EXERCISE 10.3 - DOUBLE ANGLE IDENTITIES

Complete Solutions

Question 1: Find values of sin**2** , cos**2** and tan**2**

(i) sin $=\frac{3}{5}$

Step 1: Find $\cos\alpha$:

$$\cos\alpha = \sqrt{1-\sin^2\alpha} = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

Step 2: $\sin 2\alpha = 2\sin\alpha\cos\alpha = 2\cdot\frac{3}{5}\cdot\frac{4}{5} = \frac{24}{25}$

Step 3: $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

Step 4: $\tan 2\alpha = \frac{\sin 2\alpha}{\cos 2\alpha} = \frac{24/25}{7/25} = \frac{24}{7}$

Answer: $\sin 2\alpha = \frac{24}{25}$, $\cos 2\alpha = \frac{7}{25}$, $\tan 2\alpha = \frac{24}{7}$

(ii) cos $=\frac{4}{5}$, $0 <\ \ < \frac{}{2}$

Step 1: Find $\sin\alpha$:

$$\sin\alpha = \sqrt{1-\cos^2\alpha} = \sqrt{1-\frac{16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$$

Step 2: $\sin 2\alpha = 2\sin\alpha\cos\alpha = 2\cdot\frac{3}{5}\cdot\frac{4}{5} = \frac{24}{25}$

Step 3: $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

Step 4: $\tan 2\alpha = \frac{\sin 2\alpha}{\cos 2\alpha} = \frac{24/25}{7/25} = \frac{24}{7}$

Answer: $\sin 2\alpha = \frac{24}{25}$, $\cos 2\alpha = \frac{7}{25}$, $\tan 2\alpha = \frac{24}{7}$

Question 2: Prove the following identities

(i) cot −tan = **2**cot**2**

Step 1: LHS $= \frac{\cos\alpha}{\sin\alpha} - \frac{\sin\alpha}{\cos\alpha} = \frac{\cos^2\alpha-\sin^2\alpha}{\sin\alpha\cos\alpha} = \frac{\cos 2\alpha}{\sin\alpha\cos\alpha}$

Step 2: Since $\sin 2\alpha = 2\sin\alpha\cos\alpha$:

$\frac{\cos 2\alpha}{\sin\alpha\cos\alpha} = \frac{2\cos 2\alpha}{\sin 2\alpha} = 2\cot 2\alpha$

(ii) $\frac{\sin\mathbf{2}}{\mathbf{1}+\cos\mathbf{2}}$=tan

Step 1: $\sin 2\alpha = 2\sin\alpha\cos\alpha$
Step 2: $1+\cos 2\alpha = 1 + (\cos^2\alpha - \sin^2\alpha) = 2\cos^2\alpha$

Step 3: LHS $= \frac{2\sin\alpha\cos\alpha}{2\cos^2\alpha} = \frac{\sin\alpha}{\cos\alpha} = \tan\alpha$

(iii) $\frac{\mathbf{1}-\cos}{\sin}$=tan$_{\mathbf{2}}$

Step 1: $1-\cos\alpha = 2\sin^2\frac{\alpha}{2}$
Step 2: $\sin\alpha = 2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}$

Step 3: LHS $= \frac{2\sin^2\frac{\alpha}{2}}{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}} = \frac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} = \tan\frac{\alpha}{2}$

(iv) $\frac{\cos -\sin}{\cos +\sin}=\sec\mathbf{2}\ -\tan\mathbf{2}$

Step 1: Multiply numerator and denominator by $(\cos\alpha - \sin\alpha)$:

$$\frac{(\cos\alpha-\sin\alpha)^2}{\cos^2\alpha-\sin^2\alpha} = \frac{\cos^2\alpha - 2\sin\alpha\cos\alpha+\sin^2\alpha}{\cos 2\alpha}$$

$= \frac{1-\sin 2\alpha}{\cos 2\alpha} = \frac{1}{\cos 2\alpha} - \frac{\sin 2\alpha}{\cos 2\alpha} = \sec 2\alpha - \tan 2\alpha$

(v) $\frac{\mathbf{1}+\sin}{\sqrt{\mathbf{1}-\sin}}=\frac{\sin_{\mathbf{2}}}{\sin_{\mathbf{2}}-\cos_{\mathbf{2}}}$

Step 1: Let $t = \frac{\alpha}{2}$, so $\alpha = 2t$

$$1+\sin 2t = (\sin t+\cos t)^2 1-\sin 2t = (\sin t - \cos t)^2$$

Step 2: LHS $= \frac{(\sin t+\cos t)^2}{\sqrt{(\sin t-\cos t)^2}} = \frac{(\sin t+\cos t)^2}{|sint-\cos t|}$

Step 3: The given expression has $\sin t - \cos t$ in denominator, so assuming positive:

$$= \frac{(\sin t+\cos t)^2}{\sin t-\cos t}$$

Step 4: LHS $= \frac{\sin^2 t+2\sin t\cos t+\cos^2 t}{\sin t-\cos t} = \frac{1+\sin 2t}{\sin t-\cos t}$
This doesn't match the given RHS. The problem statement may have a typo.

(vi) $\frac{\cos\cos +\mathbf{2}\cos\sin}{\sec}=\cot_{\mathbf{2}}$

Step 1: LHS $= \frac{\cos\alpha(\cos\theta+2\sin\theta)}{\sec\theta} = \cos\alpha(\cos\theta+2\sin\theta)\cos\theta$

$$= \cos\alpha(\cos^2\theta + 2\sin\theta\cos\theta)$$

This involves α which doesn't appear in the RHS. **The statement is incomplete or has a typo.**

(vii) $\mathbf{1}+\tan\tan\mathbf{2}\ =\sec\mathbf{2}$

Step 1: $\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$

Step 2: LHS $= 1+\tan\alpha\cdot\frac{2\tan\alpha}{1-\tan^2\alpha} = \frac{1-\tan^2\alpha+2\tan^2\alpha}{1-\tan^2\alpha} = \frac{1+\tan^2\alpha}{1-\tan^2\alpha}$

Step 3: $\frac{1+\tan^2\alpha}{1-\tan^2\alpha} = \frac{\sec^2\alpha}{\cos 2\alpha/\cos^2\alpha} = \frac{1}{\cos 2\alpha} = \sec 2\alpha$

(viii) $\frac{\mathbf{2}\sin\sin\mathbf{2}}{\cos +\cos\mathbf{3}}=\tan\mathbf{2}\ \tan$

Step 1: $\cos\theta + \cos 3\theta = 2\cos 2\theta\cos\theta$

Step 2: LHS $= \frac{2\sin\theta\sin 2\theta}{2\cos 2\theta\cos\theta} = \frac{\sin\theta}{\cos\theta}\cdot\frac{\sin 2\theta}{\cos 2\theta} = \tan\theta\cdot\tan 2\theta$

Summary Table

1(i)	$\sin 2\alpha = \frac{24}{25}$, $\cos 2\alpha = \frac{7}{25}$, $\tan 2\alpha = \frac{24}{7}$
1(ii)	$\sin 2\alpha = \frac{24}{25}$, $\cos 2\alpha = \frac{7}{25}$, $\tan 2\alpha = \frac{24}{7}$
2(i)	Proven

2(ii)	Proven
2(iii)	Proven
2(iv)	Proven
2(v)	Typo suspected
2(vi)	Typo suspected
2(vii)	Proven
2(viii)	Proven

EXERCISE 10.3 - DOUBLE AND HALF ANGLE IDENTITIES (Continued)

(ix) $\frac{\sin\mathbf{3}}{\sin}=\cos\mathbf{3}\ =\mathbf{2}$

Step 1: This appears to be a typo. The expression $\frac{\sin 3\theta}{\sin\theta} - \cos 3\theta = 2$ is likely intended.

Let's verify: $\frac{\sin 3\theta}{\sin\theta} = 3 - 4\sin^2\theta$ and $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

These are not equal to 2 in general. **The statement as written is not correct.**

(x) $\frac{\cos\mathbf{3}}{\cos}+\frac{\sin\mathbf{3}}{\sin}=\mathbf{4}\cos\mathbf{2}$

Step 1: $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$, so $\frac{\cos 3\theta}{\cos\theta} = 4\cos^2\theta - 3$

Step 2: $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$, so $\frac{\sin 3\theta}{\sin\theta} = 3 - 4\sin^2\theta$

Step 3: LHS $= 4\cos^2\theta - 3 + 3 - 4\sin^2\theta = 4(\cos^2\theta - \sin^2\theta) = 4\cos 2\theta$

(xi) $\frac{\tan\frac{}{\mathbf{2}}+\cot\frac{}{\mathbf{2}}}{\cot\frac{}{\mathbf{2}}-\tan\frac{}{\mathbf{2}}}=\sec$

Step 1: Let $t = \tan\frac{\theta}{2}$. Then $\cot\frac{\theta}{2} = \frac{1}{t}$

Step 2: LHS $= \frac{t+\frac{1}{t}}{\frac{1}{t}-t} = \frac{\frac{t^2+1}{t}}{\frac{1-t^2}{t}} = \frac{t^2+1}{1-t^2}$

Step 3: $\frac{1+\tan^2\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}} = \frac{1}{\cos\theta} = \sec\theta$

(xii) $\frac{\mathbf{3}+\cos\mathbf{4}}{\mathbf{1}-\cos\mathbf{4}}=\frac{\mathbf{1}}{\mathbf{2}}(\tan^{\mathbf{2}}\ +\cot^{\mathbf{2}}\ )$

Step 1: Let $t = \tan\theta$

Step 2: $\cos 4\theta = \frac{1-\tan^2 2\theta}{1+\tan^2 2\theta}$ — This gets complicated.

Step 3: Alternative: Use $\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$

But the given identity is not standard. Using the identity: $\cot^2\theta + \tan^2\theta = \frac{2}{\sin^2 2\theta} - 2$

And $\frac{3+\cos 4\theta}{1-\cos 4\theta} = \frac{3+\cos 4\theta}{2\sin^2 2\theta}$

This doesn't simplify to the given expression in general. **There may be a typo.**

(xiii) $\cos^{\mathbf{2}}\frac{}{\mathbf{8}}+\cos^{\mathbf{2}}\frac{\mathbf{3}}{\mathbf{8}}+\cos^{\mathbf{2}}\frac{\mathbf{5}}{\mathbf{8}}+\cos^{\mathbf{2}}\frac{\mathbf{7}}{\mathbf{8}}=\mathbf{2}$

Step 1: $\cos\frac{5\pi}{8} = -\cos\frac{3\pi}{8}$, so $\cos^2\frac{5\pi}{8} = \cos^2\frac{3\pi}{8}$
Step 2: $\cos\frac{7\pi}{8} = -\cos\frac{\pi}{8}$, so $\cos^2\frac{7\pi}{8} = \cos^2\frac{\pi}{8}$

Step 3: LHS $= 2\cos^2\frac{\pi}{8} + 2\cos^2\frac{3\pi}{8}$

Step 4: $\cos^2\frac{3\pi}{8} = \sin^2\frac{\pi}{8}$
Step 5: LHS $= 2(\cos^2\frac{\pi}{8} + \sin^2\frac{\pi}{8}) = 2$

(xiv) $\cos^{2}\frac{}{8}=\frac{1}{2}$

Step 1: $\cos^2\frac{\pi}{8} = \frac{1+\cos\frac{\pi}{4}}{2} = \frac{1+\frac{\sqrt{2}}{2}}{2} = \frac{2+\sqrt{2}}{4} \neq \frac{1}{2}$

The statement is incorrect.

(xv) $\cos^{2}\frac{}{8}=\frac{1-\sin 2}{1-\sin 2}$

Step 1: The RHS $= 1$ (since numerator $=$ denominator).

So $\cos^2\frac{\pi}{8} = 1 \rightarrow$ This is false since $\cos^2\frac{\pi}{8} \approx 0.8536$.

The statement is incorrect.

(xvi) $\frac{1+\sin 2}{1-\sin 2}=\tan^{2}\left(\frac{}{4}+\ \right)$

Step 1: $\tan\left(\frac{\pi}{4}+\theta\right) = \frac{1+\tan\theta}{1-\tan\theta}$

Step 2: $\tan^2\left(\frac{\pi}{4}+\theta\right) = \left(\frac{1+\tan\theta}{1-\tan\theta}\right)^2 = \frac{(1+\tan\theta)^2}{(1-\tan\theta)^2}$

Step 3: Multiply numerator and denominator by $\cos^2\theta$:

$= \frac{(\cos\theta+\sin\theta)^2}{(\cos\theta-\sin\theta)^2} = \frac{1+\sin 2\theta}{1-\sin 2\theta}$

Question 3: Show that $2\cos\ =\sqrt{2+\sqrt{2+2\cos 4}}$

Step 1: RHS $= \sqrt{2+\sqrt{2+2\cos 4\theta}}$

Step 2: $2\cos 4\theta = 2(2\cos^2 2\theta - 1) = 4\cos^2 2\theta - 2$

$$2 + 2\cos 4\theta = 2 + 4\cos^2 2\theta - 2 = 4\cos^2 2\theta$$

Step 3: $\sqrt{2+2\cos 4\theta} = \sqrt{4\cos^2 2\theta} = 2\cos 2\theta$ (assuming positive)

Step 4: RHS $= \sqrt{2+2\cos 2\theta} = \sqrt{2(1+\cos 2\theta)} = \sqrt{4\cos^2\theta} = 2\cos\theta$

Question 4: Reduce $\sin^4$ to first power

Step 1: $\sin^2\theta = \frac{1-\cos 2\theta}{2}$

Step 2: $\sin^4\theta = \left(\frac{1-\cos 2\theta}{2}\right)^2 = \frac{1-2\cos 2\theta+\cos^2 2\theta}{4}$

Step 3: $\cos^2 2\theta = \frac{1+\cos 4\theta}{2}$

Step 4: $\sin^4\theta = \frac{1-2\cos 2\theta+\frac{1+\cos 4\theta}{2}}{4} = \frac{2-4\cos 2\theta+1+\cos 4\theta}{8} = \frac{3-4\cos 2\theta+\cos 4\theta}{8}$

Answer: $\sin^4\theta = \frac{3-4\cos 2\theta+\cos 4\theta}{8}$

Question 5: Values of $\sin$ and $\cos$ for special angles

(i) $=18^\circ$

Step 1: Let $x = 18^\circ$. Then $5x = 90^\circ$, so $2x = 90^\circ - 3x$
Step 2: $\sin 2x = \sin(90^\circ - 3x) = \cos 3x$
Step 3: $2\sin x\cos x = 4\cos^3 x - 3\cos x$
Step 4: $2\sin x\cos x = \cos x(4\cos^2 x - 3)$
Step 5: Since $\cos x \neq 0$: $2\sin x = 4\cos^2 x - 3 = 4(1-\sin^2 x) - 3 = 1 - 4\sin^2 x$
Step 6: $4\sin^2 x + 2\sin x - 1 = 0$
Step 7: $\sin x = \frac{-2\pm\sqrt{4+16}}{8} = \frac{-2\pm 2\sqrt{5}}{8} = \frac{-1\pm\sqrt{5}}{4}$

Since $\sin 18^\circ > 0$, $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$

Step 8: $\cos 18^\circ = \sqrt{1-\sin^2 18^\circ} = \frac{\sqrt{10+2\sqrt{5}}}{4}$

Answer: $\sin 18^\circ = \frac{\sqrt{5}-1}{4}$, $\cos 18^\circ = \frac{\sqrt{10+2\sqrt{5}}}{4}$

(ii) $=\mathbf{36}^\circ$

Step 1: $\sin 36^\circ = \sin(2\times 18^\circ) = 2\sin 18^\circ \cos 18^\circ$

$$= 2\cdot\frac{\sqrt{5}-1}{4}\cdot\frac{\sqrt{10+2\sqrt{5}}}{4} = \frac{\sqrt{10-2\sqrt{5}}}{4}$$

Step 2: $\cos 36^\circ = \cos(2\times 18^\circ) = \cos^2 18^\circ - \sin^2 18^\circ = \frac{\sqrt{5}+1}{4}$

Answer: $\sin 36^\circ = \frac{\sqrt{10-2\sqrt{5}}}{4}$, $\cos 36^\circ = \frac{\sqrt{5}+1}{4}$

(iii) $=\mathbf{54}^\circ$

Step 1: $\sin 54^\circ = \cos 36^\circ = \frac{\sqrt{5}+1}{4}$
Step 2: $\cos 54^\circ = \sin 36^\circ = \frac{\sqrt{10-2\sqrt{5}}}{4}$

Answer: $\sin 54^\circ = \frac{\sqrt{5}+1}{4}$, $\cos 54^\circ = \frac{\sqrt{10-2\sqrt{5}}}{4}$

(iv) $=\mathbf{72}^\circ$

Step 1: $\sin 72^\circ = \cos 18^\circ = \frac{\sqrt{10+2\sqrt{5}}}{4}$
Step 2: $\cos 72^\circ = \sin 18^\circ = \frac{\sqrt{5}-1}{4}$

Answer: $\sin 72^\circ = \frac{\sqrt{10+2\sqrt{5}}}{4}$, $\cos 72^\circ = \frac{\sqrt{5}-1}{4}$

Prove: $\cos\mathbf{36}^\circ\cos\mathbf{72}^\circ\cos\mathbf{108}^\circ\cos\mathbf{144}^\circ = \mathbf{\frac{1}{16}}$

Step 1: $\cos 108^\circ = -\cos 72^\circ$, $\cos 144^\circ = -\cos 36^\circ$

Step 2: Product $= \cos 36^\circ \cos 72^\circ(-\cos 72^\circ)(-\cos 36^\circ) = \cos^2 36^\circ \cos^2 72^\circ$

Step 3: $\cos 36^\circ = \frac{\sqrt{5}+1}{4}$, $\cos 72^\circ = \frac{\sqrt{5}-1}{4}$

Step 4: $\cos 36^\circ \cos 72^\circ = \frac{(\sqrt{5}+1)(\sqrt{5}-1)}{16} = \frac{5-1}{16} = \frac{4}{16} = \frac{1}{4}$

Step 5: Product $= \left(\frac{1}{4}\right)^2 = \frac{1}{16}$

Summary Table

(x)	Proven
(xi)	Proven
(xiii)	Proven
(xvi)	Proven
3	Proven
4	$\frac{3-4\cos 2\theta+\cos 4\theta}{8}$
5(i)	$\sin 18^\circ = \frac{\sqrt{5}-1}{4}$, $\cos 18^\circ = \frac{\sqrt{10+2\sqrt{5}}}{4}$
5(ii)	$\sin 36^\circ = \frac{\sqrt{10-2\sqrt{5}}}{4}$, $\cos 36^\circ = \frac{\sqrt{5}+1}{4}$
5(iii)	$\sin 54^\circ = \frac{\sqrt{5}+1}{4}$, $\cos 54^\circ = \frac{\sqrt{10-2\sqrt{5}}}{4}$
5(iv)	$\sin 72^\circ = \frac{\sqrt{10+2\sqrt{5}}}{4}$, $\cos 72^\circ = \frac{\sqrt{5}-1}{4}$

Product	$\frac{1}{16}$