EXERCISE 11.3 - APPLICATIONS OF TRIGONOMETRIC FUNCTIONS

Complete Solutions

Question 1: Find the maximum and minimum values

Key Principle:

- For $\sin\theta$ and $\cos\theta$: Range is $[-1, 1]$
- For $a + b\sin\theta$: Range is $[a - |b|, a + |b|]$
- For $\frac{1}{a+b\sin\theta}$: Max when denominator is min, Min when denominator is max

(i) $\mathbf{3}-\sin\mathbf{3x}$

Step 1: Range of $\sin 3x$ is $[-1, 1]$
Step 2: $3 - \sin 3x$ ranges from $3 - 1$ to $3 - (-1)$
Step 3: Max $= 3 + 1 = 4$, Min $= 3 - 1 = 2$

Answer: Max = 4, Min = 2

(ii) $\mathbf{3}+\sin\mathbf{2x}$

Step 1: Range of $\sin 2x$ is $[-1, 1]$
Step 2: Max $= 3 + 1 = 4$, Min $= 3 - 1 = 2$

Answer: Max = 4, Min = 2

(iii) $\mathbf{\frac{1}{2}}+\sin(\mathbf{5x} + \ )$

Step 1: Range of $\sin(5x + \pi)$ is $[-1, 1]$
Step 2: Max $= \frac{1}{2} + 1 = \frac{3}{2}$, Min $= \frac{1}{2} - 1 = -\frac{1}{2}$

Answer: Max $= \frac{3}{2}$, Min $= -\frac{1}{2}$

(iv) $\mathbf{\frac{3}{2}}+\cos\left(\mathbf{x} - \frac{}{\mathbf{4}}\right)$

Step 1: Range of $\cos\left(x - \frac{\pi}{4}\right)$ is $[-1, 1]$
Step 2: Max $= \frac{3}{2} + 1 = \frac{5}{2}$, Min $= \frac{3}{2} - 1 = \frac{1}{2}$

Answer: Max $= \frac{5}{2}$, Min $= \frac{1}{2}$

(v) $\mathbf{1} - \mathbf{3}\cos\mathbf{2x}$

Step 1: Range of $\cos 2x$ is $[-1, 1]$
Step 2: $3\cos 2x$ ranges from -3 to 3
Step 3: $1 - 3\cos 2x$ ranges from $1 - 3$ to $1 - (-3)$
Step 4: Max $= 1 + 3 = 4$, Min $= 1 - 3 = -2$

Answer: Max = 4, Min = -2

(vi) $\mathbf{1} + \mathbf{2}\sin\left(\mathbf{x} + \frac{}{\mathbf{6}}\right)$

Step 1: Range of $2\sin\left(x + \frac{\pi}{6}\right)$ is $[-2, 2]$
Step 2: Max $= 1 + 2 = 3$, Min $= 1 - 2 = -1$

Answer: Max = 3, Min = -1

(vii) $\frac{\mathbf{1}}{\mathbf{10}-\mathbf{2}\sin\mathbf{3x}}$

Step 1: Denominator $= 10 - 2\sin 3x$

- When $\sin 3x = 1$: denominator $= 10 - 2 = 8 \rightarrow$ function $= \frac{1}{8}$ (minimum)
- When $\sin 3x = -1$: denominator $= 10 + 2 = 12 \rightarrow$ function $= \frac{1}{12}$ (maximum)

Step 2: Max $= \frac{1}{8}$, Min $= \frac{1}{12}$

Answer: Max $= \frac{1}{8}$, Min $= \frac{1}{12}$

(viii) $\frac{\mathbf{1}}{\mathbf{7+3}\cos(\mathbf{-2x})}$

Step 1: $\cos(-2x) = \cos 2x$, range is $[-1, 1]$
Step 2: Denominator $= 7 + 3\cos 2x$

- When $\cos 2x = 1$: denominator $= 10 \rightarrow$ function $= \frac{1}{10}$ (minimum)
- When $\cos 2x = -1$: denominator $= 4 \rightarrow$ function $= \frac{1}{4}$ (maximum)

Answer: Max $= \frac{1}{4}$, Min $= \frac{1}{10}$

(ix) $\frac{\mathbf{1}}{\mathbf{5-3}\cos(\mathbf{3x-1})}$

Step 1: Range of $\cos(3x - 1)$ is $[-1, 1]$
Step 2: Denominator $= 5 - 3\cos(3x - 1)$

- When $\cos(3x - 1) = 1$: denominator $= 2 \rightarrow$ function $= \frac{1}{2}$ (maximum)
- When $\cos(3x - 1) = -1$: denominator $= 8 \rightarrow$ function $= \frac{1}{8}$ (minimum)

Answer: Max $= \frac{1}{2}$, Min $= \frac{1}{8}$

Question 2: Temperature Variation

Given: $\mathbf{T(t)} = \frac{\mathbf{13}}{\mathbf{2}}\sin\left(\frac{}{\mathbf{6}}\mathbf{t} - \frac{}{\mathbf{9}}\right) + \mathbf{15}$

(a) Maximum and minimum temperature

Step 1: Range of $\sin\left(\frac{\pi}{6}t - \frac{\pi}{9}\right)$ is $[-1, 1]$
Step 2: $\frac{13}{2}$ sin ranges from $-\frac{13}{2}$ to $\frac{13}{2}$
Step 3: Max $= \frac{13}{2} + 15 = \frac{13+30}{2} = \frac{43}{2} = 21.5^{\circ}C$
Step 4: Min $= -\frac{13}{2} + 15 = \frac{-13+30}{2} = \frac{17}{2} = 8.5^{\circ}C$

Answer: Max $= 21.5^{\circ}C$, Min $= 8.5^{\circ}C$

(b) Temperature at $\mathbf{t = 9}$ hours

Step 1: $T(9) = \frac{13}{2}\sin\left(\frac{\pi}{6}(9) - \frac{\pi}{9}\right) + 15$

$$= \frac{13}{2}\sin\left(\frac{3\pi}{2} - \frac{\pi}{9}\right) + 15 = \frac{13}{2}\sin\left(\frac{27\pi - 2\pi}{18}\right) + 15 = \frac{13}{2}\sin\left(\frac{25\pi}{18}\right) + 15$$

Step 2: $\frac{25\pi}{18} = \pi + \frac{7\pi}{18}$

$$\sin\left(\frac{25\pi}{18}\right) = -\sin\left(\frac{7\pi}{18}\right) \approx -\sin 70^{\circ} \approx -0.9397$$

Step 3: $T(9) \approx \frac{13}{2}(-0.9397) + 15 = -6.108 + 15 = 8.892^{\circ}C$

Answer: Approximately $8.89^{\circ}C$

Question 3: Lighthouse Problem

Step 1: Let the height of lighthouse $= 100$ m

- Angle of depression to ship 1 = 17°
- Angle of depression to ship 2 = 19°
- Both ships on same side

Step 2: If angle of depression is θ, then angle of elevation from ship to top is also θ

Let d_1 = distance of ship 1 from lighthouse, d_2 = distance of ship 2 from lighthouse

Step 3: $\tan 17^\circ = \frac{100}{d_1} \to d_1 = \frac{100}{\tan 17^\circ}$
Step 4: $\tan 19^\circ = \frac{100}{d_2} \to d_2 = \frac{100}{\tan 19^\circ}$

Step 5: Distance between ships $= d_1 - d_2 = 100\left(\frac{1}{\tan 17^\circ} - \frac{1}{\tan 19^\circ}\right)$

Step 6: $\frac{1}{\tan 17^\circ} \approx \frac{1}{0.3057} \approx 3.271$, $\frac{1}{\tan 19^\circ} \approx \frac{1}{0.3443} \approx 2.904$

Step 7: Distance $= 100(3.271 - 2.904) = 100(0.367) = 36.7$ m

Answer: Approximately 36.7 m

Question 4: Tree Height Problem

Step 1: Let height of tree $= h$, distance from tree to point P $= x$

- Point Q is 30 m from P (so Q is closer to tree)

Step 2: From point P (farther): $\tan 12^\circ = \frac{h}{x} \to h = x \tan 12^\circ$

Step 3: From point Q (closer): $\tan 15^\circ = \frac{h}{x-30} \to h = (x-30)\tan 15^\circ$

Step 4: Equate:

$$x \tan 12^\circ = (x-30)\tan 15^\circ x \tan 12^\circ = x \tan 15^\circ - 30 \tan 15^\circ x(\tan 15^\circ - \tan 12^\circ) = 30 \tan 15^\circ x = \frac{30 \tan 15^\circ}{\tan 15^\circ - \tan 12^\circ}$$

Step 5: $\tan 15^\circ \approx 0.2679$, $\tan 12^\circ \approx 0.2126$

$$x = \frac{30(0.2679)}{0.2679 - 0.2126} = \frac{8.037}{0.0553} \approx 145.33 \text{ m}$$

Step 6: $h = x \tan 12^\circ = 145.33(0.2126) \approx 30.9$ m

Answer: Height of tree 30.9 m

Question 5: Ferris Wheel

Given:

- Diameter = 60 feet $\to$ Radius = 30 feet
- Lowest point = 6 feet above ground $\to$ Center = 36 feet above ground
- Period = 80 seconds

(a) Equation for height

Step 1: Height function:

$$h(t) = 36 - 30\cos\left(\frac{2\pi}{80}t\right)$$

or

$$h(t) = 36 + 30\sin\left(\frac{2\pi}{80}t - \frac{\pi}{2}\right)$$

Step 2: At $t = 0$ (lowest point): $h(0) = 36 - 30 = 6$

(b) Maximum and minimum heights

- Max $= 36 + 30 = 66$ feet
- Min $= 36 - 30 = 6$ feet

(c) Height at t = 20 seconds

$$h(20) = 36 - 30\cos\left(\frac{2\pi}{80} \cdot 20\right) = 36 - 30\cos\left(\frac{\pi}{2}\right) = 36 - 0 = 36 \text{ feet}$$

(d) Time to reach maximum height

- Maximum when $\cos\left(\frac{2\pi}{80}t\right) = -1 \to \frac{2\pi}{80}t = \pi \to t = 40$ seconds

Answer: (a) $h(t) = 36 - 30\cos\left(\frac{\pi}{40}t\right)$, (b) Max=66 ft, Min=6 ft, (c) 36 ft, (d) 40 seconds

Summary Table

1(i)	Max=4, Min=2
1(ii)	Max=4, Min=2
1(iii)	Max=$\frac{3}{2}$, Min=$-\frac{1}{2}$
1(iv)	Max=$\frac{5}{2}$, Min=$\frac{1}{2}$
1(v)	Max=4, Min=-2
1(vi)	Max=3, Min=-1
1(vii)	Max=$\frac{1}{8}$, Min=$\frac{1}{12}$
1(viii)	Max=$\frac{1}{4}$, Min=$\frac{1}{10}$
1(ix)	Max=$\frac{1}{2}$, Min=$\frac{1}{8}$
2(a)	Max=21.5°C, Min=8.5°C
2(b)	8.89°C
3	36.7 m
4	30.9 m
5	As derived above

EXERCISE 11.3 - APPLICATIONS OF TRIGONOMETRIC FUNCTIONS (Continued)

Question 5: Ferris Wheel (Continued from previous page)

Given:

- Diameter $= 60$ feet $\to$ Radius $= 30$ feet
- Lowest point $= 6$ feet above ground $\to$ Center $= 36$ feet above ground
- Period $= 80$ seconds

(a) Model an equation for height h(t)

Step 1: General form for sinusoidal motion starting at lowest point:

$$h(t) = 36 - 30\cos\left(\frac{2\pi}{80}t\right) = 36 - 30\cos\left(\frac{\pi}{40}t\right)$$

Answer: $h(t) = 36 - 30\cos\left(\frac{\pi}{40}t\right)$

(b) Find the maximum height

Step 1: Maximum occurs when $\cos\left(\frac{\pi}{40}t\right) = -1$

- $h_{\text{max}} = 36 - 30(-1) = 36 + 30 = 66$ feet

Answer: 66 feet

(c) Height after 35 seconds

Step 1: $h(35) = 36 - 30\cos\left(\frac{\pi}{40} \cdot 35\right) = 36 - 30\cos\left(\frac{7\pi}{8}\right)$

Step 2: $\cos\left(\frac{7\pi}{8}\right) = -\cos\left(\frac{\pi}{8}\right) \approx -0.9239$

Step 3: $h(35) = 36 - 30(-0.9239) = 36 + 27.717 = 63.717$ feet

Answer: Approximately 63.72 feet

Question 6: Swing Motion

Given: $\mathbf{h(t) = 1.5 + 1.2}\sin(\mathbf{3\ t})$

(a) Maximum height

Step 1: Range of $\sin(3\pi t)$ is [-1, 1]
Step 2: Max $= 1.5 + 1.2(1) = 2.7$ m

Answer: 2.7 m

(b) Minimum height

Step 1: Min $= 1.5 + 1.2(-1) = 0.3$ m

Answer: 0.3 m

(c) Period

Step 1: Period $= \frac{2\pi}{|k|} = \frac{2\pi}{3\pi} = \frac{2}{3}$ seconds

Answer: $\frac{2}{3}$ seconds

(d) Time(s) when height = 2.12 m

Step 1: $1.5 + 1.2\sin(3\pi t) = 2.12$

$$1.2\sin(3\pi t) = 0.62\sin(3\pi t) = \frac{0.62}{1.2} = \frac{31}{60}$$

Step 2: $3\pi t = \sin^{-1}\left(\frac{31}{60}\right)$

$$t = \frac{1}{3\pi}\sin^{-1}\left(\frac{31}{60}\right)$$

Step 3: $\sin^{-1}\left(\frac{31}{60}\right) \approx \sin^{-1}(0.5167) \approx 0.5425$ radians

Step 4: $t \approx \frac{0.5425}{3\pi} \approx \frac{0.5425}{9.4248} \approx 0.0576$ seconds

Answer: $t \approx 0.058$ seconds (first time)

Question 7: Carnival Ride

Given:

- Diameter $= 40$ feet $\rightarrow$ Radius $= 20$ feet

- Center = 28 feet above ground
- Period = 120 seconds

(a) Model equation

Step 1: Starting from lowest point:

$$h(t) = 28 - 20\cos\left(\frac{2\pi}{120}t\right) = 28 - 20\cos\left(\frac{\pi}{60}t\right)$$

Answer: $h(t) = 28 - 20\cos\left(\frac{\pi}{60}t\right)$

(b) Height after 90 seconds

Step 1: $h(90) = 28 - 20\cos\left(\frac{\pi}{60}\cdot 90\right) = 28 - 20\cos\left(\frac{3\pi}{2}\right)$

Step 2: $\cos\left(\frac{3\pi}{2}\right) = 0$

Step 3: $h(90) = 28 - 0 = 28$ feet

Answer: 28 feet

(c) Times when rider is 36 feet above ground

Step 1: $28 - 20\cos\left(\frac{\pi}{60}t\right) = 36$

$$-20\cos\left(\frac{\pi}{60}t\right) = 8\cos\left(\frac{\pi}{60}t\right) = -\frac{2}{5}$$

Step 2: $\frac{\pi}{60}t = \cos^{-1}\left(-\frac{2}{5}\right)$

$$t = \frac{60}{\pi}\cos^{-1}\left(-\frac{2}{5}\right)$$

Step 3: $\cos^{-1}(-0.4) \approx 1.9823$ radians

Step 4: $t \approx \frac{60}{3.1416}(1.9823) = 19.098 \times 1.9823 \approx 37.85$ seconds

Also at: $t = 120 - 37.85 = 82.15$ seconds (in the same period)

Answer: Approximately 37.85 seconds and 82.15 seconds

Question 8: Temperature in Lahore

Given: $\mathbf{T(t) = 64 + 8\sin\left(\frac{}{12}(t - 8)\right)}$

(a) Temperature at $\mathbf{t = 9}$ hours

Step 1: $T(9) = 64 + 8\sin\left(\frac{\pi}{12}(9-8)\right) = 64 + 8\sin\left(\frac{\pi}{12}\right)$

Step 2: $\sin\left(\frac{\pi}{12}\right) = \sin 15^\circ = \frac{\sqrt{6}-\sqrt{2}}{4} \approx 0.2588$

Step 3: $T(9) \approx 64 + 8(0.2588) = 64 + 2.0704 = 66.0704$

Answer: Approximately 66.07°F

(b) Time of maximum temperature

Step 1: Max occurs when $\sin\left(\frac{\pi}{12}(t-8)\right) = 1$

$$\frac{\pi}{12}(t-8) = \frac{\pi}{2}t - 8 = 6t = 14$$

Answer: 14:00 (2:00 PM)

(c) Maximum temperature

Step 1: $\text{Max} = 64 + 8(1) = 72$

Answer: 72°F

Question 9: Population Model

Given: $\mathbf{P(t) = 70000 + 10000}\cos\left(\frac{}{\mathbf{6}}\mathbf{t} - \frac{}{\mathbf{2}}\right)$

(a) Population at t = 7 months

Step 1: $P(7) = 70000 + 10000\cos\left(\frac{7\pi}{6} - \frac{\pi}{2}\right)$

$$= 70000 + 10000\cos\left(\frac{7\pi}{6} - \frac{3\pi}{6}\right) = 70000 + 10000\cos\left(\frac{4\pi}{6}\right) = 70000 + 10000\cos\left(\frac{2\pi}{3}\right)$$

Step 2: $\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$

Step 3: $P(7) = 70000 + 10000\left(-\frac{1}{2}\right) = 70000 - 5000 = 65000$

Answer: 65,000

(b) Maximum population

Step 1: Max when $\cos\left(\frac{\pi}{6}t - \frac{\pi}{2}\right) = 1$

Step 2: $P_{\text{max}} = 70000 + 10000(1) = 80000$

Answer: 80,000

Summary Table

5(a)	$h(t) = 36 - 30\cos\left(\frac{\pi}{40}t\right)$
5(b)	66 feet
5(c)	63.72 feet
6(a)	2.7 m
6(b)	0.3 m
6(c)	$\frac{2}{3}$ seconds
6(d)	0.058 seconds
7(a)	$h(t) = 28 - 20\cos\left(\frac{\pi}{60}t\right)$
7(b)	28 feet
7(c)	37.85 s and 82.15 s
8(a)	66.07°F
8(b)	14:00 (2:00 PM)
8(c)	72°F
9(a)	65,000
9(b)	80,000