EXERCISE 12.2 - LIMITS AND CONTINUITY

Complete Solutions

Question 1: Determine left hand limit, right hand limit, and limit

(i) $\mathbf{f(x) = 2x^2 + x - 5, c = 1}$

Step 1: Left hand limit:

$$\lim_{x\to 1^-}(2x^2 + x - 5) = 2(1)^2 + 1 - 5 = 2 + 1 - 5 = -2$$

Step 2: Right hand limit:

$$\lim_{x\to 1^+}(2x^2 + x - 5) = 2(1)^2 + 1 - 5 = -2$$

Step 3: Since LHL = RHL = -2:

$$\lim_{x\to 1} f(x) = -2$$

Answer: LHL = -2, RHL = -2, Limit = -2

(ii) $\mathbf{f(x) = \frac{x^2-9}{x-3}, c = -3}$

Step 1: Factor numerator:

$$f(x) = \frac{(x-3)(x+3)}{x-3} = x + 3(x \neq 3)$$

Step 2: Left hand limit at $x = -3$:

$$\lim_{x\to -3^-}(x+3) = -3 + 3 = 0$$

Step 3: Right hand limit at $x = -3$:

$$\lim_{x\to -3^+}(x+3) = -3 + 3 = 0$$

Step 4: Since LHL = RHL = 0:

$$\lim_{x\to -3} f(x) = 0$$

Answer: LHL = 0, RHL = 0, Limit = 0

(iii) $\mathbf{f(x) = |x - 5|, c = 5}$

Step 1: Definition of absolute value:

$$f(x) = | x - 5 | = \begin{cases} -(x-5) & \text{if } x < 5 \\ x - 5 & \text{if } x \geq 5 \end{cases}$$

Step 2: Left hand limit ($x \to 5^-$):

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$$\lim_{x \to 5^-} \mid x - 5 \mid = \lim_{x \to 5^-} -(x - 5) = 0$$

Step 3: Right hand limit $(x \to 5^+)$:

$$\lim_{x \to 5^+} \mid x - 5 \mid = \lim_{x \to 5^+} (x - 5) = 0$$

Step 4: Since LHL = RHL = 0:

$$\lim_{x \to 5} \mid x - 5 \mid = 0$$

Answer: LHL = 0, RHL = 0, Limit = 0

Question 2: Discuss continuity at x = c

(i) (f(x) =

\begin{cases}
2x + 5 & \text{if } x \leq 2 \
4x + 1 & \text{if } x > 2
\end{cases}, , c = 2)

Step 1: Check if function is defined at $x = 2$:

- For $x \leq 2$, $f(2) = 2(2) + 5 = 9$

Step 2: Left hand limit $(x \to 2^-)$:

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (2x + 5) = 4 + 5 = 9$$

Step 3: Right hand limit $(x \to 2^+)$:

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (4x + 1) = 8 + 1 = 9$$

Step 4: Check continuity:

- LHL = RHL = f(2) = 9

Answer: Function is continuous at $x = 2$

(ii) (f(x) =

\begin{cases}
3x - 1 & \text{if } x < 1 \
4 & \text{if } x = 1 \
2x & \text{if } x > 1
\end{cases}, , c = 1)

Step 1: Function value at $x = 1$:

$$f(1) = 4$$

Step 2: Left hand limit $(x \to 1^-)$:

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$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} (3x - 1) = 3(1) - 1 = 2$$

Step 3: Right hand limit ($x \to 1^+$):

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} 2x = 2(1) = 2$$

Step 4: LHL = RHL = 2, but $f(1) = 4$

Step 5: Since $\lim_{x \to 1} f(x) \neq f(1)$, function is not continuous at $x = 1$

Answer: Function is **not continuous** at $x = 1$ (discontinuity at $x = 1$)

Question 3: Discuss continuity at $\mathbf{x = 2}$ and $\mathbf{x = -2}$

Given:

$$f(x) = \begin{cases} 3x & \text{if } x \leq -2 \\ x^2 - 1 & \text{if } -2 < x < 2 \\ 3 & \text{if } x \geq 2 \end{cases}$$

(a) Continuity at $\mathbf{x = 2}$

Step 1: Function value at $x = 2$:

$$f(2) = 3$$

Step 2: Left hand limit ($x \to 2^-$):

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (x^2 - 1) = 4 - 1 = 3$$

Step 3: Right hand limit ($x \to 2^+$):

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} 3 = 3$$

Step 4: LHL = RHL = f(2) = 3

Answer: Function is **continuous** at $x = 2$

(b) Continuity at $\mathbf{x = -2}$

Step 1: Function value at $x = -2$:

$$f(-2) = 3(-2) = -6$$

Step 2: Left hand limit ($x \to -2^-$):

$$\lim_{x \to -2^-} f(x) = \lim_{x \to -2^-} 3x = 3(-2) = -6$$

Step 3: Right hand limit ($x \to -2^+$):

$$\lim_{x \to -2^+} f(x) = \lim_{x \to -2^+} (x^2 - 1) = 4 - 1 = 3$$

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Step 4: LHL = -6, RHL = 3

Step 5: Since LHL RHL, the limit does not exist at $x = -2$

Answer: Function is **not continuous** at $x = -2$

Summary Table

	LHL	RHL	Limit	Continuous?
1(i)	-2	-2	-2	N/A
1(ii)	0	0	0	N/A
1(iii)	0	0	0	N/A
2(i)	9	9	9	Yes
2(ii)	2	2	2	No (f(1)=4)
3 (x=2)	3	3	3	Yes
3 (x=-2)	-6	3	Does not exist	No

EXERCISE 12.2 - LIMITS AND CONTINUITY (Continued)

Question 4: Find "c" so that $\lim_{\mathbf{x}\to-1}\mathbf{f}(\mathbf{x})$ exists

Given:

$$f(x) = \begin{cases} x+2 & x \le -1 \\ c+2 & x > -1 \end{cases}$$

Step 1: Left hand limit at $x = -1$:

$$\lim_{x\to-1^-} f(x) = \lim_{x\to-1^-} (x+2) = -1+2 = 1$$

Step 2: Right hand limit at $x = -1$:

$$\lim_{x\to-1^+} f(x) = \lim_{x\to-1^+} (c+2) = c+2$$

Step 3: For the limit to exist:

$$\text{LHL} = \text{RHL}1 = c + 2c = -1$$

Answer: $c = -1$

Question 5: Find values of m and n so that f is continuous at x = 3

(i) (f(x) =

\begin{cases}
mx & \text{if } x < 3 \
n & \text{if } x = 3 \
-2x+9 & \text{if } x > 3
\end{cases})

Step 1: For continuity at $x = 3$:

$$\lim_{x\to3^-} f(x) = \lim_{x\to3^+} f(x) = f(3)$$

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Step 2: Left hand limit:

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} mx = 3m$$

Step 3: Right hand limit:

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (-2x+9) = -6+9 = 3$$

Step 4: $f(3) = n$

Step 5: For continuity:

$$3m = 3 \text{and} n = 3m = 1, n = 3$$

Answer: $m = 1, n = 3$

(ii) (f(x) =

\begin{cases}
mx & \text{if } x < 3 \
x^2 & \text{if } x \geq 3
\end{cases})

Step 1: Left hand limit:

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} mx = 3m$$

Step 2: Right hand limit:

$$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} x^2 = 9$$

Step 3: Function value at $x = 3$:

$$f(3) = 3^2 = 9$$

Step 4: For continuity at $x = 3$:

$$3m = 9m = 3$$

Answer: $m = 3$ (no condition on n as it's not in the function)

Question 6: Find value of k so that f is continuous at x = 2

Given:

$$f(x) = \begin{cases} \frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2} & x \neq 2 \\ k & x = 2 \end{cases}$$

Step 1: For continuity at $x = 2$:

$$\lim_{x\to 2} f(x) = f(2) = k$$

Step 2: Evaluate the limit:

$$\lim_{x\to 2}\frac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}$$

Step 3: Rationalize the numerator:

$$= \lim_{x\to 2}\frac{\left(\sqrt{2x+5}-\sqrt{x+7}\right)\left(\sqrt{2x+5}+\sqrt{x+7}\right)}{(x-2)\left(\sqrt{2x+5}+\sqrt{x+7}\right)} = \lim_{x\to 2}\frac{(2x+5)-(x+7)}{(x-2)\left(\sqrt{2x+5}+\sqrt{x+7}\right)} = \lim_{x\to 2}\frac{x-2}{(x-2)\left(\sqrt{2x+5}+\sqrt{x+7}\right)}$$

Step 4: Cancel $(x-2)$:

$$= \lim_{x\to 2}\frac{1}{\sqrt{2x+5}+\sqrt{x+7}}$$

Step 5: Substitute $x = 2$:

$$= \frac{1}{\sqrt{9}+\sqrt{9}} = \frac{1}{3+3} = \frac{1}{6}$$

Step 6: Therefore, $k = \frac{1}{6}$

Answer: $k = \frac{1}{6}$

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Question 7: Discuss limit and continuity at $x = 1$

Given:

$$f(x) = \begin{cases} 2x+3, & x \le 1 \\ -x+4, & x > 1 \end{cases}$$

Step 1: Function value at $x = 1$:

$$f(1) = 2(1) + 3 = 5$$

Step 2: Left hand limit:

$$\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}(2x+3) = 2 + 3 = 5$$

Step 3: Right hand limit:

$$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+}(-x+4) = -1 + 4 = 3$$

Step 4: Since LHL RHL, the limit does not exist at $x = 1$

Step 5: Since the limit does not exist, the function is not continuous at $x = 1$

Answer: LHL $= 5$, RHL $= 3$. Limit does not exist. Function is **not continuous** at $x = 1$.

Question 8: A function f is continuous at $x = 1$

Note: The problem statement appears to be incomplete. It only states:
"A function f is continuous at $x = 1$."

Without additional information about the function, no specific computation can be performed. If there is a specific function or condition to check, please provide the complete problem.

Summary Table

4	$c = -1$
5(i)	$m = 1, n = 3$
5(ii)	$m = 3$
6	$k = \frac{1}{6}$
7	LHL = 5, RHL = 3. Limit does not exist. Not continuous.
8	Incomplete problem

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