EXERCISE 13.2 - DIFFERENTIATION

Complete Solutions

Question 1: Differentiate w.r.t. 'x'

(i) $\mathbf{f(x) = x^4 + 2x^3 + x^2}$

Step 1: Apply power rule: $\frac{d}{dx}(x^n) = nx^{n-1}$

$$f'(x) = 4x^3 + 2(3x^2) + 2x = 4x^3 + 6x^2 + 2x$$

Answer: $4x^3 + 6x^2 + 2x$

(ii) $\mathbf{f(x) = x^3 + 2x^{\frac{3}{2}} + 3}$

Step 1: Differentiate each term:

$$f'(x) = 3x^2 + 2 \cdot \frac{3}{2}x^{\frac{1}{2}} + 0 = 3x^2 + 3x^{\frac{1}{2}}$$

Answer: $3x^2 + 3\sqrt{x}$

(iii) $\mathbf{f(x) = \frac{2x-3}{2x+1}}$

Step 1: Use quotient rule: $\left(\frac{u}{v}\right)' = \frac{u'v-uv'}{v^2}$

- $u = 2x - 3,\ u' = 2$
- $v = 2x + 1,\ v' = 2$

Step 2:

$$f'(x) = \frac{2(2x+1) - (2x-3)(2)}{(2x+1)^2} = \frac{4x + 2 - (4x-6)}{(2x+1)^2} = \frac{4x + 2 - 4x + 6}{(2x+1)^2} = \frac{8}{(2x+1)^2}$$

Answer: $\frac{8}{(2x+1)^2}$

(iv) $\mathbf{f(x) = \frac{(1+\sqrt{x})(x-x)}{\sqrt{x}}}$

Step 1: Note that $x - x = 0$

Step 2: $f(x) = \frac{(1+\sqrt{x})(0)}{\sqrt{x}} = 0$

Step 3: $f'(x) = 0$

Answer: 0

(v) $\mathbf{f(x) = \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)^2}$

Step 1: Expand first:

$$f(x) = x - 2 + \frac{1}{x} = x + x^{-1} - 2$$

Step 2: Differentiate:

$$f^{'}(x) = 1 - x^{-2} = 1 - \frac{1}{x^2}$$

Alternative using chain rule:

Step 1 (alternative): Let $u = \sqrt{x} - \frac{1}{\sqrt{x}}$

$$f^{'}(x) = 2u \cdot u^{'} u = x^{1/2} - x^{-1/2} u^{'} = \frac{1}{2}x^{-1/2} + \frac{1}{2}x^{-3/2} f^{'}(x) = 2\left(x^{1/2} - x^{-1/2}\right)\left(\frac{1}{2}x^{-1/2} + \frac{1}{2}x^{-3/2}\right) = \left(x^{1/2} - x^{-1/2}\right)\left(x^{-1/2} + x$$

Answer: $1 - \frac{1}{x^2}$

(vi) $\mathbf{f(x) = (x - 5)(3 - x)}$

Step 1: Expand first:

$$f(x) = (x - 5)(3 - x) = 3x - x^2 - 15 + 5x = -x^2 + 8x - 15$$

Step 2: Differentiate:

$$f^{'}(x) = -2x + 8$$

Alternative using product rule:

Step 1 (alternative): $u = x - 5$, $v = 3 - x$

$$u^{'} = 1, v^{'} = -1 f^{'}(x) = u^{'}v + uv^{'} = 1(3 - x) + (x - 5)(-1) = 3 - x - x + 5 = 8 - 2x$$

Answer: $8 - 2x$

Summary Table

(i)	$4x^3 + 6x^2 + 2x$
(ii)	$3x^2 + 3\sqrt{x}$
(iii)	$\frac{8}{(2x+1)^2}$
(iv)	0
(v)	$1 - \frac{1}{x^2}$
(vi)	$8 - 2x$

EXERCISE 13.2 - DIFFERENTIATION (Continued)

(vii) $\mathbf{f(x) = \frac{(x^2+1)^2}{x^2-1}}$

Step 1: Use quotient rule: $\left(\frac{u}{v}\right)^{'} = \frac{u^{'}v - uv^{'}}{v^2}$

- $u = (x^2 + 1)^2 = x^4 + 2x^2 + 1$, $u^{'} = 4x^3 + 4x = 4x(x^2 + 1)$
- $v = x^2 - 1$, $v^{'} = 2x$

Step 2:

$$f^{'}(x) = \frac{4x(x^2+1)(x^2-1)-(x^2+1)^2(2x)}{(x^2-1)^2}$$

Step 3: Factor (x^2+1):

$$= \frac{(x^2+1)[4x(x^2-1)-2x(x^2+1)]}{(x^2-1)^2} = \frac{(x^2+1)[4x^3-4x-2x^3-2x]}{(x^2-1)^2} = \frac{(x^2+1)(2x^3-6x)}{(x^2-1)^2} = \frac{2x(x^2+1)(x^2-3)}{(x^2-1)^2}$$

Answer: $\frac{2x(x^2+1)(x^2-3)}{(x^2-1)^2}$

(viii) $\mathbf{f(x) = \frac{x^2+1}{x^2-3}}$

Step 1: Use quotient rule:

- $u = x^2+1,\ u^{'} = 2x$
- $v = x^2-3,\ v^{'} = 2x$

Step 2:

$$f^{'}(x) = \frac{2x(x^2-3)-(x^2+1)(2x)}{(x^2-3)^2} = \frac{2x^3-6x-2x^3-2x}{(x^2-3)^2} = \frac{-8x}{(x^2-3)^2}$$

Answer: $-\frac{8x}{(x^2-3)^2}$

(ix) $\mathbf{f(x) = \frac{2x-1}{\sqrt{x^2+1}}}$

Step 1: Use quotient rule:

- $u = 2x-1,\ u^{'} = 2$
- $v = (x^2+1)^{1/2},\ v^{'} = \frac{1}{2}(x^2+1)^{-1/2}\cdot 2x = \frac{x}{\sqrt{x^2+1}}$

Step 2:

$$f^{'}(x) = \frac{2\sqrt{x^2+1}-(2x-1)\frac{x}{\sqrt{x^2+1}}}{x^2+1} = \frac{2(x^2+1)-x(2x-1)}{(x^2+1)^{3/2}} = \frac{2x^2+2-2x^2+x}{(x^2+1)^{3/2}} = \frac{x+2}{(x^2+1)^{3/2}}$$

Answer: $\frac{x+2}{(x^2+1)^{3/2}}$

(x) $\mathbf{f(x) = \sqrt{\frac{a-x}{a+x}}}$

Step 1: Rewrite: $f(x) = (a-x)^{1/2}(a+x)^{-1/2}$

Step 2: Use product rule and chain rule:

$$f^{'}(x) = \frac{1}{2}(a-x)^{-1/2}(-1)(a+x)^{-1/2} + (a-x)^{1/2}\left(-\frac{1}{2}\right)(a+x)^{-3/2}(1) = -\frac{1}{2}(a-x)^{-1/2}(a+x)^{-1/2} - \frac{1}{2}(a-x)^{1/2}(a+x)^{-3/2}$$

Alternative using logarithmic differentiation:

Step 1: $\ln f(x) = \frac{1}{2}[\ln(a-x) - \ln(a+x)]$

Step 2: $\frac{f'(x)}{f(x)} = \frac{1}{2}\left[-\frac{1}{a-x} - \frac{1}{a+x}\right] = -\frac{1}{2}\left[\frac{(a+x)+(a-x)}{(a-x)(a+x)}\right] = -\frac{a}{(a-x)(a+x)}$

Step 3: $f'(x) = f(x)\cdot\left[-\frac{a}{(a-x)(a+x)}\right] = \sqrt{\frac{a-x}{a+x}}\cdot\left[-\frac{a}{(a-x)(a+x)}\right] = -\frac{a}{(a-x)^{1/2}(a+x)^{3/2}}$

Answer: $-\frac{a}{(a-x)^{1/2}(a+x)^{3/2}}$

(xi) $\mathbf{f(x)} = \sqrt{\frac{x^2+1}{x^2-1}}$

Step 1: Let $y = \sqrt{\frac{x^2+1}{x^2-1}}$

Step 2: $\ln y = \frac{1}{2}[\ln(x^2+1) - \ln(x^2-1)]$

Step 3: Differentiate:

$$\frac{y'}{y} = \frac{1}{2}\left[\frac{2x}{x^2+1} - \frac{2x}{x^2-1}\right] = x\left[\frac{1}{x^2+1} - \frac{1}{x^2-1}\right] = x\left[\frac{(x^2-1)-(x^2+1)}{(x^2+1)(x^2-1)}\right] = -\frac{2x}{(x^2+1)(x^2-1)}$$

Step 4: $y' = y\cdot\left[-\frac{2x}{(x^2+1)(x^2-1)}\right] = \sqrt{\frac{x^2+1}{x^2-1}}\cdot\left[-\frac{2x}{(x^2+1)(x^2-1)}\right]$

Step 5: Simplify:

$$= -\frac{2x\sqrt{x^2+1}}{\sqrt{x^2-1}(x^2+1)(x^2-1)} = -\frac{2x}{(x^2+1)^{1/2}(x^2-1)^{3/2}}$$

Answer: $-\frac{2x}{\sqrt{x^2+1}(x^2-1)^{3/2}}$

Question 2: Find $\frac{dy}{dx}$ if $y = \frac{(\sqrt{x}+1)(x^2-1)}{x^2-1}$, $x \neq 1$

Step 1: Since $x \neq 1$, $x^2-1 \neq 0$, so we can cancel:

$$y = \sqrt{x} + 1$$

Step 2: $\frac{dy}{dx} = \frac{1}{2\sqrt{x+1}}$

Answer: $\frac{1}{2\sqrt{x+1}}$

Question 3: Differentiate $\frac{(\sqrt{x}+1)(x^2-1)}{x^2-x^2}$ with respect to x

Step 1: Denominator: $x^2 - x^2 = 0$

Step 2: The expression is undefined (division by zero).

Answer: Not defined (division by zero)

Question 4: If $y = \sqrt{x} - \frac{1}{\sqrt{x}}$, show that $2x\frac{dy}{dx} + y = 2\sqrt{x}$

Step 1: $y = x^{1/2} - x^{-1/2}$

Step 2: $\frac{dy}{dx} = \frac{1}{2}x^{-1/2} + \frac{1}{2}x^{-3/2} = \frac{1}{2\sqrt{x}} + \frac{1}{2x^{3/2}}$

Step 3: LHS $= 2x\left(\frac{1}{2\sqrt{x}} + \frac{1}{2x^{3/2}}\right) + \left(\sqrt{x} - \frac{1}{\sqrt{x}}\right)$

$$= \frac{2x}{2\sqrt{x}} + \frac{2x}{2x^{3/2}} + \sqrt{x} - \frac{1}{\sqrt{x}}$$

$= \sqrt{x} + \frac{1}{\sqrt{x}} + \sqrt{x} - \frac{1}{\sqrt{x}} = 2\sqrt{x}$

Question 5: If $\mathbf{y = x^4 + 2x^2 + 2}$, prove that $\mathbf{\frac{dy}{dx} = 4x\sqrt{x^2 - 1}}$

Step 1: The statement $y = x^4 + 2x^2 + 2$ does not match the derivative $4x\sqrt{x^2 - 1}$.

Step 2: If $y = (x^2 + 1)^2 + 1 = x^4 + 2x^2 + 2$, then:

$$\frac{dy}{dx} = 4x^3 + 4x = 4x(x^2 + 1)$$

Step 3: This is not equal to $4x\sqrt{x^2 - 1}$ unless $x^2 + 1 = \sqrt{x^2 - 1}$, which is not true.

The problem statement appears to have a typo.

Summary Table

1(vii)	$\frac{2x(x^2+1)(x^2-3)}{(x^2-1)^2}$
1(viii)	$-\frac{8x}{(x^2-3)^2}$
1(ix)	$\frac{x+2}{(x^2+1)^{3/2}}$
1(x)	$-\frac{a}{(a-x)^{1/2}(a+x)^{3/2}}$
1(xi)	$-\frac{2x}{\sqrt{x^2+1}(x^2-1)^{3/2}}$
2	$\frac{1}{2\sqrt{x+1}}$
3	Not defined
4	Proven
5	Typo in problem