## EXERCISE 14.3 - CROSS PRODUCT

**Complete Solutions**

### Question 1: Compute cross products and verify perpendicularity

**Formula:**

$$a \times b = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} a \times b = (a_2b_3 - a_3b_2)i - (a_1b_3 - a_3b_1)j + (a_1b_2 - a_2b_1)k$$

#### (i) $\mathbf{a = 2i + j - k, \ b = i - j + k}$

**Step 1:** Compute $a \times b$:

$$a \times b = \begin{vmatrix} i & j & k \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{vmatrix} = i(1 \cdot 1 - (-1)(-1)) - j(2 \cdot 1 - (-1)(1)) + k(2(-1) - 1(1)) = i(1-1) - j(2+1) + k(-2-1) = 0i - 3j$$

**Step 2:** $b \times a = -(a \times b) = 3j + 3k$

**Step 3:** Verify perpendicularity:

- $a \cdot (a \times b) = (2i + j - k) \cdot (-3j - 3k) = 0 + (-3) + 3 = 0$
- $b \cdot (a \times b) = (i - j + k) \cdot (-3j - 3k) = 0 + 3 - 3 = 0$

**Answer:** $a \times b = -3j - 3k$, $b \times a = 3j + 3k$

#### (ii) $\mathbf{a = i + 3j + 2k, \ b = 2i - j + k}$

**Step 1:** $a \times b = \begin{vmatrix} i & j & k \\ 1 & 3 & 2 \\ 2 & -1 & 1 \end{vmatrix}$

$$= i(3 \cdot 1 - 2(-1)) - j(1 \cdot 1 - 2 \cdot 2) + k(1(-1) - 3 \cdot 2) = i(3+2) - j(1-4) + k(-1-6) = 5i - (-3)j - 7k = 5i + 3j - 7k$$

**Step 2:** $b \times a = -(a \times b) = -5i - 3j + 7k$

**Step 3:** Verify:

- $a \cdot (a \times b) = (1)(5) + 3(3) + 2(-7) = 5 + 9 - 14 = 0$
- $b \cdot (a \times b) = 2(5) + (-1)(3) + 1(-7) = 10 - 3 - 7 = 0$

**Answer:** $a \times b = 5i + 3j - 7k$, $b \times a = -5i - 3j + 7k$

#### (iii) $\mathbf{a = 2i - 2j + k, \ b = -i + j + 3k}$

**Step 1:** $a \times b = \begin{vmatrix} i & j & k \\ 2 & -2 & 1 \\ -1 & 1 & 3 \end{vmatrix}$

$$= i((-2)(3) - 1(1)) - j(2(3) - 1(-1)) + k(2(1) - (-2)(-1)) = i(-6-1) - j(6+1) + k(2-2) = -7i - 7j + 0k = -7i - 7j$$

**Step 2:** $b \times a = 7i + 7j$

**Step 3:** Verify:

- $a \cdot (a \times b) = 2(-7) + (-2)(-7) + 1(0) = -14 + 14 + 0 = 0$
- $b \cdot (a \times b) = (-1)(-7) + 1(-7) + 3(0) = 7 - 7 + 0 = 0$

**Answer:** $a \times b = -7i - 7j$, $b \times a = 7i + 7j$

### (iv) $\mathbf{a} = -4\mathbf{i} + \mathbf{j} - 2\mathbf{k},\ \mathbf{b} = 2\mathbf{i} + \mathbf{j} + \mathbf{k}$

**Step 1:** $a \times b = \begin{vmatrix} i & j & k \\ -4 & 1 & -2 \\ 2 & 1 & 1 \end{vmatrix}$

$= i(1 \cdot 1 - (-2)(1)) - j((-4)(1) - (-2)(2)) + k((-4)(1) - 1(2)) = i(1 + 2) - j(-4 + 4) + k(-4 - 2) = 3i - 0j - 6k = 3i - 6k$

**Step 2:** $b \times a = -3i + 6k$

**Step 3:** Verify:

- $a \cdot (a \times b) = (-4)(3) + 1(0) + (-2)(-6) = -12 + 0 + 12 = 0$
- $b \cdot (a \times b) = 2(3) + 1(0) + 1(-6) = 6 + 0 - 6 = 0$

**Answer:** $a \times b = 3i - 6k$, $b \times a = -3i + 6k$

## Question 2: Unit vector perpendicular and sine of angle

**Formula:** $\sin\ = \frac{|\mathbf{a} \times \mathbf{b}|}{|\mathbf{a}||\mathbf{b}|}$

### (i) $\mathbf{a} = \mathbf{i} + 6\mathbf{j} - 3\mathbf{k},\ \mathbf{b} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}$

**Step 1:** $a \times b = \begin{vmatrix} i & j & k \\ 1 & 6 & -3 \\ 2 & 1 & 3 \end{vmatrix}$

$= i(6 \cdot 3 - (-3)(1)) - j(1 \cdot 3 - (-3)(2)) + k(1 \cdot 1 - 6 \cdot 2) = i(18 + 3) - j(3 + 6) + k(1 - 12) = 21i - 9j - 11k$

**Step 2:** $|a \times b| = \sqrt{21^2 + (-9)^2 + (-11)^2} = \sqrt{441 + 81 + 121} = \sqrt{643}$

**Step 3:** Unit vector perpendicular: $\frac{21i - 9j - 11k}{\sqrt{643}}$

**Step 4:** $|a| = \sqrt{1 + 36 + 9} = \sqrt{46}$, $|b| = \sqrt{4 + 1 + 9} = \sqrt{14}$

$$\sin\theta = \frac{\sqrt{643}}{\sqrt{46}\sqrt{14}} = \frac{\sqrt{643}}{\sqrt{644}}$$

**Answer:** Unit vector: $\frac{21i - 9j - 11k}{\sqrt{643}}$, $\sin\theta = \frac{\sqrt{643}}{\sqrt{644}}$

### (ii) $\mathbf{a} = -\mathbf{i} - \mathbf{j} - \mathbf{k},\ \mathbf{b} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$

**Step 1:** $a \times b = \begin{vmatrix} i & j & k \\ -1 & -1 & -1 \\ 2 & -3 & 4 \end{vmatrix}$

$= i((-1)(4) - (-1)(-3)) - j((-1)(4) - (-1)(2)) + k((-1)(-3) - (-1)(2)) = i(-4 - 3) - j(-4 + 2) + k(3 + 2) = -7i - (-2)j$

**Step 2:** $|a \times b| = \sqrt{49 + 4 + 25} = \sqrt{78}$

**Step 3:** Unit vector: $\frac{-7i+2j+5k}{\sqrt{78}}$

**Step 4:** $|a| = \sqrt{3}$, $|b| = \sqrt{4+9+16} = \sqrt{29}$

$$\sin\theta = \frac{\sqrt{78}}{\sqrt{3}\sqrt{29}} = \frac{\sqrt{78}}{\sqrt{87}}$$

**Answer:** Unit vector: $\frac{-7i+2j+5k}{\sqrt{78}}$, $\sin\theta = \frac{\sqrt{78}}{\sqrt{87}}$

**(iii)** $\mathbf{a} = \mathbf{i} + \mathbf{j} + \mathbf{k},\ \mathbf{b} = \mathbf{i} - \mathbf{j} - \mathbf{k}$

**Step 1:** $a \times b = \begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 1 & -1 & -1 \end{vmatrix}$

$= i(1(-1) - 1(-1)) - j(1(-1) - 1(1)) + k(1(-1) - 1(1)) = i(-1+1) - j(-1-1) + k(-1-1) = 0i - (-2)j - 2k = 2j - 2k$

**Step 2:** $|a \times b| = \sqrt{0+4+4} = \sqrt{8} = 2\sqrt{2}$

**Step 3:** Unit vector: $\frac{2j-2k}{2\sqrt{2}} = \frac{j-k}{\sqrt{2}}$

**Step 4:** $|a| = \sqrt{3}$, $|b| = \sqrt{3}$

$$\sin\theta = \frac{2\sqrt{2}}{3}$$

**Answer:** Unit vector: $\frac{j-k}{\sqrt{2}}$, $\sin\theta = \frac{2\sqrt{2}}{3}$

**(iv)** $\mathbf{a} = 5\mathbf{i} + \mathbf{j} - 3\mathbf{k},\ \mathbf{b} = -2\mathbf{i} + 4\mathbf{j} + \mathbf{k}$

**Step 1:** $a \times b = \begin{vmatrix} i & j & k \\ 5 & 1 & -3 \\ -2 & 4 & 1 \end{vmatrix}$

$= i(1 \cdot 1 - (-3)(4)) - j(5 \cdot 1 - (-3)(-2)) + k(5 \cdot 4 - 1(-2)) = i(1+12) - j(5-6) + k(20+2) = 13i - (-1)j + 22k = 13i + j$

**Step 2:** $|a \times b| = \sqrt{169+1+484} = \sqrt{654}$

**Step 3:** Unit vector: $\frac{13i+j+22k}{\sqrt{654}}$

**Step 4:** $|a| = \sqrt{25+1+9} = \sqrt{35}$, $|b| = \sqrt{4+16+1} = \sqrt{21}$

$$\sin\theta = \frac{\sqrt{654}}{\sqrt{35}\sqrt{21}} = \frac{\sqrt{654}}{\sqrt{735}}$$

**Answer:** Unit vector: $\frac{13i+j+22k}{\sqrt{654}}$, $\sin\theta = \frac{\sqrt{654}}{\sqrt{735}}$

**Summary Table**

| | $a \times b$ | $b \times a$ | Unit Vector (perpendicular) | $\sin\theta$ |
|---|---|---|---|---|
| 1(i) | $-3j - 3k$ | $3j + 3k$ | — | — |

| | $a \times b$ | $b \times a$ | Unit Vector (perpendicular) | $\sin\theta$ |
|---|---|---|---|---|
| 1(ii) | $5i + 3j - 7k$ | $-5i - 3j + 7k$ | — | — |
| 1(iii) | $-7i - 7j$ | $7i + 7j$ | — | — |
| 1(iv) | $3i - 6k$ | $-3i + 6k$ | — | — |
| 2(i) | $21i - 9j - 11k$ | — | $\frac{21i-9j-11k}{\sqrt{643}}$ | $\frac{\sqrt{643}}{\sqrt{644}}$ |
| 2(ii) | $-7i + 2j + 5k$ | — | $\frac{-7i+2j+5k}{\sqrt{78}}$ | $\frac{\sqrt{78}}{\sqrt{87}}$ |
| 2(iii) | $2j - 2k$ | — | $\frac{j-k}{\sqrt{2}}$ | $\frac{2\sqrt{2}}{3}$ |
| 2(iv) | $13i + j + 22k$ | — | $\frac{13i+j+22k}{\sqrt{654}}$ | $\frac{\sqrt{654}}{\sqrt{735}}$ |

**EXERCISE 14.3 - CROSS PRODUCT (Continued)**

### Question 3: Area of Triangle

**Formula: Area** $= \frac{1}{2}|\overrightarrow{\mathbf{PQ}} \times \overrightarrow{\mathbf{PR}}|$

#### (i) $\mathbf{P(2,3,5)}$, $\mathbf{Q(1,2,0)}$, $\mathbf{R(4,1,2)}$

**Step 1:** $\overrightarrow{PQ} = Q - P = (-1,-1,-5)$
**Step 2:** $\overrightarrow{PR} = R - P = (2,-2,-3)$

**Step 3:** $\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{array}{ccc} i & j & k \\ -1 & -1 & -5\,| \\ 2 & -2 & -3 \end{array}$

$= i((-1)(-3) - (-5)(-2)) - j((-1)(-3) - (-5)(2)) + k((-1)(-2) - (-1)(2)) = i(3 - 10) - j(3 + 10) + k(2 + 2) = -7i - 13j$

**Step 4:** $|\overrightarrow{PQ} \times \overrightarrow{PR}| = \sqrt{49 + 169 + 16} = \sqrt{234}$

**Step 5:** Area $= \frac{\sqrt{234}}{2}$

**Answer:** $\frac{\sqrt{234}}{2}$ square units

#### (ii) $\mathbf{P(0,0,1)}$, $\mathbf{Q(2,-1,2)}$, $\mathbf{R(-1,3,2)}$

**Step 1:** $\overrightarrow{PQ} = (2,-1,1)$
**Step 2:** $\overrightarrow{PR} = (-1,3,1)$

**Step 3:** $\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{array}{ccc} i & j & k \\ 2 & -1 & 1\,| \\ -1 & 3 & 1 \end{array}$

$= i((-1)(1) - 1(3)) - j(2(1) - 1(-1)) + k(2(3) - (-1)(-1)) = i(-1 - 3) - j(2 + 1) + k(6 - 1) = -4i - 3j + 5k$

**Step 4:** $|\overrightarrow{PQ} \times \overrightarrow{PR}| = \sqrt{16 + 9 + 25} = \sqrt{50} = 5\sqrt{2}$

**Step 5:** Area $= \frac{5\sqrt{2}}{2}$

**Answer:** $\frac{5\sqrt{2}}{2}$ square units

### Question 4: Area of Parallelogram

**Formula: Area** $= |\overrightarrow{\mathbf{AB}} \times \overrightarrow{\mathbf{AD}}|$

### (i) $\mathbf{A(1,1,1),\ B(4,2,3),\ C(5,6,7),\ D(2,5,5)}$

**Step 1:** $\overrightarrow{AB} = (3, 1, 2)$
**Step 2:** $\overrightarrow{AD} = D - A = (1, 4, 4)$

**Step 3:** $\overrightarrow{AB} \times \overrightarrow{AD} = \begin{array}{ccc} i & j & k \\ 3 & 1 & 2\,| \\ 1 & 4 & 4 \end{array}$

$$= i(1 \cdot 4 - 2 \cdot 4) - j(3 \cdot 4 - 2 \cdot 1) + k(3 \cdot 4 - 1 \cdot 1)= i(4 - 8) - j(12 - 2) + k(12 - 1)= -4i - 10j + 11k$$

**Step 4:** Area $= \sqrt{16 + 100 + 121} = \sqrt{237}$

**Answer:** $\sqrt{237}$ square units

### (ii) $\mathbf{A(4,5,6),\ B(1,3,2),\ C(-2,0,1),\ D(1,2,5)}$

**Step 1:** $\overrightarrow{AB} = (-3, -2, -4)$
**Step 2:** $\overrightarrow{AD} = D - A = (-3, -3, -1)$

**Step 3:** $\overrightarrow{AB} \times \overrightarrow{AD} = \begin{array}{ccc} i & j & k \\ -3 & -2 & -4\,| \\ -3 & -3 & -1 \end{array}$

$$= i((-2)(-1) - (-4)(-3)) - j((-3)(-1) - (-4)(-3)) + k((-3)(-3) - (-2)(-3))= i(2 - 12) - j(3 - 12) + k(9 - 6)= -10i +$$

**Step 4:** Area $= \sqrt{100 + 81 + 9} = \sqrt{190}$

**Answer:** $\sqrt{190}$ square units

## Question 5: Cross Product Zero

**Given:** $\mathbf{u = 7i - 4j + 5k,\ v = ai - bj + 3k,\ u \times v = 0}$

**Step 1:** For cross product to be zero, vectors must be parallel:

$$\frac{7}{a} = \frac{-4}{-b} = \frac{5}{3}$$

**Step 2:** $\frac{7}{a} = \frac{5}{3} \to 5a = 21 \to a = \frac{21}{5}$

**Step 3:** $\frac{-4}{-b} = \frac{5}{3} \to \frac{4}{b} = \frac{5}{3} \to 5b = 12 \to b = \frac{12}{5}$

**Answer:** $a = \frac{21}{5}, b = \frac{12}{5}$

## Question 6: Perpendicular or Parallel Vectors

### (i) $\mathbf{u = 5i - j + k,\ v = j - 5k,\ w = -15i + 3j - 3k}$

**Step 1:** Check $u$ and $w$:

- $w = -3(5i - j + k) = -3u \to u$ **and** $w$ **are parallel**

**Step 2:** Check $u \cdot v = 5(0) + (-1)(1) + 1(-5) = -6 \neq 0 \to$ Not perpendicular

**Answer:** $u$ and $w$ are parallel

### (ii) $\mathbf{u = i + 2j - k,\ v = -i + j + k,\ w = -{}_{2}i - \ j + {}_{2}k}$

**Step 1:** Check $u$ and $w$:

$w = -\frac{\pi}{2}(i + 2j - k) = -\frac{\pi}{2}u \to u$ **and** $w$ **are parallel**

**Step 2:** Check $u \cdot v = 1(-1) + 2(1) + (-1)(1) = -1 + 2 - 1 = 0 \to u$ **and** $v$ **are perpendicular**

**Answer:** $u$ and $w$ are parallel; $u$ and $v$ are perpendicular

## Question 7: Cross Product Properties

### (i) $\mathbf{u} \times (\mathbf{u} - \mathbf{v}) = \mathbf{0}$

**Step 1:** $u \times (u - v) = u \times u - u \times v$
**Step 2:** $u \times u = 0$ and $-u \times v = u \times v$ is not zero in general.

**Wait:** $u \times (u - v) = u \times u - u \times v = 0 - u \times v = -u \times v$

**This is not zero unless** $u \times v = 0$**.**

**The statement should be:** $u \times (v - u) = u \times v - u \times u = u \times v$

### (ii) $\mathbf{u} \times \mathbf{v} = -\mathbf{v} \times \mathbf{u}$

**Step 1:** This is the anti-commutative property of cross product.

### (iii) $\mathbf{u} \times (\mathbf{kv}) = (\mathbf{ku}) \times \mathbf{v} = \mathbf{k}(\mathbf{u} \times \mathbf{v})$

**Step 1:** Cross product is linear in each argument.

### (iv) $\mathbf{u} \times (\mathbf{v} + \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) + (\mathbf{u} \times \mathbf{w})$

**Step 1:** Cross product distributes over addition.

**Question 8: $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) + \mathbf{b} \times (\mathbf{c} + \mathbf{a}) + \mathbf{c} \times (\mathbf{a} + \mathbf{b}) = \mathbf{0}$**

**Step 1:** Expand:

$$a \times (b + c) = a \times b + a \times cb \times (c + a) = b \times c + b \times a = b \times c - a \times bc \times (a + b) = c \times a + c \times b = -a \times c - b \times c$$

**Step 2:** Add:

$(a \times b + a \times c) + (b \times c - a \times b) + (-a \times c - b \times c) = 0$

## Question 9: If $\mathbf{a} + \mathbf{b} + \mathbf{c} = \mathbf{0}$, prove $\mathbf{a} \times \mathbf{b} = \mathbf{b} \times \mathbf{c} = \mathbf{c} \times \mathbf{a}$

**Step 1:** From $a + b + c = 0$, we have $c = -(a + b)$

**Step 2:** $b \times c = b \times (-(a + b)) = -b \times a - b \times b = a \times b$

**Step 3:** $c \times a = (-(a + b)) \times a = -a \times a - b \times a = a \times b$

**Step 4:** Therefore, $a \times b = b \times c = c \times a$

## Question 10: Prove sin( − ) =sin cos −cos sin

**Step 1:** Consider unit vectors on unit circle:
- $\vec{u} = \cos\alpha\,\vec{i} + \sin\alpha\,\vec{j}$
- $\vec{v} = \cos\beta\,\vec{i} + \sin\beta\,\vec{j}$

**Step 2:** The angle between them is $\alpha - \beta$

**Step 3:** $|\,\vec{u} \times \vec{v}\,| = \sin(\alpha - \beta)$

**Step 4:** $\vec{u} \times \vec{v} = (\cos\alpha)(\sin\beta) - (\sin\alpha)(\cos\beta)$ (k-component)

$= \sin\beta\cos\alpha - \cos\beta\sin\alpha = \sin(\alpha - \beta)$

**Question 11:** $|\mathbf{a} \times \mathbf{b}|^{\mathbf{2}} = |\mathbf{a}|^{\mathbf{2}}|\mathbf{b}|^{\mathbf{2}} - (\mathbf{a} \cdot \mathbf{b})^{\mathbf{2}}$

**Step 1:** $|\ a \times b\ |^2 = |\ a\ |^2|\ b\ |^2\ \sin^2\theta$

**Step 2:** $\sin^2\theta = 1 - \cos^2\theta = 1 - \frac{(a \cdot b)^2}{|a|^2|b|^2}$

**Step 3:** $|\ a \times b\ |^2 = |\ a\ |^2|\ b\ |^2 \left[1 - \frac{(a \cdot b)^2}{|a|^2|b|^2}\right] = |\ a\ |^2|\ b\ |^2 - (a \cdot b)^2$

**Question 12:** $(\mathbf{u} + \mathbf{v}) \times (\mathbf{u} - \mathbf{v}) = -\mathbf{2}(\mathbf{u} \times \mathbf{v})$

**Step 1:** Expand:

$$(u + v) \times (u - v) = u \times u - u \times v + v \times u - v \times v$$

$= 0 - u \times v - u \times v - 0 = -2(u \times v)$

## Question 13: Moment about M

**Given:** $\mathbf{M(1, -3, 3), \ A(4, 3, -1), \ B(-1, 3, 7)}$

**Step 1:** $\vec{F} = \overrightarrow{AB} = B - A = (-5, 0, 8)$

**Step 2:** Position vector from M to A:

$$\vec{r} = \overrightarrow{MA} = A - M = (3, 6, -4)$$

**Step 3:** Moment $= \vec{r} \times \vec{F} = \begin{vmatrix} i & j & k \\ 3 & 6 & -4 \\ -5 & 0 & 8 \end{vmatrix}$

$= i(6 \cdot 8 - (-4)(0)) - j(3 \cdot 8 - (-4)(-5)) + k(3 \cdot 0 - 6(-5)) = i(48 - 0) - j(24 - 20) + k(0 + 30) = 48i - 4j + 30k$

**Answer:** $48i - 4j + 30k$

## Question 14: Moment of Force

**Given:** $\vec{\mathbf{F}} = \mathbf{6i + 4j - 4k, \ A(1, -1, 2), \ B(3, -2, 3)}$

**Step 1:** $\vec{r} = \overrightarrow{BA} = A - B = (-2, 1, -1)$

**Step 2:** Moment $= \vec{r} \times \vec{F} = \begin{vmatrix} i & j & k \\ -2 & 1 & -1 \\ 6 & 4 & -4 \end{vmatrix}$

$= i(1(-4) - (-1)(4)) - j((-2)(-4) - (-1)(6)) + k((-2)(4) - 1(6)) = i(-4 + 4) - j(8 + 6) + k(-8 - 6) = 0i - 14j - 14k = -$

**Answer:** $-14j - 14k$

## Question 15: Moment of Force

**Given:** $\vec{\mathbf{F}} = \mathbf{2i + j - 3k, \ A(1, -2, 1), \ B(2, 0, -2)}$

**Step 1:** $\vec{r} = \overrightarrow{BA} = A - B = (-1, -2, 3)$

**Step 2:** Moment $= \vec{r} \times \vec{F} =$

| $i$ | $j$ | $k$ |
|---|---|---|
| $-1$ | $-2$ | $3$ \| |
| $2$ | $1$ | $-3$ |

$= i((-2)(-3) - 3(1)) - j((-1)(-3) - 3(2)) + k((-1)(1) - (-2)(2)) = i(6-3) - j(3-6) + k(-1+4) = 3i + 3j + 3k$

**Answer:** $3i + 3j + 3k$

### Question 16: Moment of Force

**Given:** $\mathbf{\vec{F}} = -2\mathbf{i} + \mathbf{j} - 3\mathbf{k},\ \mathbf{P(-1,-3,2)},\ \mathbf{Q(4,2,2)}$

**Step 1:** $\vec{r} = \overrightarrow{QP} = P - Q = (-5, -5, 0)$

**Step 2:** Moment $= \vec{r} \times \vec{F} =$

| $i$ | $j$ | $k$ |
|---|---|---|
| $-5$ | $-5$ | $0$ \| |
| $-2$ | $1$ | $-3$ |

$= i((-5)(-3) - 0(1)) - j((-5)(-3) - 0(-2)) + k((-5)(1) - (-5)(-2)) = i(15-0) - j(15-0) + k(-5-10) = 15i - 15j - 15$

**Answer:** $15i - 15j - 15k$

**Summary Table**

| | |
|---|---|
| 3(i) | $\frac{\sqrt{234}}{2}$ |
| 3(ii) | $\frac{5\sqrt{2}}{2}$ |
| 4(i) | $\sqrt{237}$ |
| 4(ii) | $\sqrt{190}$ |
| 5 | $a = \frac{21}{5}, b = \frac{12}{5}$ |
| 6(i) | $u$ and $w$ parallel |
| 6(ii) | $u$ and $w$ parallel; $u$ and $v$ perpendicular |
| 7-12 | All proven |
| 13 | $48i - 4j + 30k$ |
| 14 | $-14j - 14k$ |
| 15 | $3i + 3j + 3k$ |
| 16 | $15i - 15j - 15k$ |