EXERCISE 3.1 - COMPLETE SOLUTIONS

Question 1: Find the maximum or minimum value by completing square

(i) $f(x) = x^2 + 6x + 13$

Step 1: Complete the square

- $f(x) = x^2 + 6x + 13$
- $f(x) = (x^2 + 6x + 9) + 13 - 9$
- $f(x) = (x + 3)^2 + 4$

Step 2: Identify vertex

- Since coefficient of x^2 is positive ($a = 1 > 0$), parabola opens upward
- Vertex form: $f(x) = (x - h)^2 + k$, where (h, k) is vertex
- $f(x) = (x + 3)^2 + 4 = (x - (-3))^2 + 4$
- **Vertex: $(-3, 4)$**

Step 3: Determine maximum/minimum

- Since $a > 0$, minimum value occurs at vertex
- **Minimum value = 4 at $x = -3$**

Answer: Minimum value = 4, at $x = -3$

(ii) $f(x) = x^2 + 4x$

Step 1: Complete the square

- $f(x) = x^2 + 4x$
- $f(x) = (x^2 + 4x + 4) - 4$
- $f(x) = (x + 2)^2 - 4$

Step 2: Identify vertex

- $a = 1 > 0$, parabola opens upward
- $f(x) = (x - (-2))^2 + (-4)$
- **Vertex: $(-2, -4)$**

Step 3: Determine maximum/minimum

- Since $a > 0$, minimum value occurs at vertex
- **Minimum value = -4 at $x = -2$**

Answer: Minimum value = -4, at $x = -2$

(iii) $f(x) = -x^2 + 8x + 13$

Step 1: Complete the square

- $f(x) = -(x^2 - 8x) + 13$
- $f(x) = -(x^2 - 8x + 16) + 13 + 16$
- $f(x) = -(x - 4)^2 + 29$

Step 2: Identify vertex

- $a = -1 < 0$, parabola opens downward

- **Vertex: (4, 29)**

Step 3: Determine maximum/minimum

- Since $a < 0$, maximum value occurs at vertex
- **Maximum value = 29 at x = 4**

Answer: Maximum value = 29, at x = 4

(iv) $f(x) = -x^2 - 3x - 5$

Step 1: Complete the square

- $f(x) = -(x^2 + 3x) - 5$
- $f(x) = -(x^2 + 3x + 9/4) - 5 + 9/4$
- $f(x) = -(x + 3/2)^2 - 20/4 + 9/4$
- $f(x) = -(x + 3/2)^2 - 11/4$

Step 2: Identify vertex

- $a = -1 < 0$, parabola opens downward
- **Vertex: (-3/2, -11/4)**

Step 3: Determine maximum/minimum

- Since $a < 0$, maximum value occurs at vertex
- **Maximum value = -11/4 at x = -3/2**

Answer: Maximum value = -11/4, at x = -3/2

(v) $f(x) = 3x^2 + 6x - 13$

Step 1: Complete the square

- $f(x) = 3(x^2 + 2x) - 13$
- $f(x) = 3(x^2 + 2x + 1) - 13 - 3$
- $f(x) = 3(x + 1)^2 - 16$

Step 2: Identify vertex

- $a = 3 > 0$, parabola opens upward
- **Vertex: (-1, -16)**

Step 3: Determine maximum/minimum

- Since $a > 0$, minimum value occurs at vertex
- **Minimum value = -16 at x = -1**

Answer: Minimum value = -16, at x = -1

(vi) $f(x) = -2x^2 - x + 21$

Step 1: Complete the square

- $f(x) = -2(x^2 + x/2) + 21$
- $f(x) = -2(x^2 + x/2 + 1/16) + 21 + 1/8$
- $f(x) = -2(x + 1/4)^2 + 169/8$

Step 2: Identify vertex

- $a = -2 < 0$, parabola opens downward
- **Vertex: (-1/4, 169/8)**

Step 3: Determine maximum/minimum

- Since $a < 0$, maximum value occurs at vertex
- **Maximum value = 169/8 at x = -1/4**

Answer: Maximum value = 169/8, at x = -1/4

Summary Table for Question 1:

Part	Function	Vertex	Max/Min	Value
(i)	$x^2 + 6x + 13$	(-3, 4)	Min	4
(ii)	$x^2 + 4x$	(-2, -4)	Min	-4
(iii)	$-x^2 + 8x + 13$	(4, 29)	Max	29
(iv)	$-x^2 - 3x - 5$	(-3/2, -11/4)	Max	-11/4
(v)	$3x^2 + 6x - 13$	(-1, -16)	Min	-16
(vi)	$-2x^2 - x + 21$	(-1/4, 169/8)	Max	169/8

Question 2: Find maximum/minimum point by sketching, domain and range

(i) $f(x) = x^2 - 4x$

Step 1: Find vertex

- $f(x) = x^2 - 4x + 4 - 4 = (x - 2)^2 - 4$
- **Vertex: (2, -4)**

Step 2: Find x-intercepts (where $f(x) = 0$)

- $x^2 - 4x = 0$
- $x(x - 4) = 0$
- **$x = 0, x = 4$**

Step 3: Find y-intercept ($x = 0$)

- $f(0) = 0$

Step 4: Determine shape

- $a = 1 > 0$, parabola opens upward (minimum)

x	-1	0	1	2	3	4	5
f(x)	5	0	-3	-4	-3	0	5

Domain: All real numbers = **$(-\infty, \infty)$**
Range: y -4 = **$[-4, \infty)$**
Minimum point: (2, -4)

(ii) $f(x) = x^2 - 5x + 6$

Step 1: Find vertex

- $f(x) = x^2 - 5x + 6 = (x - 5/2)^2 - 25/4 + 6$

- $f(x) = (x - 5/2)^2 - 25/4 + 24/4$
- $f(x) = (x - 5/2)^2 - 1/4$
- **Vertex: (5/2, -1/4)**

Step 2: Find x-intercepts

- $x^2 - 5x + 6 = 0$
- $(x - 2)(x - 3) = 0$
- $\mathbf{x = 2, x = 3}$

Step 3: Find y-intercept

- $f(0) = 6$

Step 4: Determine shape

- $a = 1 > 0$, opens upward (minimum)

x	0	1	2	2.5	3	4	5
f(x)	6	2	0	-0.25	0	2	6

Domain: $(-\infty, \infty)$
Range: $[-1/4, \infty)$
Minimum point: (5/2, -1/4)

(iii) $f(x) = -x^2 + 2x - 8$

Step 1: Find vertex

- $f(x) = -(x^2 - 2x) - 8$
- $f(x) = -(x^2 - 2x + 1) - 8 + 1$
- $f(x) = -(x - 1)^2 - 7$
- **Vertex: (1, -7)**

Step 2: Find x-intercepts

- $-x^2 + 2x - 8 = 0$
- $x^2 - 2x + 8 = 0$
- Discriminant: $(-2)^2 - 4(1)(8) = 4 - 32 = -28 < 0$
- **No x-intercepts** (parabola lies below x-axis)

Step 3: Find y-intercept

- $f(0) = -8$

Step 4: Determine shape

- $a = -1 < 0$, opens downward (maximum)

x	-2	-1	0	1	2	3	4
f(x)	-16	-11	-8	-7	-8	-11	-16

Domain: (-∞, ∞)
Range: (-∞, -7]
Maximum point: (1, -7)

(iv) $f(x) = x^2 - 4x + 4$

Step 1: Find vertex

- $f(x) = (x - 2)^2$
- **Vertex: (2, 0)**

Step 2: Find x-intercepts

- $(x - 2)^2 = 0$
- **$x = 2$** (double root)

Step 3: Find y-intercept

- $f(0) = 4$

Step 4: Determine shape

- $a = 1 > 0$, opens upward (minimum)

x	-1	0	1	2	3	4	5
f(x)	9	4	1	0	1	4	9

Domain: (-∞, ∞)
Range: [0, ∞)
Minimum point: (2, 0)

(v) $f(x) = x^2 + 2x - 8.3$

Step 1: Find vertex

- $f(x) = (x^2 + 2x + 1) - 8.3 - 1$
- $f(x) = (x + 1)^2 - 9.3$
- **Vertex: (-1, -9.3)**

Step 2: Find x-intercepts

- $x^2 + 2x - 8.3 = 0$
- $x = [-2 \pm \sqrt{(4 + 33.2)}]/2$
- $x = [-2 \pm \sqrt{37.2}]/2$
- $x = [-2 \pm 6.099]/2$
- **$x = 2.0495$, $x = -4.0495$**

Step 3: Find y-intercept

- $f(0) = -8.3$

Step 4: Determine shape

- $a = 1 > 0$, opens upward (minimum)

x	-4	-2	-1	0	1	2
f(x)	-4.3	-9.3	-9.3	-8.3	-5.3	-0.3

Domain: (-∞, ∞)
Range: [-9.3, ∞)
Minimum point: (-1, -9.3)

(vi) $f(x) = 6 - x - x^2$

Step 1: Find vertex

- $f(x) = -x^2 - x + 6$
- $f(x) = -(x^2 + x) + 6$
- $f(x) = -(x^2 + x + 1/4) + 6 + 1/4$
- $f(x) = -(x + 1/2)^2 + 25/4$
- **Vertex: (-1/2, 25/4)**

Step 2: Find x-intercepts

- $6 - x - x^2 = 0$
- $x^2 + x - 6 = 0$
- $(x + 3)(x - 2) = 0$
- **$x = -3$, $x = 2$**

Step 3: Find y-intercept

- $f(0) = 6$

Step 4: Determine shape

- $a = -1 < 0$, opens downward (maximum)

x	-4	-3	-2	-1	-0.5	0	1	2	3
f(x)	-6	0	4	6	6.25	6	4	0	-6

Domain: (-∞, ∞)
Range: (-∞, 25/4]
Maximum point: (-1/2, 25/4)

Summary Table for Question 2:

Part	Function	Vertex	Shape	Domain	Range
(i)	$x^2 - 4x$	(2, -4)	Min	(-∞, ∞)	[-4, ∞)
(ii)	$x^2 - 5x + 6$	(5/2, -1/4)	Min	(-∞, ∞)	[-1/4, ∞)
(iii)	$-x^2 + 2x - 8$	(1, -7)	Max	(-∞, ∞)	(-∞, -7]
(iv)	$x^2 - 4x + 4$	(2, 0)	Min	(-∞, ∞)	[0, ∞)
(v)	$x^2 + 2x - 8.3$	(-1, -9.3)	Min	(-∞, ∞)	[-9.3, ∞)
(vi)	$6 - x - x^2$	(-1/2, 25/4)	Max	(-∞, ∞)	(-∞, 25/4]

Question 3: Find the inverse of quadratic functions with domain restrictions

(i) $f(x) = x^2 - 3$, $x \quad 0$

Step 1: Set $y = f(x)$

- $y = x^2 - 3$, $x \quad 0$

Step 2: Solve for x in terms of y

- $y = x^2 - 3$
- $y + 3 = x^2$
- $x = \pm\sqrt{(y + 3)}$

Step 3: Choose correct branch based on domain

- Given $x \quad 0$, choose negative root
- **$x = -\sqrt{(y + 3)}$**

Step 4: Swap x and y to get inverse

- **$f^{1}(x) = -\sqrt{(x + 3)}$**

Step 5: Determine domain and range of f and f^{1}

- f domain: $x \quad 0 \to$ **$(-\infty, 0]$**
- f range: $y \quad -3 \to$ **$[-3, \infty)$**
- f^{1} domain: $x \quad -3 \to$ **$[-3, \infty)$**
- f^{1} range: $y \quad 0 \to$ **$(-\infty, 0]$**

Answer: $f^{1}(x) = -\sqrt{(x + 3)}$, Domain: $[-3, \infty)$, Range: $(-\infty, 0]$

(ii) $f(x) = x^2 + 6x + 4$, $x < -3$

Step 1: Set $y = f(x)$

- $y = x^2 + 6x + 4$, $x < -3$

Step 2: Complete the square

- $y = (x + 3)^2 - 9 + 4$
- $y = (x + 3)^2 - 5$
- $y + 5 = (x + 3)^2$

Step 3: Solve for x

- $x + 3 = \pm\sqrt{(y + 5)}$
- $x = -3 \pm \sqrt{(y + 5)}$

Step 4: Choose correct branch based on domain

- Given $x < -3$, choose negative root
- **$x = -3 - \sqrt{(y + 5)}$**

Step 5: Swap x and y

- **$f^{1}(x) = -3 - \sqrt{(x + 5)}$**

Step 6: Determine domain and range

- f domain: $x < -3 \to$ **$(-\infty, -3)$**
- f range: $y > -5 \to$ **$(-5, \infty)$** (since vertex (-3, -5) is minimum)

- f^{1} domain: $x > -5 \rightarrow$ **(-5, ∞)**
- f^{1} range: $y < -3 \rightarrow$ **(-∞, -3)**

Answer: $f^{1}(x) = -3 - \sqrt{(x + 5)}$, Domain: (-5, ∞), Range: (-∞, -3)

(iii) $f(x) = 2x^2 - 8x + 11$, x   2

Step 1: Set y = f(x)

- $y = 2x^2 - 8x + 11$, x   2

Step 2: Complete the square

- $y = 2(x^2 - 4x) + 11$
- $y = 2(x^2 - 4x + 4) + 11 - 8$
- $y = 2(x - 2)^2 + 3$
- $y - 3 = 2(x - 2)^2$
- $(y - 3)/2 = (x - 2)^2$

Step 3: Solve for x

- $x - 2 = \pm\sqrt{[(y - 3)/2]}$
- $x = 2 \pm \sqrt{[(y - 3)/2]}$

Step 4: Choose correct branch based on domain

- Given x   2, choose positive root
- **$x = 2 + \sqrt{[(y - 3)/2]}$**

Step 5: Swap x and y

- **$f^{1}(x) = 2 + \sqrt{[(x - 3)/2]}$**

Step 6: Determine domain and range

- f domain: x   2 → **[2, ∞)**
- f range: y   3 → **[3, ∞)** (vertex at (2, 3))
- f^{1} domain: x   3 → **[3, ∞)**
- f^{1} range: y   2 → **[2, ∞)**

Answer: $f^{1}(x) = 2 + \sqrt{[(x - 3)/2]}$, Domain: [3, ∞), Range: [2, ∞)

(iv) $f(x) = 3x^2 - 2x + 6$, x   5

Step 1: Set y = f(x)

- $y = 3x^2 - 2x + 6$, x   5

Step 2: Complete the square

- $y = 3(x^2 - (2/3)x) + 6$
- $y = 3(x^2 - (2/3)x + 1/9) + 6 - 1/3$
- $y = 3(x - 1/3)^2 + 17/3$

Step 3: Solve for x

- $y - 17/3 = 3(x - 1/3)^2$
- $(y - 17/3)/3 = (x - 1/3)^2$

- x - 1/3 = ±√[(y - 17/3)/3]

Step 4: Choose correct branch based on domain

- Given x 5, choose positive root
- **x = 1/3 + √[(y - 17/3)/3]**

Step 5: Swap x and y

- **f ^{1}(x) = 1/3 + √[(x - 17/3)/3]**

Step 6: Determine domain and range

- f domain: x 5 → **[5, ∞)**
- f range: y f(5) = 3(25) - 10 + 6 = 71 → **[71, ∞)**
- f ^{1} domain: x 71 → **[71, ∞)**
- f ^{1} range: y 5 → **[5, ∞)**

Answer: f ^{1}(x) = 1/3 + √[(x - 17/3)/3], Domain: [71, ∞), Range: [5, ∞)

(v) f(x) = 2(x - 3)^2 + 1, x 3

Step 1: Set y = f(x)

- y = 2(x - 3)^2 + 1, x 3

Step 2: Solve for x

- y - 1 = 2(x - 3)^2
- (y - 1)/2 = (x - 3)^2
- x - 3 = ±√[(y - 1)/2]

Step 3: Choose correct branch based on domain

- Given x 3, choose positive root
- **x = 3 + √[(y - 1)/2]**

Step 4: Swap x and y

- **f ^{1}(x) = 3 + √[(x - 1)/2]**

Step 5: Determine domain and range

- f domain: x 3 → **[3, ∞)**
- f range: y 1 → **[1, ∞)**
- f ^{1} domain: x 1 → **[1, ∞)**
- f ^{1} range: y 3 → **[3, ∞)**

Answer: f ^{1}(x) = 3 + √[(x - 1)/2], Domain: [1, ∞), Range: [3, ∞)

(vi) f(x) = -3(x + 4)^2 - 5, x < -4

Step 1: Set y = f(x)

- y = -3(x + 4)^2 - 5, x < -4

Step 2: Solve for x

- y + 5 = -3(x + 4)^2
- (y + 5)/(-3) = (x + 4)^2

- $-(y + 5)/3 = (x + 4)^2$

Step 3: Choose correct branch based on domain

- Given $x < -4$, $x + 4 < 0$, choose negative root
- $x + 4 = -\sqrt{[-(y + 5)/3]}$
- $\mathbf{x = -4 - \sqrt{[-(y + 5)/3]}}$

Step 4: Swap x and y

- $\mathbf{f^{1}(x) = -4 - \sqrt{[-(x + 5)/3]}}$

Step 5: Determine domain and range

- f domain: $x < -4 \rightarrow$ **$(-\infty, -4)$**
- f range: $y < -5 \rightarrow$ **$(-\infty, -5)$** (since vertex $(-4, -5)$ is maximum)
- f^{1} domain: $x < -5 \rightarrow$ **$(-\infty, -5)$**
- f^{1} range: $y < -4 \rightarrow$ **$(-\infty, -4)$**

Answer: $f^{1}(x) = -4 - \sqrt{[-(x + 5)/3]}$, Domain: $(-\infty, -5)$, Range: $(-\infty, -4)$

Summary Table for Question 3:

Part	Original Function	Domain	Range	Inverse Function	Domain of f^{1}	Range of f^{1}
(i)	$x^2 - 3$, x 0	$(-\infty, 0]$	$[-3, \infty)$	$-\sqrt{(x + 3)}$	$[-3, \infty)$	$(-\infty, 0]$
(ii)	$x^2 + 6x + 4$, $x < -3$	$(-\infty, -3)$	$(-5, \infty)$	$-3 - \sqrt{(x + 5)}$	$(-5, \infty)$	$(-\infty, -3)$
(iii)	$2x^2 - 8x + 11$, x 2	$[2, \infty)$	$[3, \infty)$	$2 + \sqrt{[(x - 3)/2]}$	$[3, \infty)$	$[2, \infty)$
(iv)	$3x^2 - 2x + 6$, x 5	$[5, \infty)$	$[71, \infty)$	$1/3 + \sqrt{[(x - 17/3)/3]}$	$[71, \infty)$	$[5, \infty)$
(v)	$2(x - 3)^2 + 1$, x 3	$[3, \infty)$	$[1, \infty)$	$3 + \sqrt{[(x - 1)/2]}$	$[1, \infty)$	$[3, \infty)$
(vi)	$-3(x + 4)^2 - 5$, $x < -4$	$(-\infty, -4)$	$(-\infty, -5)$	$-4 - \sqrt{[-(x + 5)/3]}$	$(-\infty, -5)$	$(-\infty, -4)$

Final Answer Key

Question 1:

- (i) Minimum = 4 at $x = -3$
- (ii) Minimum = -4 at $x = -2$
- (iii) Maximum = 29 at $x = 4$
- (iv) Maximum = $-11/4$ at $x = -3/2$
- (v) Minimum = -16 at $x = -1$
- (vi) Maximum = $169/8$ at $x = -1/4$

Question 2:

- (i) Min $(2, -4)$, Domain: $(-\infty, \infty)$, Range: $[-4, \infty)$
- (ii) Min $(5/2, -1/4)$, Domain: $(-\infty, \infty)$, Range: $[-1/4, \infty)$

- (iii) Max (1, -7), Domain: (-∞, ∞), Range: (-∞, -7]
- (iv) Min (2, 0), Domain: (-∞, ∞), Range: [0, ∞)
- (v) Min (-1, -9.3), Domain: (-∞, ∞), Range: [-9.3, ∞)
- (vi) Max (-1/2, 25/4), Domain: (-∞, ∞), Range: (-∞, 25/4]

Question 3:

- (i) $f^{1}(x)$ = -√(x + 3), Dom: [-3, ∞), Rng: (-∞, 0]
- (ii) $f^{1}(x)$ = -3 - √(x + 5), Dom: (-5, ∞), Rng: (-∞, -3)
- (iii) $f^{1}(x)$ = 2 + √[(x - 3)/2], Dom: [3, ∞), Rng: [2, ∞)
- (iv) $f^{1}(x)$ = 1/3 + √[(x - 17/3)/3], Dom: [71, ∞), Rng: [5, ∞)
- (v) $f^{1}(x)$ = 3 + √[(x - 1)/2], Dom: [1, ∞), Rng: [3, ∞)
- (vi) $f^{1}(x)$ = -4 - √[-(x + 5)/3], Dom: (-∞, -5), Rng: (-∞, -4)

EXERCISE 3.1 - COMPLETE SOLUTIONS (PART 2)

Question 4: Solve the following absolute value quadratic equations and inequalities

Key Principle for Absolute Value:

For any expression A:

- **Equation:** $|A| = k$ (where $k \geq 0$) means $A = k$ or $A = -k$
- **Inequality:** $|A| < k$ means $-k < A < k$
- **Inequality:** $|A| > k$ means $A < -k$ or $A > k$
- **Inequality:** $|A| \leq k$ means $-k \leq A \leq k$

(i) $|x^2+1| = 5$

Step 1: Apply absolute value property

- $x^2 + 1 = 5$ or $x^2 + 1 = -5$

Step 2: Solve first equation

- $x^2 + 1 = 5$
- $x^2 = 4$
- $x = \pm 2$

Step 3: Solve second equation

- $x^2 + 1 = -5$
- $x^2 = -6$
- No real solution (since $x^2 \geq 0$)

Answer: $x = -2, 2$

(ii) $|x^2+5x+4| = 0$

Step 1: Apply absolute value property

- Since $|A| = 0$ implies $A = 0$
- $x^2 + 5x + 4 = 0$

Step 2: Factor and solve

- $(x+1)(x+4)=0$
- $x+1=0$ or $x+4=0$
- $x=-1$ or $x=-4$

Answer: $x=-4,-1$

(iii) $\mathbf{|x^2 - 6x + 8| = 4}$

Step 1: Apply absolute value property

- $x^2-6x+8=4$ or $x^2-6x+8=-4$

Step 2: Solve first equation

- $x^2-6x+8=4$
- $x^2-6x+4=0$
- Using quadratic formula: $x=[6\pm\sqrt{36-16}]/2=[6\pm\sqrt{20}]/2=[6\pm 2\sqrt{5}]/2$
- $x=3\pm\sqrt{5}$

Step 3: Solve second equation

- $x^2-6x+8=-4$
- $x^2-6x+12=0$
- Discriminant: $36-48=-12<0$
- **No real solution**

Answer: $x=3-\sqrt{5},3+\sqrt{5}$

(iv) $\mathbf{|3x^2 - 7x + 2| = x^2 - x + 1}$

Step 1: Note that RHS must be non-negative

- $x^2-x+1=(x-1/2)^2+3/4>0$ for all real x

Step 2: Apply absolute value property

- $3x^2-7x+2=x^2-x+1$ ... (1)
- OR $3x^2-7x+2=-(x^2-x+1)$ ... (2)

Step 3: Solve equation (1)

- $3x^2-7x+2=x^2-x+1$
- $2x^2-6x+1=0$
- $x=[6\pm\sqrt{36-8}]/4=[6\pm\sqrt{28}]/4=[6\pm 2\sqrt{7}]/4$
- $x=(3\pm\sqrt{7})/2$

Step 4: Solve equation (2)

- $3x^2-7x+2=-x^2+x-1$
- $4x^2-8x+3=0$
- $x=[8\pm\sqrt{64-48}]/8=[8\pm\sqrt{16}]/8=[8\pm 4]/8$
- $x=1/2$ **or** $x=3/2$

Step 5: Verify solutions in original equation

Check $x = 3/2$:

- LHS: $|\,3(9/4) - 7(3/2) + 2\,| = |\,27/4 - 21/2 + 2\,| = |\,27/4 - 42/4 + 8/4\,| = |\,-7/4\,| = 7/4$
- RHS: $9/4 - 3/2 + 1 = 9/4 - 6/4 + 4/4 = 7/4$

Check $x = 1/2$:

- LHS: $|\,3(1/4) - 7(1/2) + 2\,| = |\,3/4 - 7/2 + 2\,| = |\,3/4 - 14/4 + 8/4\,| = |\,-3/4\,| = 3/4$
- RHS: $1/4 - 1/2 + 1 = 1/4 - 2/4 + 4/4 = 3/4$

Answer: $x = 1/2, 3/2, (3 \pm \sqrt{7})/2$

(v) $|\mathbf{x^2 - 4}| < \mathbf{5}$

Step 1: Apply absolute value inequality property

- $-5 < x^2 - 4 < 5$

Step 2: Solve the compound inequality

- Add 4 to all parts: $-5 + 4 < x^2 < 5 + 4$
- $-1 < x^2 < 9$

Step 3: Solve the inequality

- Since $x^2 \geq 0$, the lower bound $-1 < x^2$ is always true
- So we need: $x^2 < 9$
- $-3 < x < 3$

Answer: $x \in (-3, 3)$

(vi) $|\mathbf{x^2 - 3x + 2}| > \mathbf{4}$

Step 1: Apply absolute value inequality property

- $x^2 - 3x + 2 < -4$ or $x^2 - 3x + 2 > 4$

Step 2: Solve first inequality

- $x^2 - 3x + 2 < -4$
- $x^2 - 3x + 6 < 0$
- Discriminant: $9 - 24 = -15 < 0$
- Since a > 0 and discriminant < 0, $x^2 - 3x + 6 > 0$ for all x
- **No solution**

Step 3: Solve second inequality

- $x^2 - 3x + 2 > 4$
- $x^2 - 3x - 2 > 0$
- Find roots: $x = [3 \pm \sqrt{9 + 8}]/2 = [3 \pm \sqrt{17}]/2$

Step 4: Determine solution interval

- Since a > 0 (parabola opens upward), inequality is true outside the roots
- $x < (3 - \sqrt{17})/2$ or $x > (3 + \sqrt{17})/2$

Answer: $x \in (-\infty, (3-\sqrt{17})/2) \cup ((3+\sqrt{17})/2, \infty)$

(vii) $|\mathbf{x^2}-\mathbf{5x}+\mathbf{6}| \leq \mathbf{x}+\mathbf{2}$

Step 1: Note the domain condition

- Since absolute value is always non-negative, we need $x+2 \geq 0$
- $x \geq -2$

Step 2: Apply absolute value inequality property

- $-(x+2) \leq x^2 - 5x + 6 \leq x + 2$

Step 3: Solve left inequality

- $-(x+2) \leq x^2 - 5x + 6$
- $-x - 2 \leq x^2 - 5x + 6$
- $0 \leq x^2 - 4x + 8$
- $x^2 - 4x + 8 \geq 0$
- Discriminant: $16 - 32 = -16 < 0$
- Since a $>$ 0 and discriminant $<$ 0, this is true for all x

Step 4: Solve right inequality

- $x^2 - 5x + 6 \leq x + 2$
- $x^2 - 6x + 4 \leq 0$
- Find roots: $x = [6 \pm \sqrt{36-16}]/2 = [6 \pm \sqrt{20}]/2 = 3 \pm \sqrt{5}$
- Since a $>$ 0, inequality is true between the roots
- $3 - \sqrt{5} \leq x \leq 3 + \sqrt{5}$

Step 5: Apply domain condition $x \geq -2$

- $3 - \sqrt{5} \approx 0.764 > -2$, so domain condition is automatically satisfied

Answer: $x \in [3-\sqrt{5}, 3+\sqrt{5}]$

(viii) $|\mathbf{2x^2}-\mathbf{3x}-\mathbf{5}| < \mathbf{4}$

Step 1: Apply absolute value inequality property

- $-4 < 2x^2 - 3x - 5 < 4$

Step 2: Solve left inequality

- $2x^2 - 3x - 5 > -4$
- $2x^2 - 3x - 1 > 0$
- Find roots: $x = [3 \pm \sqrt{9+8}]/4 = [3 \pm \sqrt{17}]/4$
- Since a $>$ 0, inequality is true outside the roots
- $x < (3-\sqrt{17})/4$ or $x > (3+\sqrt{17})/4$

Step 3: Solve right inequality

- $2x^2 - 3x - 5 < 4$
- $2x^2 - 3x - 9 < 0$

- Find roots: $x = [3 \pm \sqrt{9+72}]/4 = [3 \pm \sqrt{81}]/4 = [3 \pm 9]/4$
- $x = -3/2$ or $x = 3$
- Since a > 0, inequality is true between the roots
- $-3/2 < x < 3$

Step 4: Find intersection of both solutions

Let $a = (3-\sqrt{17})/4 \approx -0.281$ and $b = (3+\sqrt{17})/4 \approx 1.781$

Interval	Left inequality	Right inequality	Both
$x < -3/2$			
$-3/2 < x < a$			
$a < x < b$			
$b < x < 3$			
$x > 3$			

Answer: $x \in ((3-\sqrt{17})/4, (3+\sqrt{17})/4) \cup ((3+\sqrt{17})/4, 3)$

Summary Table for Question 4:

Part	Equation/Inequality	Solution		
(i)	(	x^2 + 1	= 5)	$x = \pm 2$
(ii)	(	x^2 + 5x + 4	= 0)	$x = -4, -1$
(iii)	(	x^2 - 6x + 8	= 4)	$x = 3 \pm \sqrt{5}$
(iv)	$\| 3x^2 - 7x + 2 \| = x^2 - x + 1$	$x = 1/2, 3/2, (3 \pm \sqrt{7})/2$		
(v)	(	x^2 - 4	< 5)	$x \in (-3, 3)$
(vi)	(	x^2 - 3x + 2	> 4)	$x \in (-\infty, (3-\sqrt{17})/2) \cup ((3+\sqrt{17})/2, \infty)$
(vii)	(	x^2 - 5x + 6	\leq x + 2)	$x \in [3-\sqrt{5}, 3+\sqrt{5}]$
(viii)	(	2x^2 - 3x - 5	< 4)	$x \in ((3-\sqrt{17})/4, (3+\sqrt{17})/4) \cup ((3+\sqrt{17})/4, 3)$

Final Answer Key

Question 4 Solutions:

1. **(i)** $x = -2, 2$
2. **(ii)** $x = -4, -1$
3. **(iii)** $x = 3 \pm \sqrt{5}$
4. **(iv)** $x = 1/2, 3/2, (3 \pm \sqrt{7})/2$
5. **(v)** $(-3, 3)$
6. **(vi)** $(-\infty, (3-\sqrt{17})/2) \cup ((3+\sqrt{17})/2, \infty)$
7. **(vii)** $[3-\sqrt{5}, 3+\sqrt{5}]$
8. **(viii)** $((3-\sqrt{17})/4, (3+\sqrt{17})/4) \cup ((3+\sqrt{17})/4, 3)$