$

Let $z = x + iy$ then
$\bar{z} = x - iy$, where $x, y \in R$ and $i = \sqrt{-1}$
Now L.H.S.

$$= \frac{z + \bar{z}}{2} = \frac{x + iy + (x - iy)}{2} = \frac{x + iy + x - iy}{2} = \frac{2x}{2} = x = Re(z) =$$

R.H.S
Hence proved:
(ii) $\frac{z-\bar{z}}{2i} = Im(z)$

Solution:

$$\because z \in C$$

Let $z = x + iy$ then $\bar{z} = x - iy$, where
$x, y \in R$ and $i = \sqrt{-1}$
Now, L.H.S

$$= \frac{z - \bar{z}}{2i} = \frac{x + iy - (x - iy)}{2i} = \frac{x + iy - x + iy}{2i} = \frac{2iy}{2i} = y = Im(z) =$$

R.H.S
Hence proved.
(iii) $|z|^2 = z \cdot \bar{z}$

Solution:

$$\because z \in C$$

Let $z = x + iy$ then $\bar{z} = x - iy$,where $x, y \in R$ and $i = \sqrt{-1}$
Now R.H.S $= z \cdot \bar{z} = (x + iy)(x - iy)$

$$= x^2 - (iy)^2 = x^2 - i^2 y^2 = x^2 - (-1)y^2 = x^2 + y^2 = \left(\sqrt{x^2 + y^2}\right)^2 = |z|^2 =$$

R.H.S
Hence proved.
Q. 5 If $z_1 = 2 + i, z_2 = 3 - 2i, z_3 = 1 + 3i$ then express $\frac{\bar{z}_1 \bar{z}_3}{z_2}$ in form of $a + ib$.

11301008

Solution:

$$\begin{aligned}
\frac{\bar{z}_1 \bar{z}_3}{z_2} &= \frac{(\overline{2+i}) \cdot (\overline{1+3i})}{3-2i} \\
&= \frac{(2-i)(1-3i)}{3-2i} \\
&= \frac{2-6i-i+3i^2}{3-2i} \\
&= \frac{2-7i+3(-1)}{3-2i} \\
&= \frac{2-7i-3}{3-2i} \\
&= \frac{-1-7i}{3-2i} \\
&= \frac{-1-7i}{3-2i} \times \frac{3+2i}{3+2i} \\
&= \frac{-3-2i-21i-14i^2}{(3)^2-(2i)^2} \\
&= \frac{-3-23i-14(-1)}{9-4i^2} \\
&= \frac{-3-23i+14}{9-4(-1)} \\
&= \frac{11-23i}{9+4} \\
&= \frac{11}{13} - \frac{23}{13} i
\end{aligned}$$

So, Real part $= \frac{11}{13}$
imaginary part $= -\frac{23}{13}$
Q. 6 If $z_1 = 2 + 7i$ and $z_2 = -5 + 3i$, then evaluate the following:

11301009
(i) $|2z_1 - 4z_2|$

Solution:

$$
\begin{aligned}
|2z_1 - 4z_2| &= |2(2+7i) - 4(-5+3i)| \\
&= |4 + 14i + 20 - 12i| \\
&= |24 + 2i| \\
&= \sqrt{(24)^2 + (2)^2} \\
&= \sqrt{576 + 4} \\
&= \sqrt{580} \\
&= \sqrt{4 \times 145} \\
&= 2\sqrt{145}
\end{aligned}
$$

(ii) $|3z_1 + 2\bar{z}_1|$

Solution:

$$
\begin{aligned}
|3z_1 + 2z_1| &= |3(2+7i) + 2(\overline{2+7i})| \\
&= |3(2+7i) + 2(2-7i)| \\
&= |6 + 21i + 4 - 14i| \\
&= |10 + 7i| \\
&= \sqrt{(10)^2 + (7)^2} \\
&= \sqrt{100 + 49} \\
&= \sqrt{149}
\end{aligned}
$$

(iii) $|-7z_2 + 2\overline{z_2}|$

Solution:

$$
\begin{aligned}
|-7z_2 + 2\overline{z_2}| &= |-7(-5+3i) + 2(\overline{-5+3i})| \\
&= |-7(-5+3i) + 2(-5-3i)| \\
&= |35 - 21i - 10 - 6i| \\
&= |25 - 27i|
\end{aligned}
$$

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$$
\begin{aligned}
&= \sqrt{(25)^2 + (-27)^2} \\
&= \sqrt{625 + 729} \\
&= \sqrt{1354}
\end{aligned}
$$

(iv) $\left|(z_1 + z_2)^3\right|$

Solution:

$$
\begin{aligned}
\left|(z_1 + z_2)^3\right| &= \left|(2 + 7i - 5 + 3i)^3\right| \\
&= \left|(-3 + 10i)^3\right| \\
&= \left|(10i - 3)^3\right| \\
&= \left|(10i)^3 - (3)^3 - 3(10i)(3)(10i - 3)\right| \\
&= \left|1000i^3 - 27 - 90i(10i - 3)\right| \\
&= \left|1000i^2 \cdot i - 27 - 900i^2 + 270i\right| \\
&= |1000(-1) \cdot i - 27 - 900(-1) + 270i| \\
&= |-1000i - 27 + 900 + 270i| \\
&= |873 - 730i| \\
&= \sqrt{(873)^2 + (-730)^2} \\
&= \sqrt{762129 + 532900} \\
&= \sqrt{1295029} \\
&= 109\sqrt{109}
\end{aligned}
$$

Q. 7 Show that: $i^{n+1}+i^{n+2}+i^{n+3}+i^{n+4}=0$ for all $n \in N$.

Solution:

To show that:

$$i^{n+1}+i^{n+2}+i^{n+3}+i^{n+4}=0$$

L.H.S:

$= i^{n+1}+i^{n+2}+i^{n+3}+i^{n+4} = i^n.i+i^n.i^2+i^n.i^3+i^n.i^4 = i^n.i+i^n.i^2+i^n.i^2.i+i^n.\left(i^2\right)^2 = i^n\cdot i+i^n\cdot(-1)+i^n(-1)\cdot i+i^n\cdot$

Because $i^2=-1$
=R.H.S
Hence proved.
Q. 8 Find the least positive value of n,
if: $\left(\frac{1+i}{1-i}\right)^{2n}=1$
11301011

Solution:

Given that:

$$\begin{aligned}
\left(\tfrac{1+i}{1-i}\right)^{2n} &= 1\\
\left(\tfrac{1+i}{1-i}\times\tfrac{1+i}{1+i}\right)^{2n} &= 1\\
\left[\tfrac{(1+i)^2}{1-i^2}\right]^{2n} &= 1\\
\left[\tfrac{1+i^2+2i}{1-(-1)}\right]^{2n} &= 1\\
\left[\tfrac{1+(-1)+2i}{1+1}\right]^{2n} &= 1\\
\left[\tfrac{1-1+2i}{2}\right]^{2n} &= 1\\
\left[\tfrac{2i}{2}\right]^{2n} &= 1\\
[i]^{2n} &= 1\\
i^{2n} &= 1
\end{aligned}$$

$$i^{2n}=i^4$$

by comparing, we have

$$2n=4n=\frac{4}{2}n=2$$

.

$$=i^n\cdot i-i^n-i^n\cdot i+i^n=0$$

So, the least positive value of n is 2 .
Q. 9 Show that, the value of i^n for $n \in N$ and $n>4$ is i^r,where r is the remainder when n is divided by 4 .

Solution:

Let $n=4k+r$,where k is positive integer and r is the remainder when n is divisible by 4 .
Now,

$$
\begin{aligned}
i^n &= i^{4k+r} \\
&= i^{4k} \cdot i^r \\
&= \left(i^2\right)^{2k} \cdot i^r \\
&= (-1)^{2k} \cdot i^r \\
&= 1 i^r \because 2k \text{ is even.} \\
i^n &= i^r
\end{aligned}
$$

Hence proved.
Equality of two complex numbers
The two complex numbers $z_1 = a + ib$ and $z_2 = c + id$ are said to be equal iff their real and imaginary parts are equal. i.e. $a + ib = c + id \Longleftrightarrow a = c$ and $b = d$.
Example 4: If $(3 + 2i)(x + iy) = 5 + 12i$,
where $x, y \in R$, then find the values of x and y.

11301013

Solution:

Given that:

$$
\begin{aligned}
& (3+2i)(x+iy) = 5 + 12i \\
\Rightarrow \quad & 3x + 3iy + 2ix + 2yi^2 = 5 + 12i \\
\Rightarrow \quad & (3x - 2y) + (2x + 3y)i = 5 + 12i
\end{aligned}
$$

Comparing real and imaginary parts, we have

$$
\begin{aligned}
3x - 2y &= 5 \\
2x + 3y &= 1?
\end{aligned}
$$

Add the

$$9x - 6y$$

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Squal
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$$p^2 - 2pq \because (a \therefore (1 = ($$

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