

EXERCISE 11.1 - EVEN/ODD FUNCTIONS AND PERIODS**Complete Solutions****Question 1: Determine whether the functions are even, odd, or neither****Definitions:**

- **Even function:** $f(-x) = f(x)$
- **Odd function:** $f(-x) = -f(x)$

(i) $\sin^2 x$

Step 1: $f(-x) = \sin^2(-x) = (-\sin x)^2 = \sin^2 x = f(x)$

Answer: Even function(ii) $\sin x + \cos x$

Step 1: $f(-x) = \sin(-x) + \cos(-x) = -\sin x + \cos x$

Step 2: $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$ (since $-f(x) = -\sin x - \cos x$)

Answer: Neither odd nor even(iii) $\sin^4 x + \cos^4 x$

Step 1: $f(-x) = \sin^4(-x) + \cos^4(-x) = (-\sin x)^4 + (\cos x)^4 = \sin^4 x + \cos^4 x = f(x)$

Answer: Even function(iv) $\tan x + \sec x$

Step 1: $f(-x) = \tan(-x) + \sec(-x) = -\tan x + \sec x$

Step 2: $f(-x) \neq f(x)$ and $f(-x) \neq -f(x)$

Answer: Neither odd nor even(v) $\frac{1}{\csc x} = \sin x$

Step 1: $f(-x) = \sin(-x) = -\sin x = -f(x)$

Answer: Odd function(vi) $\sin x + \sin 3x$

Step 1: $f(-x) = \sin(-x) + \sin(-3x) = -\sin x - \sin 3x = -(\sin x + \sin 3x) = -f(x)$

Answer: Odd function(vii) $\cos x + \cos 3x$

Step 1: $f(-x) = \cos(-x) + \cos(-3x) = \cos x + \cos 3x = f(x)$

Answer: Even function(viii) $\sec x + \sec^3 x$

Step 1: $f(-x) = \sec(-x) + \sec^3(-x) = \sec x + \sec^3 x = f(x)$

Answer: Even function(ix) $\sec x + \cot^2 x$

Step 1: $f(-x) = \sec(-x) + \cot^2(-x) = \sec x + (-\cot x)^2 = \sec x + \cot^2 x = f(x)$

Answer: Even function

Question 2: Find the periods of the following functions

Period Formulas:

- For $\sin(kx)$, $\cos(kx)$, $\sec(kx)$, $\csc(kx)$: Period = $\frac{2\pi}{|k|}$
- For $\tan(kx)$, $\cot(kx)$: Period = $\frac{\pi}{|k|}$
- Adding constants or vertical shifts does not change the period.

(i) $\sin 5x$

Step 1: $k = 5$, Period = $\frac{2\pi}{5}$

Answer: $\frac{2\pi}{5}$

(ii) $\cos 7x$

Step 1: $k = 7$, Period = $\frac{2\pi}{7}$

Answer: $\frac{2\pi}{7}$

(iii) $\tan 3x$

Step 1: $k = 3$, Period = $\frac{\pi}{3}$

Answer: $\frac{\pi}{3}$

(iv) $\cot \frac{x}{2}$

Step 1: $k = \frac{1}{2}$, Period = $\frac{\pi}{1/2} = 2\pi$

Answer: 2π

(v) $19\sin\left(\frac{x}{20}\right)$

Step 1: $k = \frac{\pi}{20}$, Period = $\frac{2\pi}{\pi/20} = 40$

Answer: 40

(vi) $\cos\left(\frac{2x}{5}\right)$

Step 1: $k = \frac{2}{5}$, Period = $\frac{2\pi}{2/5} = 5\pi$

Answer: 5π

(vii) $\frac{1}{2}\sin\left(\frac{3x}{2} - \frac{\pi}{2}\right)$

Step 1: $k = \frac{3}{2}$, Period = $\frac{2\pi}{3/2} = \frac{4\pi}{3}$

Answer: $\frac{4\pi}{3}$

(viii) $-5 - 3\sec\left(7x + \frac{\pi}{4}\right)$

Step 1: $k = 7$, Period = $\frac{2\pi}{7}$

Answer: $\frac{2\pi}{7}$

(ix) $12 + 10\tan\left(\frac{x}{30}\right)$

Step 1: $k = \frac{\pi}{30}$, Period = $\frac{\pi}{\pi/30} = 30$

Answer: 30

$$(x) \quad 6 - 4\cot\left(\frac{7x}{4} + \frac{\pi}{4}\right)$$

Step 1: $k = \frac{7}{4}$, Period = $\frac{\pi}{7/4} = \frac{4\pi}{7}$

Answer: $\frac{4\pi}{7}$

$$(xi) \quad 9 + 30\sec\left(\frac{x}{15} + \frac{2\pi}{15}\right)$$

Step 1: $k = \frac{1}{15}$, Period = $\frac{2\pi}{1/15} = 30\pi$

Answer: 30π

Summary Table

Question 1: Even/Odd Functions

Function	Type
$\sin^2 x$	Even
$\sin x + \cos x$	Neither
$\sin^4 x + \cos^4 x$	Even
$\tan x + \sec x$	Neither
$\frac{1}{\csc x}$	Odd
$\sin x + \sin 3x$	Odd
$\cos x + \cos 3x$	Even
$\sec x + \sec^3 x$	Even
$\sec x + \cot^2 x$	Even

Question 2: Periods

Function	Period
$\sin 5x$	$\frac{2\pi}{5}$
$\cos 7x$	$\frac{2\pi}{7}$
$\tan 3x$	$\frac{\pi}{3}$
$\cot \frac{x}{2}$	2π
$19 \sin\left(\frac{\pi}{20}x\right)$	40
$\cos\left(\frac{2x}{5}\right)$	5π
$\frac{1}{2} \sin\left(\frac{3x}{2} - \frac{\pi}{2}\right)$	$\frac{4\pi}{3}$
$-5 - 3 \sec\left(7x + \frac{\pi}{4}\right)$	$\frac{2\pi}{7}$
$12 + 10 \tan\left(\frac{\pi}{30}x\right)$	30
$6 - 4 \cot\left(\frac{7x}{4} + \frac{\pi}{4}\right)$	$\frac{4\pi}{7}$
$9 + 30 \sec\left(\frac{x}{15} + \frac{2\pi}{15}\right)$	30π