EXERCISE 10.1 - TRIGONOMETRIC FUNCTIONS

Complete Solutions

Question 1: Find values without using tables

(i) $\cos(-\mathbf{1230}^\circ)$

Step 1: $\cos(-\theta) = \cos\theta$, so $\cos(-1230^\circ) = \cos(1230^\circ)$

Step 2: Subtract multiples of 360°: $1230^\circ - 3(360^\circ) = 1230^\circ - 1080^\circ = 150^\circ$

Step 3: $\cos 150^\circ = \cos(180^\circ - 30^\circ) = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$

Answer: $-\frac{\sqrt{3}}{2}$

(ii) $\tan(-\mathbf{1035}^\circ)$

Step 1: $\tan(-\theta) = -\tan\theta$, so $\tan(-1035^\circ) = -\tan(1035^\circ)$

Step 2: $1035^\circ - 2(360^\circ) = 1035^\circ - 720^\circ = 315^\circ$

Step 3: $\tan 315^\circ = \tan(360^\circ - 45^\circ) = -\tan 45^\circ = -1$

Step 4: $\tan(-1035^\circ) = -(-1) = 1$

Answer: 1

(iii) $\sec(\mathbf{1140}^\circ)$

Step 1: $1140^\circ - 3(360^\circ) = 1140^\circ - 1080^\circ = 60^\circ$

Step 2: $\sec 60^\circ = \frac{1}{\cos 60^\circ} = \frac{1}{1/2} = 2$

Answer: 2

(iv) $\csc(-\mathbf{690}^\circ)$

Step 1: $\csc(-\theta) = -\csc\theta$, so $\csc(-690^\circ) = -\csc(690^\circ)$

Step 2: $690^\circ - 360^\circ = 330^\circ$

Step 3: $\csc 330^\circ = \frac{1}{\sin 330^\circ} = \frac{1}{-\sin 30^\circ} = \frac{1}{-1/2} = -2$

Step 4: $\csc(-690^\circ) = -(-2) = 2$

Answer: 2

(v) $\cot(\mathbf{1320}^\circ)$

Step 1: $1320^\circ - 3(360^\circ) = 1320^\circ - 1080^\circ = 240^\circ$

Step 2: $\cot 240^\circ = \frac{\cos 240^\circ}{\sin 240^\circ} = \frac{-\cos 60^\circ}{-\sin 60^\circ} = \frac{-1/2}{-\sqrt{3}/2} = \frac{1}{\sqrt{3}}$

Answer: $\frac{1}{\sqrt{3}}$

(vi) $\cos(-\mathbf{240}^\circ)$

Step 1: $\cos(-240^\circ) = \cos(240^\circ)$

Step 2: $\cos 240^\circ = -\cos 60^\circ = -\frac{1}{2}$

Answer: $-\frac{1}{2}$

Question 2: Express as trigonometric function of angle $< 45^\circ$

(i) $\cos\mathbf{168^\circ}$

Step 1: $168^\circ = 180^\circ - 12^\circ$

Step 2: $\cos(180^\circ - \theta) = -\cos\theta$

- $\cos 168^\circ = -\cos 12^\circ$

Answer: $-\cos 12^\circ$

(ii) $\sin\mathbf{192^\circ}$

Step 1: $192^\circ = 180^\circ + 12^\circ$

Step 2: $\sin(180^\circ + \theta) = -\sin\theta$

- $\sin 192^\circ = -\sin 12^\circ$

Answer: $-\sin 12^\circ$

(iii) $\cos\mathbf{333^\circ}$

Step 1: $333^\circ = 360^\circ - 27^\circ$

Step 2: $\cos(360^\circ - \theta) = \cos\theta$

- $\cos 333^\circ = \cos 27^\circ$

Answer: $\cos 27^\circ$

(iv) $\tan\mathbf{213^\circ}$

Step 1: $213^\circ = 180^\circ + 33^\circ$

Step 2: $\tan(180^\circ + \theta) = \tan\theta$

- $\tan 213^\circ = \tan 33^\circ$

Answer: $\tan 33^\circ$

(v) $\cos(-\mathbf{435^\circ})$

Step 1: $\cos(-435^\circ) = \cos(435^\circ)$

Step 2: $435^\circ - 360^\circ = 75^\circ$

Step 3: $\cos 75^\circ = \sin 15^\circ$

Answer: $\sin 15^\circ$

(vi) $\sin\mathbf{219^\circ}$

Step 1: $219^\circ = 180^\circ + 39^\circ$

Step 2: $\sin(180^\circ + \theta) = -\sin\theta$

- $\sin 219^\circ = -\sin 39^\circ$

Answer: $-\sin 39^\circ$

(vii) $\tan(-\mathbf{597^\circ})$

Step 1: $\tan(-597^\circ) = -\tan(597^\circ)$

Step 2: $597^\circ - 360^\circ = 237^\circ$

Step 3: $237^\circ = 180^\circ + 57^\circ$

Step 4: $\tan(180^\circ + \theta) = \tan\theta$, so $\tan 237^\circ = \tan 57^\circ$

Step 5: $\tan(-597^\circ) = -\tan 57^\circ$

Answer: $-\tan 57^\circ$

(viii) $\cos(-\mathbf{111}^\circ)$

Step 1: $\cos(-111^\circ) = \cos(111^\circ)$

Step 2: $111^\circ = 90^\circ + 21^\circ$

Step 3: $\cos(90^\circ + \theta) = -\sin\theta$

- $\cos 111^\circ = -\sin 21^\circ$

Answer: $-\sin 21^\circ$

(ix) $\sin(-\mathbf{390}^\circ)$

Step 1: $\sin(-390^\circ) = -\sin(390^\circ)$

Step 2: $390^\circ - 360^\circ = 30^\circ$

Step 3: $\sin(-390^\circ) = -\sin 30^\circ$

Answer: $-\sin 30^\circ$

Question 3: Prove the following

(i) $\sin(\mathbf{180}^\circ + \ )\sin(\mathbf{90}^\circ - \ ) = -\sin\ \cos$

Step 1: $\sin(180^\circ + \alpha) = -\sin\alpha$
Step 2: $\sin(90^\circ - \alpha) = \cos\alpha$

Step 3: LHS $= (-\sin\alpha)(\cos\alpha) = -\sin\alpha\cos\alpha$

(ii) $\sin\mathbf{810}^\circ\sin\mathbf{630}^\circ + \cos\mathbf{135}^\circ\sin\mathbf{225}^\circ = \mathbf{\frac{1}{2}}$

Step 1: $810^\circ = 720^\circ + 90^\circ \to \sin 810^\circ = \sin 90^\circ = 1$
Step 2: $630^\circ = 720^\circ - 90^\circ \to \sin 630^\circ = \sin(-90^\circ) = -1$
Step 3: $\cos 135^\circ = -\cos 45^\circ = -\frac{\sqrt{2}}{2}$
Step 4: $\sin 225^\circ = -\sin 45^\circ = -\frac{\sqrt{2}}{2}$

Step 5: LHS $= 1(-1) + \left(-\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) = -1 + \frac{2}{4} = -1 + \frac{1}{2} = -\frac{1}{2}$

The statement as written gives $-\frac{1}{2}$, not $\frac{1}{2}$.

(iii) $\tan\mathbf{150}^\circ\cot\mathbf{330}^\circ - \mathbf{2}\sec\mathbf{135}^\circ\csc\mathbf{225}^\circ = -\mathbf{3}$

Step 1: $\tan 150^\circ = -\tan 30^\circ = -\frac{1}{\sqrt{3}}$
Step 2: $\cot 330^\circ = -\cot 30^\circ = -\sqrt{3}$
Step 3: $\sec 135^\circ = \frac{1}{\cos 135^\circ} = -\sqrt{2}$
Step 4: $\csc 225^\circ = \frac{1}{\sin 225^\circ} = -\sqrt{2}$

Step 5: LHS $= \left(-\frac{1}{\sqrt{3}}\right)(-\sqrt{3}) - 2(-\sqrt{2})(-\sqrt{2}) = 1 - 2(2) = 1 - 4 = -3$

(iv) $\sin\mathbf{210}^\circ + \cos\mathbf{240}^\circ + \tan\mathbf{225}^\circ + \cot\mathbf{225}^\circ = \mathbf{1}$

Step 1: $\sin 210^\circ = -\sin 30^\circ = -\frac{1}{2}$
Step 2: $\cos 240^\circ = -\cos 60^\circ = -\frac{1}{2}$

Step 3: $\tan 225^\circ = \tan 45^\circ = 1$
Step 4: $\cot 225^\circ = \cot 45^\circ = 1$

Step 5: LHS $= -\frac{1}{2} - \frac{1}{2} + 1 + 1 = -1 + 2 = 1$

Question 4: Prove the identities

(i) $\frac{\tan(\mathbf{180}^\circ +)\cot(\mathbf{90}^\circ -)}{\sin(\mathbf{360}^\circ -)\cos(\mathbf{270}^\circ +)} = -\sec^{\mathbf{2}}$

Step 1: $\tan(180^\circ + \alpha) = \tan\alpha$
Step 2: $\cot(90^\circ - \alpha) = \tan\alpha$
Step 3: $\sin(360^\circ - \alpha) = -\sin\alpha$
Step 4: $\cos(270^\circ + \alpha) = \sin\alpha$

Step 5: LHS $= \frac{\tan\alpha \cdot \tan\alpha}{(-\sin\alpha)(\sin\alpha)} = \frac{\tan^2\alpha}{-\sin^2\alpha} = -\frac{\sin^2\alpha/\cos^2\alpha}{\sin^2\alpha} = -\frac{1}{\cos^2\alpha} = -\sec^2\alpha$

(ii) $\frac{\cos(\mathbf{90}^\circ +)\sec(-)\tan(\mathbf{180}^\circ -)}{\sec(\mathbf{360}^\circ -)\sin(\mathbf{180}^\circ +)\cot(\mathbf{90}^\circ -)} = -\mathbf{1}$

Step 1: $\cos(90^\circ + \theta) = -\sin\theta$
Step 2: $\sec(-\theta) = \sec\theta$
Step 3: $\tan(180^\circ - \theta) = -\tan\theta$
Step 4: $\sec(360^\circ - \theta) = \sec\theta$
Step 5: $\sin(180^\circ + \theta) = -\sin\theta$
Step 6: $\cot(90^\circ - \theta) = \tan\theta$

Step 7: LHS $= \frac{(-\sin\theta)(\sec\theta)(-\tan\theta)}{(\sec\theta)(-\sin\theta)(\tan\theta)} = \frac{\sin\theta\sec\theta\tan\theta}{-\sin\theta\sec\theta\tan\theta} = -1$

Question 5: Show that

$$\sec\left(\frac{3\pi}{2} - \theta\right)\csc\left(\frac{5\pi}{2} - \theta\right) = -\tan\left(\frac{3\pi}{2} - \theta\right)\tan\left(\frac{5\pi}{2} + \theta\right) = -1$$

Step 1: $\sec\left(\frac{3\pi}{2} - \theta\right) = \frac{1}{\cos(\frac{3\pi}{2} - \theta)}$
Step 2: $\cos(\frac{3\pi}{2} - \theta) = -\sin\theta$, so $\sec(\frac{3\pi}{2} - \theta) = -\csc\theta$

Step 3: $\csc(\frac{5\pi}{2} - \theta) = \frac{1}{\sin(\frac{5\pi}{2} - \theta)}$
Step 4: $\sin(\frac{5\pi}{2} - \theta) = \sin(2\pi + \frac{\pi}{2} - \theta) = \sin(\frac{\pi}{2} - \theta) = \cos\theta$

- So $\csc(\frac{5\pi}{2} - \theta) = \sec\theta$

Step 5: First product $= (-\csc\theta)(\sec\theta) = -\frac{1}{\sin\theta\cos\theta}$

Step 6: $\tan(\frac{3\pi}{2} - \theta) = \tan(2\pi - \frac{\pi}{2} - \theta)$ — This is complex.

Actually, $\tan(\frac{3\pi}{2} - \theta) = \cot\theta$
And $\tan(\frac{5\pi}{2} + \theta) = \tan(2\pi + \frac{\pi}{2} + \theta) = \tan(\frac{\pi}{2} + \theta) = -\cot\theta$

Step 7: Product $= (\cot\theta)(-\cot\theta) = -\cot^2\theta$

This doesn't match -1 unless $\cot^2\theta = 1$.

The statement as written may have a typo.

Question 6: Triangle ABC with angles , ,

Given: $\alpha + \beta + \gamma = 180^\circ$

(i) sin(+) =sin

Step 1: $\alpha + \beta = 180^\circ - \gamma$
Step 2: $\sin(\alpha + \beta) = \sin(180^\circ - \gamma) = \sin\gamma$

(ii) sec $\left(\frac{\ +\ }{\mathbf{2}}\right)$ =csc$\frac{}{\mathbf{2}}$

Step 1: $\frac{\alpha+\beta}{2} = \frac{180^\circ-\gamma}{2} = 90^\circ - \frac{\gamma}{2}$
Step 2: $\sec(90^\circ - \frac{\gamma}{2}) = \csc\frac{\gamma}{2}$

(iii) csc $=\frac{\mathbf{1}}{\sin(\ +\ )}$

Step 1: $\beta + \gamma = 180^\circ - \alpha$
Step 2: $\sin(\beta + \gamma) = \sin(180^\circ - \alpha) = \sin\alpha$
Step 3: $\frac{1}{\sin(\beta+\gamma)} = \frac{1}{\sin\alpha} = \csc\alpha$

(iv) tan(+)+tan $=\mathbf{0}$

Step 1: $\alpha + \beta = 180^\circ - \gamma$
Step 2: $\tan(\alpha + \beta) = \tan(180^\circ - \gamma) = -\tan\gamma$
Step 3: $\tan(\alpha + \beta) + \tan\gamma = -\tan\gamma + \tan\gamma = 0$

Summary Table

1(i)	$-\frac{\sqrt{3}}{2}$
1(ii)	1
1(iii)	2
1(iv)	2
1(v)	$\frac{1}{\sqrt{3}}$
1(vi)	$-\frac{1}{2}$
2(i)	$-\cos 12^\circ$
2(ii)	$-\sin 12^\circ$
2(iii)	$\cos 27^\circ$
2(iv)	$\tan 33^\circ$
2(v)	$\sin 15^\circ$
2(vi)	$-\sin 39^\circ$
2(vii)	$-\tan 57^\circ$
2(viii)	$-\sin 21^\circ$
2(ix)	$-\sin 30^\circ$
3(i)	Proven
3(ii)	$-\frac{1}{2}$ (typo in problem)
3(iii)	Proven
3(iv)	Proven
4(i)	Proven
4(ii)	Proven
6(i)-(iv)	All proven